An Introduction to the theory of Dynamic Economics

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AN INTRODUCTION TO THE THEORY OF DYNAMIC ECONOMICS

A Theoretical Study in Long Run Disequilibrium Analysis

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To
Nicky
and
Clare

'Lending and the *ad hoc* creation of the means of payment are essential elements of an economic process the model of which would be incomplete without them.'

J. A. Schumpeter, *Business Cycles*
(New York, 1939), p. 114.

PREFACE

About fifteen years ago, Professor Adrian Roberts, of the Engineering Mathematics Department at Queen's University, introduced me to the methods of control engineering. I am deeply indebted to him; without his help and encouragement this book could never have been written. He is not, of course, responsible for any shortcomings of the mathematical analysis as it is presented here.

I applied my new-found tool to an analysis of multiplier-accelerator models, but I soon discovered that the results I obtained were too far-fetched to be taken seriously. The problems I encountered are summarized in Chapter 2. It took me quite some time to come up with a multiplier-accelerator model which satisfied me: this model is outlined in Chapter 3.

In order to develop beyond this model, I had to introduce, in particular, factor prices. For the reasons outlined in Chapter 4, I felt that I had to reject the neo-Keynesian approach to production relationships, so I was left with the neo-classical. I then developed what is here called the quantity-of-money controlled model. I benefited very greatly from the critical help that Sir John Hicks gave me at this stage in the development of my ideas. I am very grateful to him for making me aware that I had developed only a purely monetarist model. This set me searching for Keynesian features in my thought and, more by accident than design, I discovered what is here called the rate-of-interest controlled model. Having got that far, it was then a relatively easy matter to compare the monetarist and Keynesian features of my system (see Chapter 10). In addition, again more by accident than design, I discovered that I could theoretically distinguish between those cases where the rate of interest has no effect on (actual) investment from those where it has; I have thus attempted to fill a gap in contemporary thought.

As Professor John Black, of the University of Exeter, has pointed out to me, much of my theory depends vitally on the particular definition of saving which I adopt. I am grateful to him for making this clear to me and I hope that my brief justification of my definition in Chapter 1 shows that I have benefited from his critical comments. However, I do not think that I have yet been able to convince him of the usefulness of my approach.

Throughout, the argument is essentially mathematical. However, in an attempt to make the argument more generally understood, I have followed the advice Sir Alec Cairncross gave me many years ago and have deliberately separated the purely mathematical argument from the verbal (and simple algebraic) argument, placing it in appendices.

I am, finally, very grateful, firstly, for all the help given to me in reproducing the many diagrams and earlier typescripts of my work by Mr J. R. Watson of the Reprographic Unit in Queen's University and, secondly, to Miss Annette Smith who drew the final versions of my diagrams as they appear in the text here.

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DYNAMIC ECONOMICS

1. DYNAMIC ECONOMICS

There is little point at this stage in the development of dynamic economic theory in attempting yet another definition of 'dynamic economics'. Here it must suffice to point out that 'dynamic economics' involves two distinctly separate aspects. Firstly, the economic *state*, or model, that is to be analysed must be dynamic. Following Baumol's[1] definition of a stationary state (or equilibrium), a dynamic state may be defined as one where forces producing change are automatically (or endogenously) set in motion. In particular, a dynamic state is not one where the only change that can take place is a response to an exogenous disturbance. Secondly, the *method of analysis* applied to this dynamic state must itself be dynamic in the sense that 'it explains how one situation grows out of the foregoing'.[2] Specifically, in dynamic economic terms, the operationally significant relationship is between, putting it in discontinuous terms for clarity, today's income, and not today's total expenditure, but tomorrow's total expenditure. Mathematically, the method of analysis consists of certain key dynamic relations which contain 'at least one of the variables as related to *different points of time*'.[3] In other words, the method of analysis involves 'economically significant variables at different points of time in an *irremovable* way'.[4]

Two simple examples should serve to clarify these ideas. Firstly, consider the classical approach. Here, the presence of profits at one point of time ensures the expansion of the capital stock at a later point of time. This, in turn, results in rents rising at the expense of profits. Since the values of the variables are capable of changing automatically, the classical state is dynamic. Since the method of analysis involves time in an irremovable way, this, too, is dynamic. In the limit of the classical 'stationary' state, the dynamic method of analysis

[1] See W. J. Baumol [6], p. 40.
[2] R. Frisch [26], p. 171.
[3] R. Frisch [27], p. 100, italics in original.
[4] P. A. Samuelson [66], p. 314, italics in original.

explains the stationariness. It should be noticed that the stationary state appears as a particular case of the general dynamic state. On the other hand, secondly, the neo-classical 'stationary' state is an entirely different concept. Here we *assume* that a position of stationariness exists. What we have is 'a state made static [that is, stationary] by arbitrary abstraction as a methodological device'.[5] Automatic change is thus ruled out by definition. In other words, 'equilibrium is not something which is attained: it is something which, *if* attained, displays certain properties'.[6] The timeless analysis of these properties is static analysis: in fact, it is the maximization properties obtained from static analysis which *justify* the assumption of stationariness. Neither the economic state nor the (static) method of analysis is, then, dynamic.

Throughout this book we are concerned with dynamic economics of the general type used by the classical economists. The states, or models, which will be examined are capable of automatic change and are, therefore, dynamic; and the method of analysis employed is dynamic in the sense that it involves time in an irremovable way.

2. THE CAUSE OF CHANGE IN A DYNAMIC ECONOMIC SYSTEM

Any general theory of dynamics contains two essential elements. First and foremost, the theory must explain the *cause* of change in a system: that is, it must explain *why* change takes place. Secondly, the theory must explain the *nature* of change in a system: that is, it must explain *how* the changing variables are related together. To these we must add a third element which distinguishes an economic dynamic system from a general dynamic system. A dynamic economic theory must examine the monetary conditions which enable change to take place. Something must be briefly said about these three elements.

Conventional dynamic (that is, growth) economics has deliberately embraced, from the static analysis of the neo-classical stationary equilibrium, the idea that the growth system is in *equilibrium* at least when it is on its steady-growth-state time-path. In other words, when the system is on its steady-growth-state time-path, since everyone, particularly investing entrepreneurs, is at optimum, everyone is as happy as he can be. Consequently, as in the neo-classical equilibrium case, it follows that there is no a priori happiness reason for entrepreneurs to

[5] F. H. Knight [50], p. 143.
[6] P. A. Samuelson [66], p. 9, italics added.

alter their capital stock. In other words, loosely speaking there is no happiness cause present in the system making for further change in general, or growth in particular. Two possibilities follow. In one, the neo-classical growth theory case, there is no investment demand function and investing entrepreneurs are (irrationally) *forced* to behave conveniently in such a way that causes growth to persist. In the other case, typically the Harrod model, when an investment demand function is explicitly introduced into the system, this demand function must be framed in such a way that entrepreneurs are (unexplainedly) assumed to want growth rather than maximum happiness.

The dynamic system developed here is entirely different. Here, as it turns out, except in the particular possible case of the steady stationary state, the system is always, particularly on its steady-growth-state time-path, in long-run *disequilibrium* and it is the system's unsuccessful attempt to reach a long-run equilibrium that maintains change in general and growth in particular. In other words, everyone, and particularly investing entrepreneurs, in their unsuccessful attempt to attain maximum happiness, generates growth. Since this idea is unconventional it must be slightly elaborated. This is most easily done by briefly examining the most important aspect of change as it occurs in the dynamic system to be developed here. For investing entrepreneurs, in broad terms, equality of the rate of interest with the rate of profit on capital ensures an equilibrium when, since the capital stock is at its optimum level, no further investment will take place. However, suppose, instead, that the model is constructed in such a way that, with suitable money-supply conditions (one of the monetary conditions enabling change to take place) the rate of interest is always kept below the rate of profit. (How this can be done is explained in Chapter 4.) Then investing entrepreneurs, in their attempts to reach maximum happiness, will cause change to take place automatically and continuously. There is no need to introduce any additional explanations of entrepreneurial behaviour. This is the essence of the disequilibrium theory of change. It can be summed up by saying that the relentless pursuit of maximum happiness is the main-spring of change.

3. THE NATURE OF CHANGE IN A DYNAMIC ECONOMIC SYSTEM

When we come to examine the nature of change in a dynamic system we again part company with conventional growth theory in so far as here the relationships between certain key economic variables involve time in an irremovable way. All the models constructed here are framed in terms of continuous (and not discontinuous) time. As Phillips has argued,[7] in a macro-system the discrete (or discontinuous) response periods of thousands of micro-units all overlap so that a mathematical analysis based upon continuous change gives a more appropriate representation of reality. Consequently, a differential (and not a difference) equation system is used throughout. The relationships between the key variables that involve irremovable time can then be generally expressed by introducing the *dynamic transfer operator*, $\lambda/(D + \lambda)$, where the constant λ (>0) is the speed of response and D ($\equiv \mathrm{d}/\mathrm{d}t$) is the differential operator with respect to time. It can be noticed here that throughout the entire analysis the unit of time is taken to be one year. We can then write the general dynamic relationship between two variables as

$$X = \frac{\lambda}{D + \lambda} Z \tag{1.1}$$

where X can be regarded as the output of this simple system and Z as the input.

In economics, this operator is most generally interpreted as the *continuously distributed (or exponential) lag*.[8] Then $1/\lambda$ can be regarded as the time-constant, or length, of the lag, so that if λ is infinite there is no lag and the complete response of X to (changes in) Z is instantaneous. The smaller is λ, that is, the greater is the length of the lag, the greater is the length of time that it takes for the complete response of X to (changes in) Z. There is also a second interpretation of the dynamic transfer operator. Here the operator is regarded as embodying the *adaptive expectations hypothesis*. Probably the originator of this view was Cagan.[9] And more particularly, Friedman has used the operator to define permanent income in time-series analysis.[10]

[7] See A. W. Phillips [60], p. 291.
[8] See, for example, R. G. D. Allen [3], pp. 25–8.
[9] See P. Cagan [10], p. 37.
[10] See M. Friedman [18], pp. 143–4.