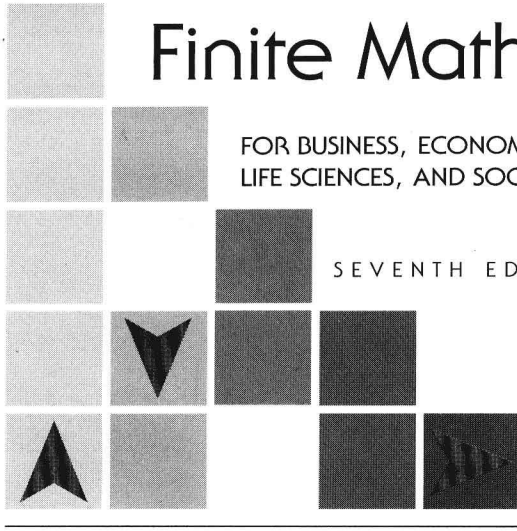
SEVENTH EDITION

FINITE MATHEMATICS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES, AND SOCIAL SCIENCES

RAYMOND A. BARNETT

MICHAEL R. ZIEGLER



Finite Mathematics

FOR BUSINESS, ECONOMICS,
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SEVENTH EDITION

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Upper Saddle River, NJ 07458

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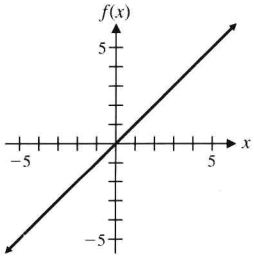
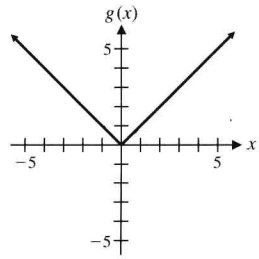
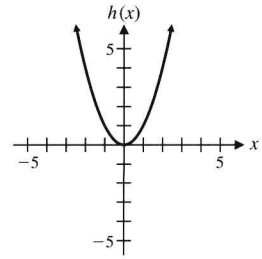
BASIC FUNCTIONS



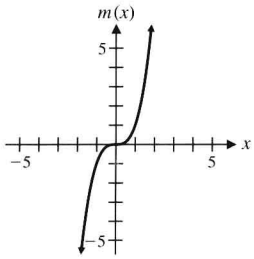
Identity function
$f(x) = x$



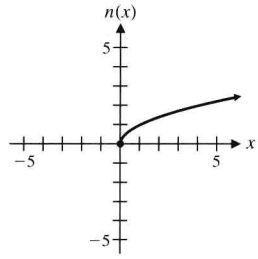
Absolute value function
$g(x) = |x|$



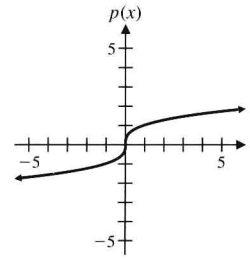
Square function
$h(x) = x^2$



Cube function
$m(x) = x^3$

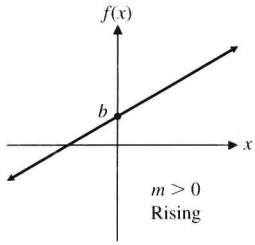


Square root function
$n(x) = \sqrt{x}$

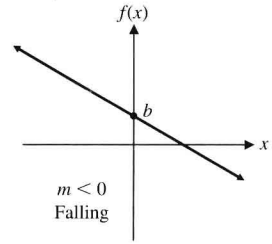


Cube root function
$p(x) = \sqrt[3]{x}$

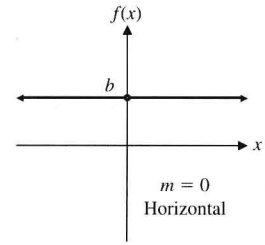
FIRST-DEGREE POLYNOMIAL FUNCTIONS



Linear function
$f(x) = mx + b$

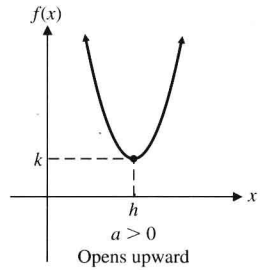


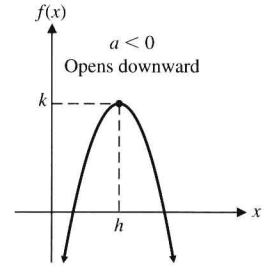
Linear function
$f(x) = mx + b$



Constant function
$f(x) = b$

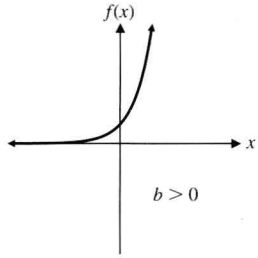
SECOND-DEGREE POLYNOMIAL FUNCTIONS



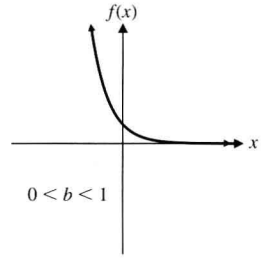


$f(x) = ax^2 + bx + c = a(x - h)^2 + k$

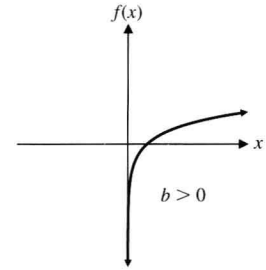
EXPONENTIAL AND LOGARITHMIC FUNCTIONS



Exponential function
$f(x) = b^x$

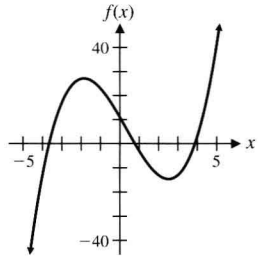


Exponential function
$f(x) = b^x$

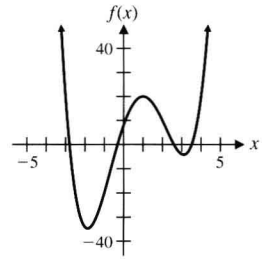


Logarithmic function
$f(x) = \log_b x$

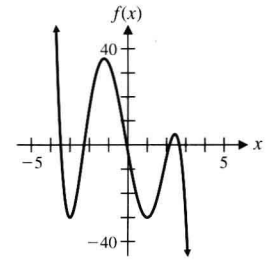
REPRESENTATIVE POLYNOMIAL FUNCTIONS (DEGREE > 2)



Third-degree polynomial
$f(x) = x^3 - x^2 - 14x + 11$

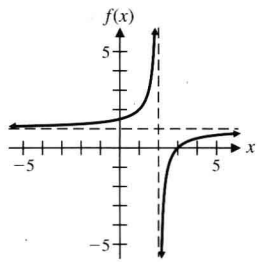


Fourth-degree polynomial
$f(x) = x^4 - 3x^3 - 9x^2 + 23x + 8$

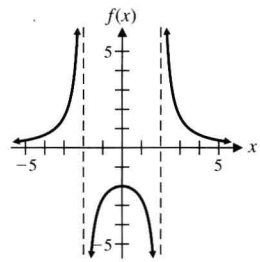


Fifth-degree polynomial
$f(x) = -x^5 - x^4 + 14x^3 + 6x^2 - 45x - 3$

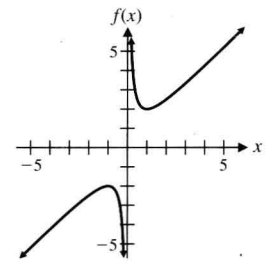
REPRESENTATIVE RATIONAL FUNCTIONS



$f(x) = \dfrac{x-3}{x-2}$

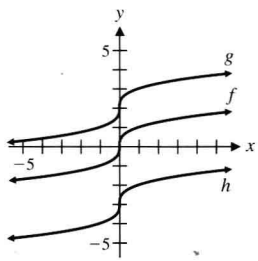


$f(x) = \dfrac{8}{x^2-4}$

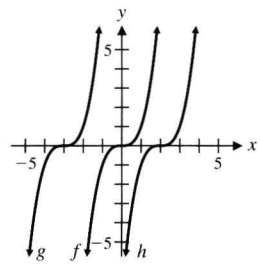


$f(x) = x + \dfrac{1}{x}$

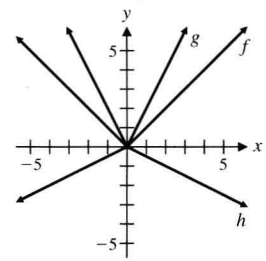
GRAPH TRANSFORMATIONS



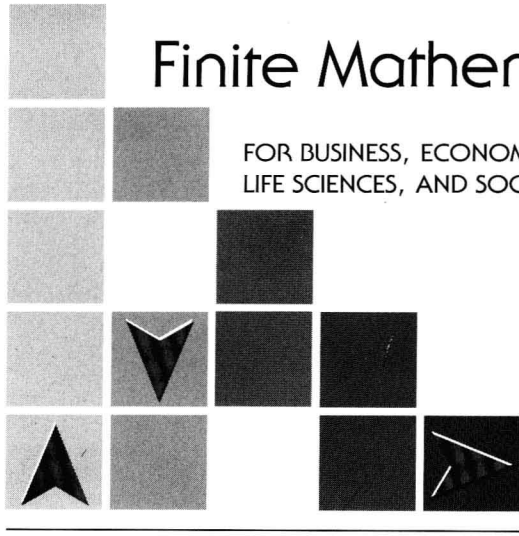
Vertical translation
$g(x) = f(x) + 2$
$h(x) = f(x) - 3$



Horizontal translation
$g(x) = f(x + 3)$
$h(x) = f(x - 2)$



Expansion, contraction, and reflection
$g(x) = 2f(x)$
$h(x) = -0.5f(x)$



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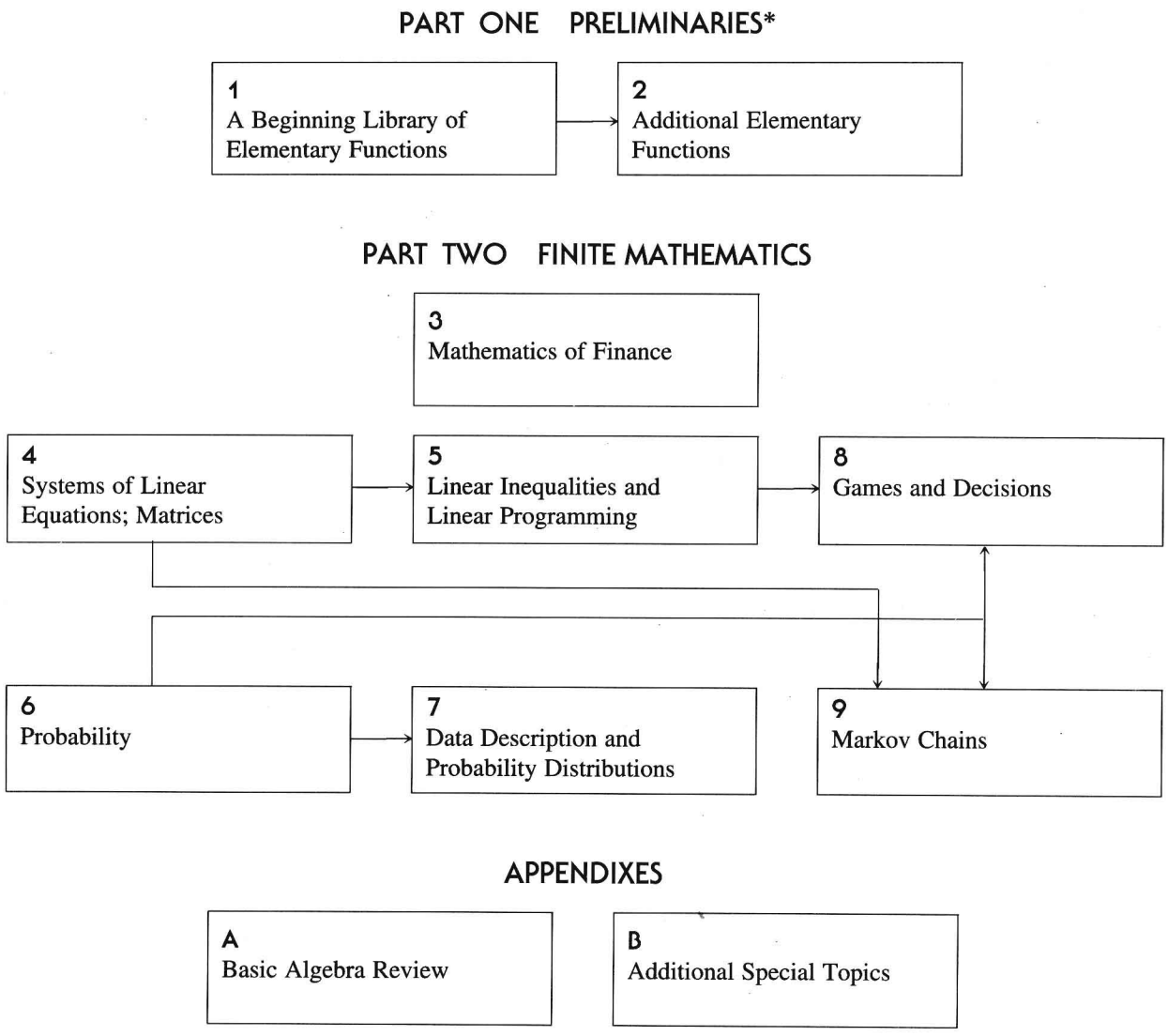
CHAPTER DEPENDENCIES



* Selected topics from Part One may be referred to as needed in Part Two or reviewed systematically before starting Part Two.

PREFACE

The seventh edition of *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences* is designed for a one-term course in finite mathematics and for students who have had $1\frac{1}{2}-2$ years of high school algebra or the equivalent. The choice and independence of topics make the text readily adaptable to a variety of courses (see the chapter dependency chart on the preceding page). It is one of six books in the authors' college mathematics series.

Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors, mathematics departments, course outlines, and college catalogs. Fundamental to a book's growth and effectiveness is classroom use and feedback. Now in its seventh edition, *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences* has had the benefit of having a substantial amount of both.

■ EMPHASIS AND STYLE

The text is **written for student comprehension.** Great care has been taken to write a book that is mathematically correct and accessible to students. Emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. Most derivations and proofs are omitted except where their inclusion adds significant insight into a particular concept. General concepts and results are usually presented only after particular cases have been discussed.

■ EXAMPLES AND MATCHED PROBLEMS

Over 260 completely worked examples are used to introduce concepts and to demonstrate problem-solving techniques. Many examples have multiple parts, significantly increasing the total number of worked examples. Each example is followed by a similar **matched problem for the student to work** while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

■ EXPLORATION AND DISCUSSION

Every section contains **Explore–Discuss** boxes interspersed at appropriate places to encourage the student to think about a relationship or process before a result is stated, or to investigate additional consequences of a development in the text. **Verbalization** of mathematical concepts, results, and processes is encouraged in these Explore–Discuss boxes, as well as in some matched problems, and in some problems in almost every exercise set. The Explore–Discuss material also can be used as

in-class or out-of-class **group activities.** In addition, at the end of every chapter (before the chapter review), we have included a special **chapter group activity** that involves several of the concepts discussed in the chapter. All these special activities are highlighted with color shading to emphasize their importance.

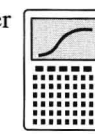
■ EXERCISE SETS

The book contains over 2,900 problems. Many problems have multiple parts, significantly increasing the total number of problems. Each exercise set is designed so that an average or below-average student will experience success and a very capable student will be challenged. Exercise sets are mostly divided into A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) levels.

■ APPLICATIONS

A major objective of this book is to give the student substantial experience in **modeling and solving real-world problems.** Enough applications are included to convince even the most skeptical student that mathematics is really useful (see the Applications Index inside the back cover). Worked examples involving applications are identified by ▶. **Almost every exercise set contains application problems,** usually divided into business and economics, life science, and social science groupings. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems taken from professional journals and books. No specialized experience is required to solve any of the applications.

■ TECHNOLOGY

The generic term **graphing utility** is used to refer to any of the various graphing calculators or computer software packages that might be available to a student using this book. (See the description of the software accompanying this book later in this Preface.) Although **access to a graphing utility is not assumed,** it is likely that many students will want to make use of one of these devices. To assist these students, **optional graphing utility activities** are included in appropriate places in the book. These include brief discussions in the text, examples or portions of examples solved on a graphing utility, and problems for the student to solve. All the optional graphing utility material is clearly identified by either  or C and can be omitted without loss of continuity, if desired.

■ GRAPHS

All graphs are new and are computer-generated to ensure mathematical accuracy. Graphing utility screens displayed in the text are actual output from a graphing calculator.

■ STUDENT AIDS

Annotation of examples and developments, in small color type, is found throughout the text to help students through critical stages (see Sections 1-1 and 4-2). **Think boxes** (dashed boxes) are used to enclose steps that are usually performed mentally (see Sections 1-1 and 4-1). **Boxes** are used to highlight important definitions, theorems, results, and step-by-step processes (see Sections 1-1 and 1-4). **Caution** statements appear throughout the text where student errors often occur (see Sections 4-3 and 4-5). **Functional use of color** improves the clarity of many illustrations, graphs, and developments, and guides students through certain critical steps (see Sections 1-1 and 4-2). **Boldface type** is used to introduce new terms and highlight important comments. **Chapter review** sections include a review of all important terms and symbols and a comprehensive review exercise. **Answers to most review exercises,** keyed to appropriate sections, are included in the back of the book. Answers to all other odd-numbered problems are also in the back of the book. Answers to application problems in linear programming include both the mathematical model and the numeric answer.

■ CONTENT

The text begins with the development of a library of elementary functions in Chapters 1 and 2, including their properties and uses. We encourage students to investigate mathematical ideas and processes **graphically** and **numerically,** as well as **algebraically.** This development lays a firm foundation for studying mathematics both in this book and in future endeavors. Depending on the syllabus for the course and the background of the students, some or all of this material can be covered at the beginning of a course, or selected portions can be referred to as needed later in the course.

The material in Part Two (Finite Mathematics) can be thought of as four units: **mathematics of finance** (Chapter 3); linear algebra, including **matrices, linear systems, and linear programming** (Chapters 4 and 5); **probability and statistics** (Chapters 6 and 7); and applications of linear algebra and probability to **game theory and Markov chains** (Chapters 8 and 9). The first three units are independent of each other, while the last two chapters are dependent on some of the earlier chapters (see the chapter dependency chart preceding this Preface).

Chapter 3 presents a thorough treatment of simple and compound interest and present and future value of ordinary annuities. Appendix B contains a section on progressions that can be covered in conjunction with this chapter, if desired.

Chapter 4 covers linear systems and matrices with an **emphasis on using row operations and Gauss–Jordan elimination** to solve systems and to find matrix inverses. The row operations discussed in Sections 4-2 and 4-3 are required for the simplex method in Chapter 5. Matrix multiplication, matrix inverses, and systems of equations are required for Markov chains in Chapter 9.

Chapter 5 provides **broad and flexible coverage of linear programming.** The first two sections cover two-variable graphing techniques. Instructors who wish to emphasize techniques can cover the basic simplex method in Sections 5-3 and 5-4 and then discuss any or all of the following: the dual method (Section 5-5), the big *M* method (Section 5-6), or the two-phase simplex method (Chapter 5 Group Activity). Those who want to emphasize modeling can discuss the formation of the mathematical model for any of the application examples in Sections 5-4, 5-5, and 5-6, and either omit the solution or use software to find the solution (see the description of the software that accompanies this text later in this preface). To facilitate this approach, **all the answers in the back of the book to application problems** in Exercises 5-4, 5-5, 5-6, and the Chapter 5 Review Exercise **contain both the mathematical model and the numeric solution.** Geometric, simplex, and dual solution methods are required for portions of Chapter 8.

Chapter 6 covers **counting techniques and basic probability,** including Bayes' formula and random variables. Appendix B contains a review of basic set theory and notation to support the use of sets in probability. Most of the topics discussed in Chapter 6 are required for Chapter 7.

Chapter 7 deals with basic **descriptive statistics** and more advanced probability distributions, including the important **normal distribution.** Appendix B contains a short discussion of the binomial theorem that can be used in conjunction with the development of the binomial distribution in Section 7-5.

Each of the last two chapters ties together concepts developed in earlier chapters and applies them to two interesting topics: **game theory** (Chapter 8) and **Markov chains** (Chapter 9). Either chapter provides an excellent unifying conclusion to a finite mathematics course.

Appendix A contains a **self-test** and a **concise review of basic algebra** that also may be covered as part of the course or referred to as needed. As mentioned above, Appendix B contains additional topics that can be covered in conjunction with certain sections in the text, if desired.

■ PRINCIPAL CHANGES FROM THE SIXTH EDITION

As mentioned earlier, exploration and discussion activities have been distributed uniformly throughout the book. These new elements include Explore–Discuss questions in the text and exercise sets, and chapter group activities. The optional material on graphing utilities is also more uniformly distributed, but the major emphasis of the book is still on solving problems without the aid of technology.

Part One has been revised extensively. Basic algebraic operations have been moved to Appendix A, and the remaining material has been reorganized and mostly rewritten in order to present the student with a library of elementary functions and to

encourage viewing mathematical ideas and processes graphically, numerically, and algebraically. A self-test has been added to Appendix A to help identify areas that need review.

In Chapter 4, all matrix operations are now covered in a single section. The two chapters on probability have been merged into a single chapter (Chapter 6). The discussion of Markov chains has been expanded to form a new chapter (Chapter 9) and now includes a discussion of absorbing chains.

■ SUPPLEMENTS FOR THE STUDENT

1. A **Student's Solutions Manual** by Garret J. Etgen is available through a book store. The manual includes detailed solutions to all odd-numbered problems and all review exercises.
2. **Computer software** for IBM-compatible computers is available at a nominal cost through a book store. *Explorations in Finite Mathematics* by David Schneider contains over twenty routines that provide additional insight into the topics discussed in the text. Although this software has much of the computing power of standard mathematical software packages, it is primarily a teaching tool that focuses on understanding mathematical concepts, rather than on computing. Included are routines for Gaussian elimination, matrix inversion, solution of linear programming problems by both the geometric method and the simplex method, Markov chains, probability and statistics, and mathematics of finance. All the routines in this software package are menu-driven and are very easy to use. The matrix routines use and display rational numbers, and matrices may be saved and printed. The software is accompanied by a manual with instructions and additional exercises for the student. Hardware requirements are an IBM-compatible computer with at least 384K of memory and a graphics adapter: CGA, EGA, VGA, or Hercules.
3. A **Graphics Calculator Manual** by Carolyn L. Meitler contains examples illustrating the use of a graphics calculator to solve problems similar to those discussed in the text. The manual follows the chapter organization of the text, making it easy to find examples in the manual illustrating appropriate calculator solution methods for problems in the text. The manual includes keystrokes for the TI-81, TI-82, and TI-85 calculators. However, the examples and techniques can be used with any graphing utility.
4. A **Supplemental Applications and Topics** manual by Jon E. Baum is available at a nominal cost through a book store. Part I of the manual expands the application exercises in the text and reinforces the important role of the mathematics presented. These exercises provide the student with a richer and more varied experience in solving real-world problems. Part II of the manual presents some applications that are not covered in the text, including transportation problems, assignment problems, sensitivity analysis, and a variety of finance topics. After completing the prerequisite material in the text, students interested in these more specialized topics will realize substantial benefits by studying this portion of the manual.

■ SUPPLEMENTS FOR THE INSTRUCTOR

For a summary of all available supplementary materials and detailed information regarding examination copy requests and orders, see page xix.

1. **PH Custom Test, a menu-driven random test system** for either IBM-compatible or Macintosh computers is available to instructors without cost. The test system has been greatly expanded and now offers **on-line testing.** Carefully constructed algorithms use random-number generators to produce different, yet equivalent, versions of each of these problems. In addition, the system incorporates a unique **editing function** that allows the instructor to create additional problems, or alter any of the existing problems in the test, using a full set of mathematical notation. The test system offers **free-response, multiple-choice, and mixed exams.** An almost unlimited number of quizzes, review exercises, chapter tests, midterms, and final examinations, each different from the other, can be generated quickly and easily. At the same time, the system will produce answer keys, student worksheets, and a gradebook for the instructor, if desired.
2. An **Instructor's Resource Manual** provides over 100 transparency masters and all the answers not included in the text, as well as hard copy of test items available in PH Custom Test. This manual is available to instructors without charge.
3. A **Student's Solutions Manual** by Garret J. Etgen (see Student Aids) is available to instructors without charge from the publisher.
4. **Computer software** and accompanying **manual** for *Explorations in Finite Mathematics* by David Schneider (see Student Aids) are available to instructors without charge. The manual contains complete instructions for using the software, eliminating the need to spend class time discussing these details, and examples and exercises for the student. In addition to providing students with the opportunity to use the computer as an effective tool in the learning process, instructors will find the software very useful for activities such as preparing examples for class, constructing test questions, and classroom demonstrations.
5. A **Graphics Calculator Manual** by Carolyn L. Meitler (see Student Aids) is available to instructors without charge from the publisher. The manual contains all the necessary information for a student with no previous experience with a graphic calculator, eliminating the need for the instructor to prepare materials related to calculator usage. In particular, separate appendixes for the TI-81, TI-85, HP-48, and Casio fx-7700G graphic calculators contain detailed instructions, including calculator-specific keystrokes, for performing the various operations required to effectively use each of these calculators to solve problems in the text. Furthermore, the methods illustrated for these calculators are easily adapted to other graphing utilities. The manual is very effective both for a class where all students purchase the same calculator and in a setting where students are using a variety of different calculators—an important consideration as more and more students arrive at college having already purchased a graphic calculator.
6. A **Supplemental Applications and Topics** manual by Jon E. Baum (see Student Aids) is available to instructors without charge from the publisher. Instructors can use Part I of this manual to supplement the exercise sets in the text, providing

students with additional experience in solving applications utilizing the mathematics presented in the text. Part II of the manual can be used to provide coverage of applications not covered in the text, such as transportation problems, assignment problems, sensitivity analysis, and a variety of finance topics, either as part of the syllabus for a course or as subjects for independent study.

■ ERROR CHECK

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error-free. For any errors remaining, the authors would be grateful if they were sent to: Michael R. Ziegler, 509 W. Dean Court, Fox Point, WI 53217; or, by e-mail, to michael@mscs.mu.edu

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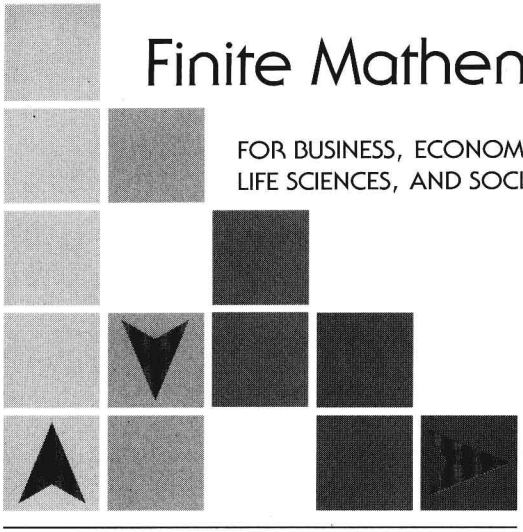

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