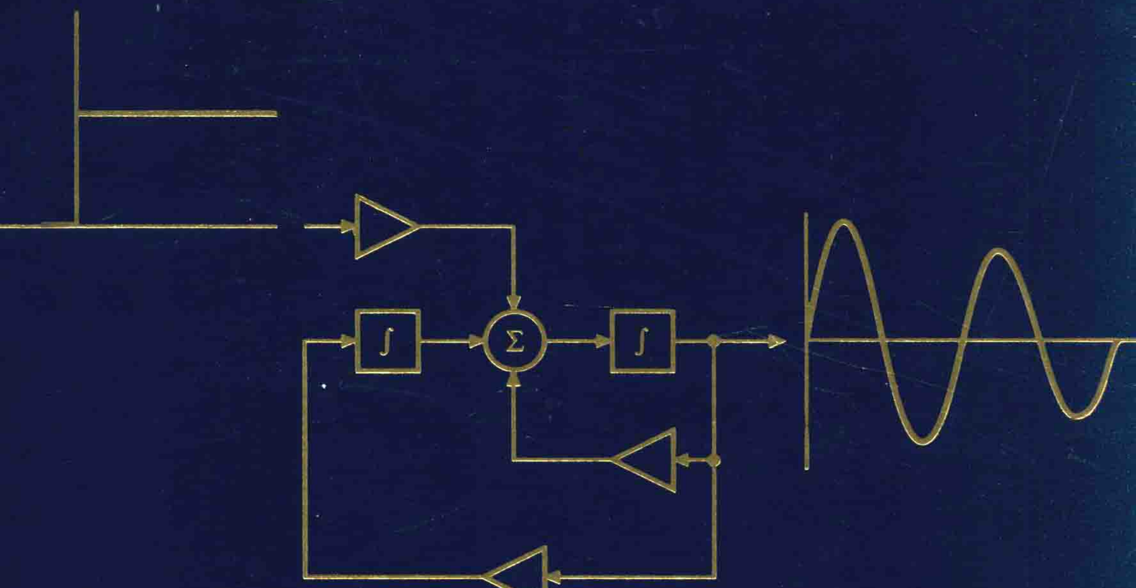
SIGNALS AND SYSTEMS: Continuous and Discrete

THIRD EDITION



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Preface

The philosophy of the previous edition of *Signals and Systems* is retained in the third edition by continuing to stress the systems approach so that students are provided the tools and techniques for understanding and analyzing both continuous-time and discrete-time linear systems. While the systems approach is applicable to a very broad class of problems, liberal examples based on traditional circuit theory are included in the book to illustrate the various systems analysis techniques that are introduced.

An important feature of the third edition of *Signals and Systems* is the inclusion of several "computer exercises" in each chapter. These problems require the use of a computer and are placed in specifically identified sections following the regular end-of-chapter problems. There are several motivations for including these problems. First, it is clear that computer use is now recognized as a valuable component in engineering education and, as a result, is being fully integrated into engineering curricula. Most departments provide for student access to extensive computing resources. Through the use of the computer the student is able to explore more advanced problems having significant computational requirements. The student is also exposed to, and gains confidence in, new problem solving tools. Many of the computer problems allow the student the choice of solving the problem by writing a program using a general-purpose high-level language such as FORTRAN, Pascal, or C, or by using a scientific problem-solving program such as Matlab® or MathCAD®. We feel that it is important for students to have experience in computer-aided analysis tools and to use them as appropriate in their day-to-day problem solving activities.

Another feature of the third edition of *Signals and Systems* is that all of the end-of-chapter summaries are written as a point-by-point review of the important topics covered in the chapter. Such a listing can be a valuable study guide for the student. In addition, a number of new problems are contained in the third edition.

The organization of the third edition is identical to that of the second edition. The first seven chapters deal with continuous-time linear systems in both the time domain and the frequency domain. References to discrete-time signals and systems are made where appropriate so that the students can appreciate the relationship between discrete-time and continuous-time signals and systems. The principal tool developed for time-domain analysis is the convolution integral. Frequency-domain techniques include the Fourier and the Laplace transforms. An introduction to state-variable techniques is also included, along with treatments of both continuous-time and discrete-time state equations. The remainder of the book deals with discrete-time systems, including z-transform analysis techniques, digital filter analysis and synthesis, and the discrete Fourier transform and fast Fourier transform (FFT) algorithms.

This organization allows the book to be covered in two three-semester-hour courses, with the first course being devoted to continuous-time signals and systems and the second course to discrete-time signals and systems. Alternatively, the material can be used as a basis for three quarter-length courses. With this format, the first course could cover time- and frequency-domain analysis of continuous-time systems. The second course could cover state variables, sampling, and an introduction to the z-transform and discrete-time systems. The third course could deal with the analysis and synthesis of digital filters and provide an introduction to the discrete Fourier transform and its applications. Other groupings of topics are possible.

The assumed background of the student is mathematics through differential equations and the usual introductory circuit theory course or courses. Knowledge of the basic concepts of matrix algebra would be helpful but is not essential. An appendix is included to bring together the pertinent matrix relations that are used in Chapters 6 and 7. We feel that in most electrical engineering curricula the material presented in this book is best taught at the junior level.

Plan of the Text

We begin the book by introducing the basic concepts of signal and system models and system classifications. The idea of spectral representations of periodic signals is first introduced in Chapter 1 because we feel that it is important for the student to think in terms of both the time and the frequency domains from the outset.

The convolution integral and its use in fixed, linear system analysis by means of the principle of superposition are treated in Chapter 2. This chapter deals with system modeling and analysis in the time domain. The evaluation of the convolution integral is treated in detailed examples to provide reinforcement of the basic concepts. Calculation of the impulse response and its relation to the step and ramp responses of a system are discussed. The concepts of system modeling and system simulation are also treated in Chapter 2. This chapter concludes with a section devoted to the numerical solution of system equations.

The Fourier series is introduced in Chapter 3. We have emphasized the elementary approach of approximating a periodic function by means of a trigonometric series and obtaining the expansion coefficients by using the orthogonality of sines and cosines. We do this because this is the first time many students are introduced to the Fourier series. The alternative generalized orthogonal function approach is included as a section at the end of this chapter for those who prefer it. The concept of the transfer function in terms of sinusoidal steady-state response of a system is discussed in relation to signal distortion.

The Fourier transform is the subject of Chapter 4, with its applications to spectral analysis and system analysis in the frequency domain. The concept of an ideal filter, as motivated by the idea of distortionless transmission, is also introduced at this point. The Gibbs phenomenon, window functions, and convergence properties of the Fourier coefficients are treated in optional closing sections.

The Laplace transform and its properties are introduced in Chapter 5. Again, we have tried to keep the treatment as simple as possible because this is assumed to be a first exposure to the material for a majority of students, although a summary of complex variable theory is provided in Appendix C so that additional rigor may be used at the instructor's option. The derivation of Laplace transforms from elementary pairs is illustrated by example, as is the technique of inverse Laplace transformation using partial-fraction expansion. Optional sections on the evaluation of inverse Laplace transforms by means of the complex inversion integral and an introduction to the two-sided Laplace transform are also provided.

The application of the Laplace transform to network analysis is treated in detail in Chapter 6. The technique of writing Laplace-transformed network equations by inspection is covered and used to review the ideas of impedance and admittance matrices, which the student will have learned in earlier circuits courses for resistive networks. The concepts of zero-state response and zero-input response are discussed along with their relationship to transient and steady-state responses. The transfer function is treated in detail, and the Routh test for determining stability is presented. The chapter closes with a treatment of Bode plots and block diagram algebra for fixed, linear systems.

In Chapter 7 the concepts of a state variable and the formation of the state-variable approach to system analysis are developed. The state equations are solved using both time-domain and Laplace transform techniques, and the important properties of the solution are examined. As an example, we show how the state-variable method can be applied to the analysis of circuits. Discrete-time state equations and the concepts of controllability and observability are briefly introduced.

The final three chapters provide coverage of the topics of discrete-time signal and system analysis. Chapter 8 begins with a study of sampling and the representation of discrete-time systems. The sampling operation is covered in considerable detail. This is accomplished in the context of formulating a model for an analog-to-digital (A/D) converter so that the operation of quantizing can be given some physical basis. A brief analysis of the effect of quantizing sample values in the A/D conversion process is included as an introduction to quantizing errors. As a bonus, the student is given a basis upon which to select an appropriate wordlength of an A/D converter. Both ideal and approximate methods for reconstructing a signal from a sequence of samples are treated in detail. The z-transform, difference equations, and discrete-time transfer functions are developed with sufficient rigor to allow for competent problem solving but without the complications of contour integration. The material on the classification of discrete-time systems has been written in a way that parallels the similar material for continuous-time systems.

Chapter 9 allows students to use their knowledge of discrete-time analysis techniques to solve an important class of interesting problems. The idea of system synthesis, as opposed to system analysis, is introduced. Discrete-time integration is covered in considerable detail for several reasons. First, the idea of integration will be a familiar one. Thus students can appreciate the different information gained by a frequency-domain analysis as opposed to a time-domain analysis. In addition, the integrator is a basic building block for many analog systems. Finally, the relationship between trapezoidal integration and the bilinear z-transform is of sufficient importance to warrant a discussion of trapezoidal integration. The synthesis techniques for digital filters covered in this chapter are the standard ones. These are synthesis by time-domain invariance, bilinear z-transform synthesis, and synthesis through Fourier series expansion. The section on the design of FIR digital filters using the Fourier series and windowing technique was rewritten for the third edition. The result is a more general treatment of this technique that allows for the design of digital filters having a complex unit pulse response. Through the application of these techniques, the student is able to gain confidence in the previously developed theory. Since several synthesis techniques depend on knowledge of analog filter prototypes, Appendix D, which discusses several different filter prototypes, is included.

The discrete Fourier transform (DFT) and its realization through the use of fast Fourier transform (FFT) algorithms is the subject of Chapter 10. Both decimation-in-time and decimation-in-frequency algorithms are discussed. Several examples are provided to give the student practice in performing the FFT operations. We believe that this approach best leads to a good understanding of the FFT algorithms and their function. Basic properties of the DFT are summarized and a comparison of the number of operations required for the FFT as compared to the DFT is made. Several applications of the DFT are summarized and the use of windows in supressing leakage is discussed. This chapter closes with a discussion and illustration of FFT algorithms with arbitrary radixes and the chirp-z transform.

The book also contains a number of optional sections, denoted by an asterisk, that go deeper into specific topics than is often customary for a junior-level course. These topics can be eliminated without loss of continuity.

A complete solutions manual, which contains solutions to all problems, is available from the publisher as an aid to the instructor. The solutions manual for the third edition is in a typeset format and makes extensive use of MathCAD for a large number of problems. The instructor can make use of the MathCAD solutions to develop templates from which problem parameters can be easily changed. Answers to selected problems are provided in Appendix F as an aid to the student.

Acknowledgments

We wish to thank the many people who have contributed, both knowingly and unknowingly, to the development of this textbook. Special thanks go to our many students who have provided a living laboratory within which we could test-teach many sections of this textbook. Their comments and criticisms have been very valuable and are gratefully appreciated. Special thanks are also due to our many faculty colleagues, now too many to mention, who taught from the first and second editions of this book and have offered many suggestions for improvement. Thanks are also due to the reviewers who provided many insightful comments and suggestions for improvement for the third edition.

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R. E. Z.
W. H. T.
D. R. F.

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CHAPTER 1

Signal and System Modeling Concepts

1-1 Introduction

This book deals with *systems* and the interaction of *signals* in systems. A system, in its most general form, is defined as a combination and interconnection of several components to perform a desired task.† Such a task might be the measurement of the acceleration of a rocket or the transmission of a message from New York to Los Angeles. The measurement of the acceleration might make use of visual observation of its position versus time. An equally unsophisticated solution to the message delivery problem might use a horse and rider. Obviously, more complex solutions are possible (and probably better). Note, however, that our definition is sufficiently general to include them all.

We will be concerned primarily with *linear* systems. Such a restriction is reasonable because many systems of engineering interest are closely approximated by linear systems and very powerful techniques exist for analyzing them. We consider several methods for analyzing linear systems in this book. Although each of the methods to be considered is general, not all of them are equally convenient for any particular case. Therefore, we will attempt to point out the usefulness of each.

A *signal* may be considered to be a function of time that represents a physical variable of interest associated with a system. In electrical systems, signals usually represent currents and voltages, whereas in mechanical systems, they might represent forces and velocities (or positions).‡ In the example mentioned above, one of the signals of interest represents the acceleration, but this could be integrated to yield a signal proportional to velocity. Since electrical voltages and currents are relatively easy to process, the original signal representing acceleration, which is a mechanical signal, would probably be converted to an electrical one before further *signal processing* takes place. Examples illustrating these remarks will be given in the next section.

Just as there are several methods of systems analysis, there are several different ways of representing and analyzing signals. They are not all equally convenient in

† The Institute of Electrical and Electronics Engineers dictionary defines a system as "an integrated whole even though composed of diverse, interacting structures or subjunctions."

‡ More generally, a signal can be a function of more than one independent variable, such as the pressure on the surface of an airfoil, which is a function of three spatial variables *and* time.