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Karl H. Guenther
Chairman / Editor

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Karl H. Guenther
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OPHTHALMIC OPTICS

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OPHTHALMIC OPTICS

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INTRODUCTION

This is the first time that Ophthalmic Optics have been handled within the framework of an SPIE/ANRT technical symposium. This is somehow surprising in view of the long history of these symposia and the fact that the optics of the eye, sight, and related ancillary devices (e.g., glasses) supposedly play an important role in any visual application of an optical or electro-optical instrument.

Session 1, Ophthalmic Lens Design, was dominated by papers on the development and application of aspheric lens-making techniques in order to reduce distortions and provide glasses with continuously varying focal length (progressive lenses).

In Session 2, Optical and Protective Coatings for Ophthalmic Lenses, coatings for Ophthalmic lenses were reviewed in two papers with respect to the state of the art in coating technology and with respect to protective eye-wear for laser radiation. The abrasion and scratch resistance of coatings for plastic lenses and related test methods were dealt with in four original papers, and further discussed with a panel consisting of the authors of these papers. In this discussion, the limited significance of almost any abrasion or scratch test was emphasized. It was concluded that a universally applicable test method does not yet exist and will likely never exist, which makes standardization of these coating properties extremely difficult if not impossible. In assessing coating durability relative to abrasion and scratching, the nature of the particular test employed and its relevance to actual wear, as well as the nature of the coating failures involved, should be noted carefully.

Some interesting new approaches for the remote and quick evaluation of the human eye highlighted Session 3, Instrumentation for Refraction and Optometry. It may well be that a number of the instrumental solutions presented will come into common use for both clinical and field surveys in the near future.

The presentation and actual working demonstration of a bifocal, electrically switched intraocular and eyeglass molecular lens was the spectacular opening of Session 4, New Developments in Spectacle Lenses, which also included solar ultraviolet protection and the biocompatibility of a new polymer for contact lenses.

In summary, this conference provided an excellent overview of the state of the art in some important (by far not all) fields of Ophthalmic Optics, and their potential prospects for the future.

It is hoped that the quality of the papers published in these proceedings will inspire further interdisciplinary activities and the mutual understanding of the various fields in Ophthalmic Optics. It is desired also that optical engineers active in the visible will take the opportunity to refresh their knowledge in Ophthalmic Optics. It is also hoped that these Proceedings will encourage many more people to attend the next conference of this kind in person.

Karl H. Guenther
Balzers AG, Liechtenstein

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OPHTHALMIC OPTICS

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Session 1

Ophthalmic Lens Design

Chairmen

William Lenne

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Third order distortion and spectacle lens design

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Abstract

Third order (primary) aberration theory has had little application to the design of aspheric spectacle lenses. Such an application would be useful because:

1. Third order theory is useful in designing simple optical systems, as relatively simple equations can be used to obtain approximate magnitudes of aberrations and to show how these aberrations change with variations in design parameters without recourse to a large mass of data.
2. Aspherising one or both surfaces of spectacle lenses allows the correction of off-axis power errors in high positive power lenses where this would be otherwise impossible, and enables other factors, such as distortion correction, to be considered simultaneously with off-axis power error correction over the total range of lens powers.

Third order formulae are developed for calculation of distortion in thin spectacle lenses, when one or both surfaces are conicoid aspherics. Results are presented which show the validity of using third order theory. Solutions which allow correction of rotatory or peripheral distortion, when one lens surface is a conicoid aspheric, are illustrated. A study of these solutions shows that one of the off-axis power errors (eg. oblique astigmatism) and one of the distortions can be simultaneously eliminated, but the lens forms required are too curved to be cosmetically feasible.

Introduction

In ophthalmic optics, approximations are used to simplify the relationships between quantities. The simplest approximation made is that the sine of an angle can be replaced by the angle itself i.e.

$$\sin A = A \quad (1)$$

The actual relationship between an angle and its sine is given by the series

$$\sin A = A - A^3/3! + A^5/5! - A^7/7! + \dots \quad (2)$$

When the angle A is very small, it is justifiable to neglect the second and successive terms. This approximation is the basis of first order (gaussian) theory from which are calculated quantities such as lens power and lens prismatic effects. If all rays of light in an optical system behaved according to first order theory, an optical system would be perfect - an image would be an exact copy of its object, except for size and luminance changes. The failure of rays to behave in a "perfect" fashion results in aberrations of the image.

As angles increase in an optical system, because objects further from the optical axis are considered or because the aperture size of the system is increasing, the first order approximation is eventually no longer adequate for raytracing purposes, and better results are obtained using the approximation

$$\sin A = A - A^3/3! \quad (3)$$

Theory derived using this approximation is referred to as third order (also primary and Seidel) aberration theory. Such theory can predict the change in clarity and position of images. Raytracing and aberration calculations are more accurate if the first three terms of the expansion are used, but the algebra required becomes much more complicated.

Third order theory was very useful for early lens designers. Extensive use of finite raytracing is now feasible with modern computing facilities, but third order theory is still useful because relatively simple equations can be used to obtain approximate magnitudes of aberrations and to show how these aberrations change with variations in design parameters without recourse to a large mass of data. Third order theory may be sufficient for the design of systems of small aperture and field size. It is commonly believed that third order theory is not sufficiently accurate for the design of spectacle lenses because of the higher order aberration contribution. However, for small object angles of up to 10-20 degrees from the optical axis it is possible that third order theory is sufficient. In any case, the application of third order theory can show the trends in bending and aspherising needed to reduce certain aberrations.

- The major aberrations which are of concern to spectacle lens designers are:
1. The aberrations affecting image quality when the eye rotates to look at objects that are not on the lens optical axis - oblique astigmatism and mean oblique error. They are sometimes referred to as the off-axis power errors.

2. Distortion

- a) Peripheral (ordinary or stationary)

This is a measure of the displacement of the image, of an off-axis point object, from its ideal position. It is referred to the eye looking through the centre of the lens, and viewing a point object in the peripheral visual field. In raytracing, the stop of the system is the eye's entrance pupil. Peripheral distortion affects spatial perception by causing the image of an extended object to be altered in shape.

- b) Rotatory

This is a measure of the failure of eye rotation to match the rotation predicted by gaussian theory, when an eye turns to look at an object not on the lens optical axis. In raytracing, the stop of the system is the eye's centre of rotation. Rotatory distortion affects spatial perception by causing apparent movement of objects.

This classification of distortion is a simplification of the distortions which exist when looking through a spectacle lens e.g. objects in the periphery may look out of shape, and this shape changes according to the degree of eye rotation.

Smith and Atchison¹ and Atchison² derived third-order solutions for the elimination of oblique astigmatism and mean oblique error. They showed that for thin spherical lenses, the aberrations are a quadratic function of shape (bending). Hence, there are two bending solutions within a limited range of lens powers (approximately -22D to +7D), and none outside this range. When one surface is made aspheric as a conicoid, the aberrations become cubic functions of bending. Thus, there is always one or three bending solutions.

There has been little investigation of third order distortion in spectacle lenses. Distortion is a considerable problem in high power lenses, particularly high power positive lenses used for the correction of aphakia. With the advent of cheap, castable plastics for ophthalmic lenses, it has become feasible to consider aspherising as a variable. As well as using asphericity to correct off-axis power errors in high positive power lenses where this would be otherwise impossible, distortion correction can now be considered simultaneously with the reduction of the off-axis power errors.

In this paper, third order formulae will be developed for the calculation of distortion in thin spectacle lenses. These formulae will be used to present solutions for the elimination of the distortions when one surface is aspherised as a conicoid aspheric. The validity of the third order solutions will be assessed by finite raytracing. The feasibility of simultaneously correcting the distortions and the off-axis power errors, with one surface as a conicoid aspheric, will be investigated.

Formulae

The third order aberrations of a thin lens in air, with spherical surfaces, are quadratic functions of the bending X , i.e.

$$S_i = A_i + B_i X + C_i X^2 \quad (4)$$

X is given by

$$X = 1 - 2F_2/F \quad (5)$$

where F_2 and F are the back surface power, and equivalent power, respectively. Using the third order aberration formulae of Hopkins³, the co-efficients of distortion S_5 are

$$A_5 = \frac{E^3 h^4 F^3}{4} \left\{ \frac{n^2}{(n-1)^2} + \frac{(3n+2)}{n} Y^2 \right\} - \frac{3E^2 H h^2 F^2 (2n+1)}{2n} Y + E(H^2 F/n + 3H^2 F) \quad (6a)$$

$$B_5 = - \frac{E^3 h^4 F^3 (n+1)}{n(n-1)} Y + \frac{3E^2 H h^2 F^2 (n+1)}{2n(n-1)} \quad (6b)$$

$$C_5 = \frac{E^3 h^4 F^3 (n+2)}{4n(n-1)^2} \quad (6c)$$

For these co-efficients, h is the paraxial marginal ray height at the lens, \bar{h} is the paraxial pupil ray height at the lens, E is defined by \bar{h}/h , n is the lens refractive index, H is the optical invariant, and Y is the conjugate factor, given by

$$(1 + 1')/(1' - 1) \quad (7)$$

where l and l' are object and image distances, respectively.

If a spherical surface is made aspheric as a conicoid of the form

$$c(x^2 + y^2 + (1 + Q)z^2) - 2z = 0 \quad (8)$$

where c is the vertex curvature and z is measured from the vertex along the optical axis, the third order aberrations change by an amount depending upon the asphericity factor Q . S_5 changes by

$$S_5 = E^3 \alpha \quad (9)$$

where, from Hopkins³

$$\alpha = c^3 Q h^4 (n_1 - n_0) \quad (10)$$

Here, n_0 and n_1 are the refractive indices on the object and image sides of the surface, respectively. S_5 is now a cubic function of the shape factor X , i.e.

$$S_5 = A_5 + B_5 X + C_5 X^2 + D_5 X^3 \quad (11)$$

where the changes to the co-efficients of equation (6) are

$$\Delta A_5 = E^3 G, \Delta B_5 = 3E^3 G, \Delta C_5 = 3E^3 G, \Delta D_5 = D_5 = E^3 G \quad (12a)$$

for front surface aspherising,

$$\Delta A_5 = E^3 G, \Delta B_5 = -3E^3 G, \Delta C_5 = 3E^3 G, \Delta D_5 = D_5 = -E^3 G \quad (12b)$$

for back surface aspherising,
and

$$G = F^3 Q h^4 / (8(n - 1)^2) \quad (13)$$

Results

Solutions for the elimination of third order rotatory distortion in thin spectacle lenses are shown in Figure 1. Back surface power is plotted as a function of equivalent power for distance vision and front surface asphericity. The stop is 27mm behind the lens. Third order distortion is quadratic in shape factor for spherical lenses, but there are no solutions for typical parameters. The dotted line gives the spherical forms with minimum distortion - very high back surface powers are required. When one surface is aspherised, there is now one bending solution for any power.

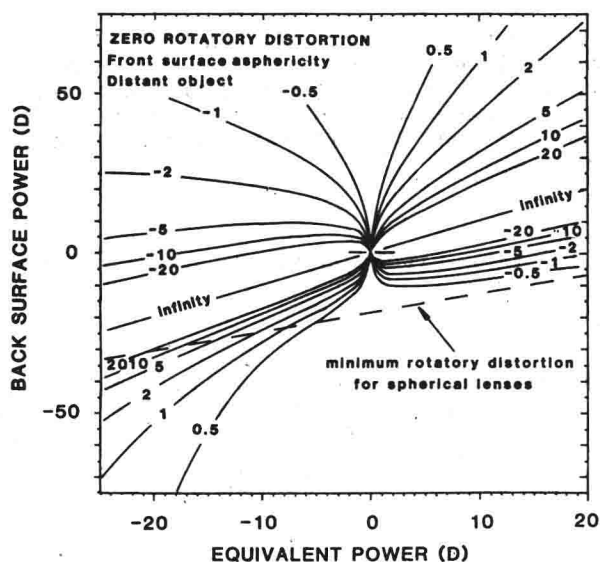


Figure 1. Solutions for third order rotatory distortion of thin lenses for distance vision and front surface asphericity. Q values are marked.

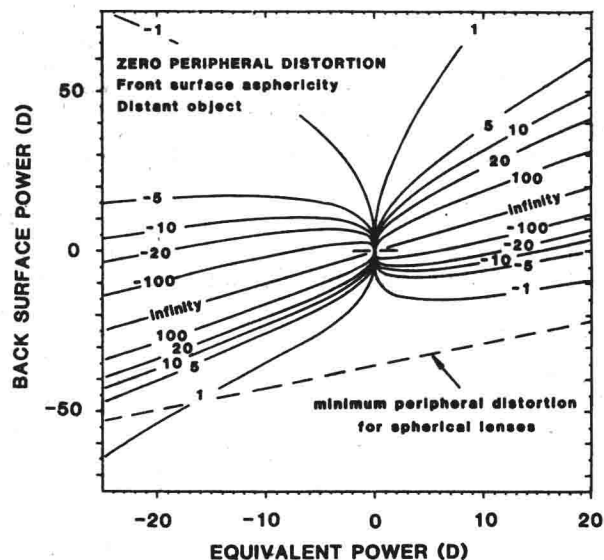


Figure 2. Solutions for third order peripheral distortion of thin lenses for distance vision and front surface asphericity. Q values are marked.

Solutions for the elimination of third order peripheral distortion in thin spectacle lenses are shown in Figures 2 and 3. Figure 2 shows solutions for front surface asphericity and Figure 3 shows solutions for back surface asphericity. The stop is 15mm behind the lens. Again, there are no solutions for spherical lenses. The minimum peripheral distortion forms for spherical lenses have much steeper curves than those required for minimum rotatory distortion forms. Also, the bending at any front surface

asphericity Q is much greater for elimination of peripheral distortion than for elimination of rotatory distortion (compare Figures 1 and 2).

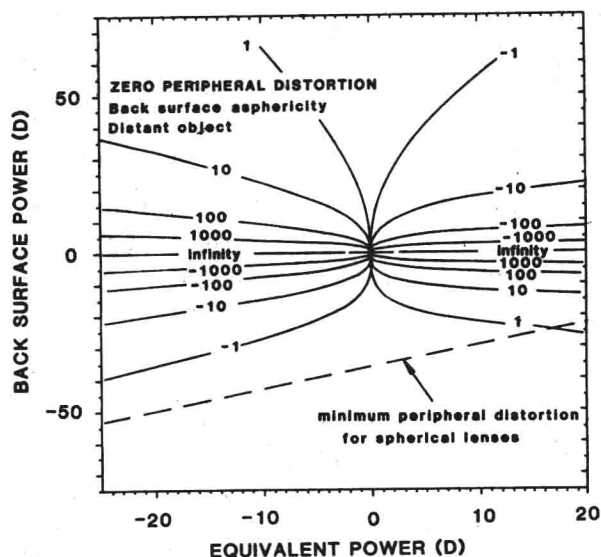


Figure 3. Solutions for third order peripheral distortion of thin lenses for distance vision and back surface asphericity. Q values are marked.

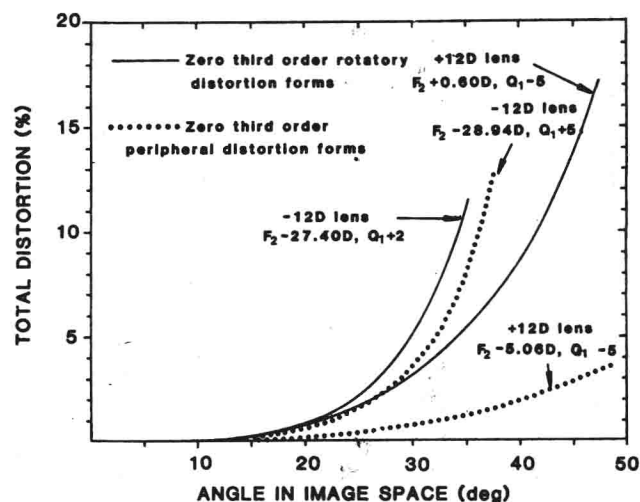


Figure 4. Total distortion of thin lenses with front surfaces aspherised for zero third order peripheral distortion or zero third order rotatory distortion.

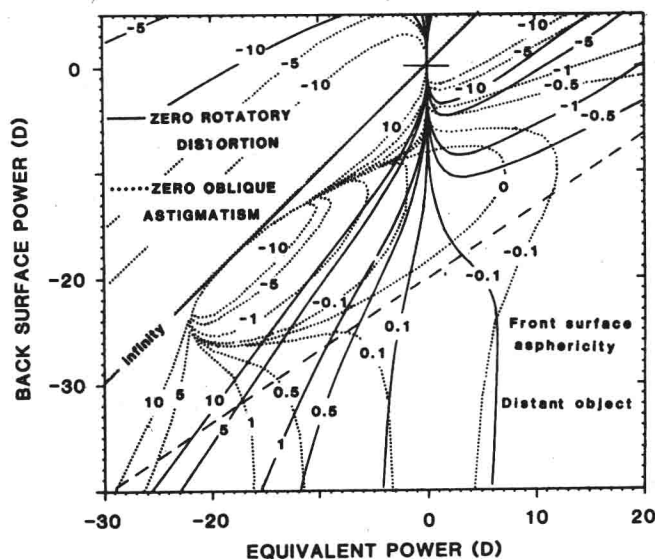


Figure 5. Solutions for third order oblique astigmatism and rotatory distortion for distance vision and front surface asphericity. Q values are marked.

The third order theory will only accurately quantify aberrations for small field angles. Figure 4 shows the level of total distortion for thin lenses with zero third order distortion plotted as a function of image angle. Two +12D lens forms with front surface asphericity, one of which corrects third order peripheral distortion ($F_2 = -5.06D$, $Q_1 = -5$) and the other which corrects third order rotatory distortion ($F_2 = +0.60D$, $Q_1 = -5$), have little total distortion up to 30° image angle. A measure of the error can be obtained by comparing the total distortion of these forms with the total distortion of the unaspherised forms. This gives proportional errors of only 20% and 10% at 30° image angle for the third order peripheral distortion-free and third order rotatory distortion-free forms, respectively. Two -12D lens forms with front surface asphericity

are not as well corrected for total distortion. At 20° object angle, a third order peripheral distortion-free form ($F_2 = -28.94D$, $Q_1 = 5$) has a proportional error of 29%, and a third order rotatory distortion-free form ($F_2 = -27.40D$, $Q_1 = 2$) has a proportional error of 34%.

Figure 5 compares the third order rotatory distortion solutions with third order oblique astigmatism solutions. Solutions for mean oblique error are similar to those for oblique astigmatism. For a given level of asphericity, in general the bendings required for elimination of rotatory distortion are much greater than those required for the elimination of oblique astigmatism. It is possible to eliminate them simultaneously, but the lens forms required to do this are very curved. For example, it can be seen from the figure that for a -12D power, the solution is approximately $F_2 = -40D$, $Q = +0.5$. Also, for a +6D power, the solution is approximately $F_2 = -24D$, $Q_1 = -0.1$. The solution for a +12D power (not shown) is approximately $F_2 = -14D$, $Q = -0.16$. These solutions are not feasible from a cosmetic viewpoint. The bendings required for elimination of peripheral distortion (not shown) are greater than the rotatory distortion solutions. Again, peripheral distortion and oblique astigmatism can be corrected concurrently, but the lens forms would be even more curved than the above solutions.

Figures 6 to 9 give some quantification of the other aberration when either distortion or oblique astigmatism is corrected in high power lenses. Figure 6 shows the total oblique astigmatism present when third order distortion is corrected for the +12D lenses shown in Figure 4. These forms have been chosen as they have reasonable cosmetic forms (fairly flat back surfaces). The peripheral distortion-free form produces extremely high oblique astigmatism (-3.3D at 15° image angle). This would be unacceptable to patients. When the front surface asphericity is removed, this lens has small oblique astigmatism (+0.8D at 30° image angle). The rotatory distortion-free form has reasonably high oblique astigmatism (-2.0D at 30° image angle). When the front surface asphericity is removed, the lens has similar oblique astigmatism but of the opposite sign, suggesting that the ideal asphericity for correction of oblique astigmatism for this form will partially correct the rotatory distortion.

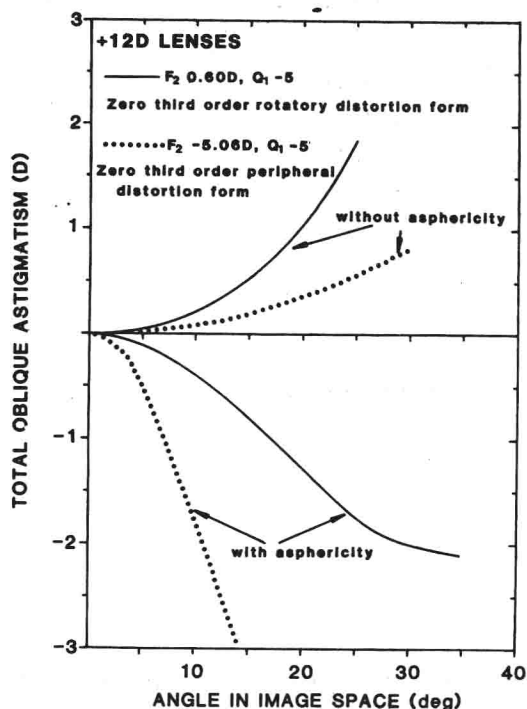


Figure 6. Total oblique astigmatism of thin +12D lenses with and without front surfaces aspherised for zero third order peripheral distortion ($F_2 = -5.06D$, $Q_1 = -5$) or zero third order rotatory distortion ($F_2 = +0.60D$, $Q_1 = -5$).

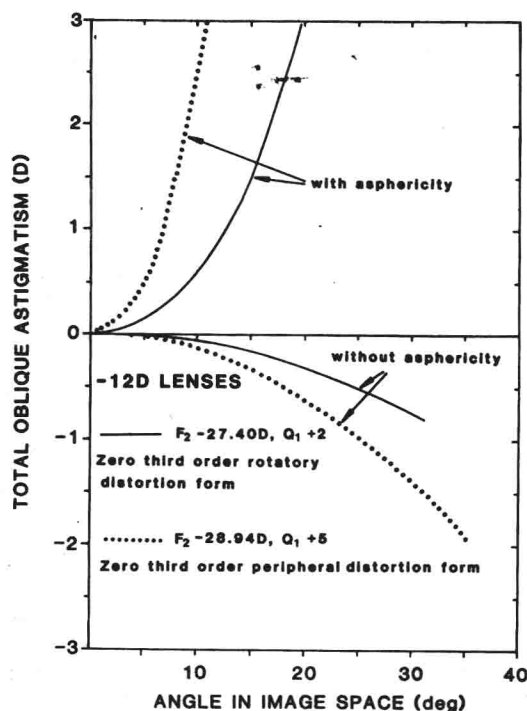


Figure 7. Total oblique astigmatism of thin -12D lenses with and without front surfaces aspherised for zero third order peripheral distortion ($F_2 = -28.94D$, $Q_1 = +5$) or zero third order rotatory distortion ($F_2 = -27.40D$, $Q_1 = +2$).

Figure 7 shows the total oblique astigmatism present when third order distortion is corrected for the -12D lenses shown in figure 4. Again, the peripheral distortion-free form produces unacceptable oblique astigmatism (+2.5D at 10° image angle). When the front

surface asphericity is removed, it is reasonably corrected for oblique astigmatism ($-1.4D$ at 30° image angle). The rotatory distortion-free form has high, probably unacceptable oblique astigmatism ($+3.2D$ at 20° image angle). This becomes quite small when the asphericity is removed ($-0.7D$ at 30° image angle).

Figure 8 shows the total distortion present when third order oblique astigmatism is corrected for a thin $+12D$ lens with front surface asphericity ($F_2 = -2.38D$, $Q_1 = -0.5$). This lens has been chosen as it has a reasonable cosmetic form. The levels of both rotatory and peripheral distortion are quite high. If the negative asphericity is removed, the rotatory distortion is increased by about 25% but there is little change in peripheral distortion. Other examples would also show that the negative asphericity on the front surface of high positive power lenses required to eliminate oblique astigmatism will result in substantially reduced rotatory distortion, but not peripheral distortion.

Figure 9 shows the total distortion present when third order oblique astigmatism is corrected for a spherical $-12D$ lens ($F_2 = -25.06D$). It is the more bent of the two spherical lens forms which will correct oblique astigmatism. Both rotatory and peripheral distortion are high. This lens has a similar bending, but not the asphericity, of the $-12D$ lenses examples which corrected the distortions. The bending of these lenses is probably cosmetically unacceptable.

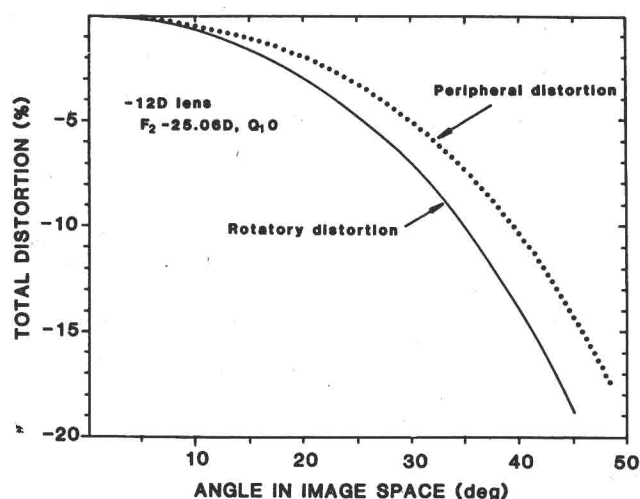
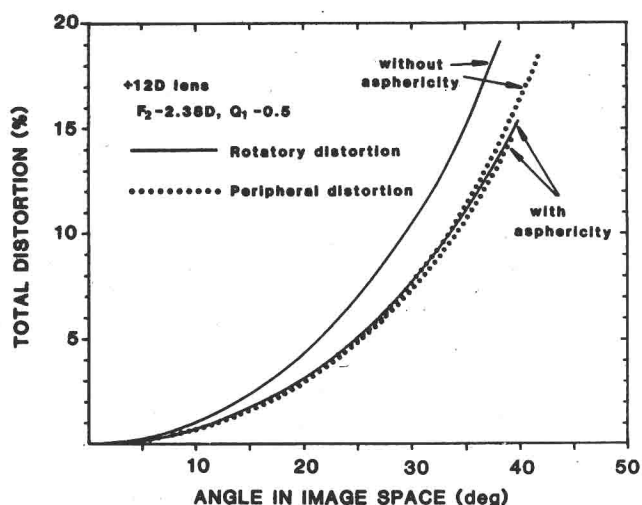


Figure 8. Total distortions of a thin $+12D$ lens with and without the front surface aspherised for zero third order oblique astigmatism ($F_2 = -2.38D$, $Q_1 = -0.5$).

Figure 9. Total distortions of a thin spherical $-12D$ lens corrected for zero third order oblique astigmatism ($F_2 = -25.06D$).

Discussion

Third order theory is useful in quickly obtaining approximate magnitudes of aberrations and in showing how the solutions change with parameters such as lens power and asphericity. When one surface is aspherised as a conicoid aspheric, it is possible to eliminate third order off-axis power errors (such as oblique astigmatism), third order rotatory distortion or third order peripheral distortion. The oblique astigmatism and one of the distortion types may be eliminated simultaneously, but the lens forms required to do this are very curved. For a given level of asphericity, in general the bendings required for elimination of the distortions are much greater than those required for the elimination of the off-axis power errors. The elimination of the distortions in high power lenses with cosmetically acceptable forms tends to produce large off-axis power errors which will probably be unacceptable to patients, and this is best not attempted. However, at least for high positive power lenses, the asphericities required to eliminate the off-axis power errors will result in substantially reduced rotatory distortion (but not peripheral distortion).

The third order solutions for the distortions are valid to 20° - 30° image angles. For assessment at large angles, finite raytracing with thick lenses is essential. One possible approach to lens design of high power lenses is to correct off-axis power errors out to some suitable angle of rotation e.g. 30° , and beyond this to use aspherising with figuring co-efficients in the lens equation to achieve other goals such as reduction of distortion and lens thickness. This approach has been adopted in the design of high

positive blended lenticular lenses such as the American Optical Fulvue lens⁴.

For an in-depth analysis of optimising lens forms, one must also use thick lenses and finite raytracing. Katz⁵ used sophisticated optimising procedures together with figured conicoids on both surfaces to obtain lens forms optimised for oblique astigmatism, mean oblique error, and rotatory distortion together. He was able to achieve considerable success for high negative lens powers, but could not do this for high positive lens powers. The optimised negative lenses were very curved and very probably cosmetically unacceptable.

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Ophthalmic lens design with splines

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Abstract

Spline surfaces can be formed at will. They are always twice continuously differentiable, i. e. they have no jumps, bends or ripples. In the field of optics splines are employed for the spectacle lens, Gradal HS. The form of the spline surface is defined by optical requirements. Splines have contributed to the successful reduction to a minimum of the unavoidable aberrations present in progressive addition lenses and to the binocular equilibrium of the remaining aberrations. Conventional surface structures will be used as a comparison.

Introduction

When we are over the tender age of forty our sight begins to deteriorate; our eyes become less and less able to focus at close range and we need a pair of spectacles for reading. Single vision reading glasses only enable good near vision, however; they have to be taken off for distance vision. Bifocals allow good vision in a near portion and a distance portion. The most elegant visual aid for presbyopia is, however, the progressive addition lens: here, lenses are used which not only permit clear vision for near and distance, but also for all intermediate ranges. This progressive transition of good vision is made possible by a progressive addition surface on the object side of the lens.

Any special form given to the progressive addition surface with a given, e. g. spherical, back surface clearly defines the optical properties of the entire progressive lens. It is therefore the aim of optical construction to create a progressive surface which embodies all those optical properties recognized as being desirable.

The progressive surface

The progress made in optical construction using spline surfaces can only be assessed against a background of conventional surface designs.

Conventional surface constructions

In progressive lenses there is a continuous transition of the lens power from the stable distance power in the upper area of the lens to the stable near power in the lower area. This increase in power results from an increase in curvature in the progressive surface; there is a constant decrease in the size of the radii of curvature between the distance radius and the near radius.

An obvious construction principle for progression surfaces, illustrated in Figure 1, is therefore:

- the calculation of the surface form from the desired progression of radii in a vertical plane section. The line of intersection is known as the progression meridian.
- the lateral extension of this progression meridian by suitably formed sections.



Figure 1 Construction of a progressive addition surface consisting of progression meridians and sections

The calculation of the progression meridian

For the desired progression of radii $r(x)$ along the progression meridian the function $f(x)$ of the progression meridian must itself be calculated. Function f and the radius of

curvature r are linked by a simple formula¹. After setting $f(0)=f'(0)=0$ as the initial condition, a solution is sought for the initial value problem:

$$\frac{f'''}{(1+f'^2)^{3/2}} = \frac{1}{r}; \quad f(0)=f'(0)=0$$

Simple integration² will provide the result:

$$f(x) = \int_0^x Q(x) \cdot (1-Q^2(x))^{-1/2} dx \quad \text{with} \quad Q(x) = \int_0^x \frac{1}{r(x)} dx$$

The lateral extension of the progression meridian

Circular sections. An American patent specification contains the following sentence:³ For the central portion of the lower half of the lens or block, including both the intermediate and reading levels

$$\begin{aligned} z &= f_{04}(x, y) \\ &= \int_0^x \frac{Q}{(1-Q^2)^{1/2}} dx + (r^2 - u^2)^{1/2} - (r^2 - u^2 - y^2)^{1/2} \\ \text{where } u &= Q \cdot r \text{ and } Q = \int_0^x \frac{dx}{r} \end{aligned}$$

For $y = 0$ this is exactly the mathematical description of our progression meridian - and for $x = \text{const}$ we have a circle equation. The surface is described by horizontally lying, circular sections with radius $r_h = r \cdot (1-Q^2)^{1/2}$. This special choice of r_h provides principal radii of curvature, what makes every point of the progression meridian a so-called umbilical point surrounded by an infinitesimally small spherical surface element. From the optical standpoint this means that paraxial imaging along the progression meridian is made identical to that of appropriately positioned spherical surface elements.

It should, however, be noted that the radii r_h define the entire surface of the lens: Progressive addition lenses with circular sections and an umbilical progression meridian are determined fully by the choice of radii along the line of progression. This would not be bad in itself if these surfaces were to offer good optical imaging. The circular sections lead to large differences in principal curvature in the lateral transition areas from distance to near, however, as can be seen in Figure 2, and therefore to pronounced astigmatic blur. It was for this reason that the circles were replaced by more suitable sections in subsequent surface constructions.

Conic sections. Every circle is a special conic section. What can be better than to replace the circles by conic sections with the appropriate vertex radii of curvature but with variable eccentricity? The lateral distance portion can be bent backwards using ellipses which become more and more curved towards the lateral areas, and the lateral near portion can be bent forwards using ellipses, parabolas and hyperbolas which become flatter towards the lateral areas, thus facilitating lateral transition⁴. This does indeed result in a clear reduction of lateral blurring.

If we consider the general representation of the conic section function

$$f(x) = \frac{1}{r} \cdot x^2 \cdot (1 + (1-k) \cdot \frac{1}{r^2} \cdot x^2)^{\frac{1}{2}} - 1$$

with the numeric eccentricity $\varepsilon = (1-k)^{1/2}$ for $k \leq 1$, it becomes clear how this advance is achieved. Admittedly, radius r continues to be determined by the umbilical requirement, but at the same time k introduces 1 new degree of freedom for each horizontal section. The increase from 0 degrees of freedom in circles to 1 degree of freedom in conic sections brings about a considerable improvement. Would an increase from 1 to 2 or more degrees of freedom not signify a further optimization of the optical surface?

Periodic functions. The initial steps towards a very general formulation of the conic sections are once again found in patent literature⁵. In quite a recent specification the progressive surface is characterized by the fact that the surface fits the following equation in a cylindrical coordinate system (y, s, φ) :