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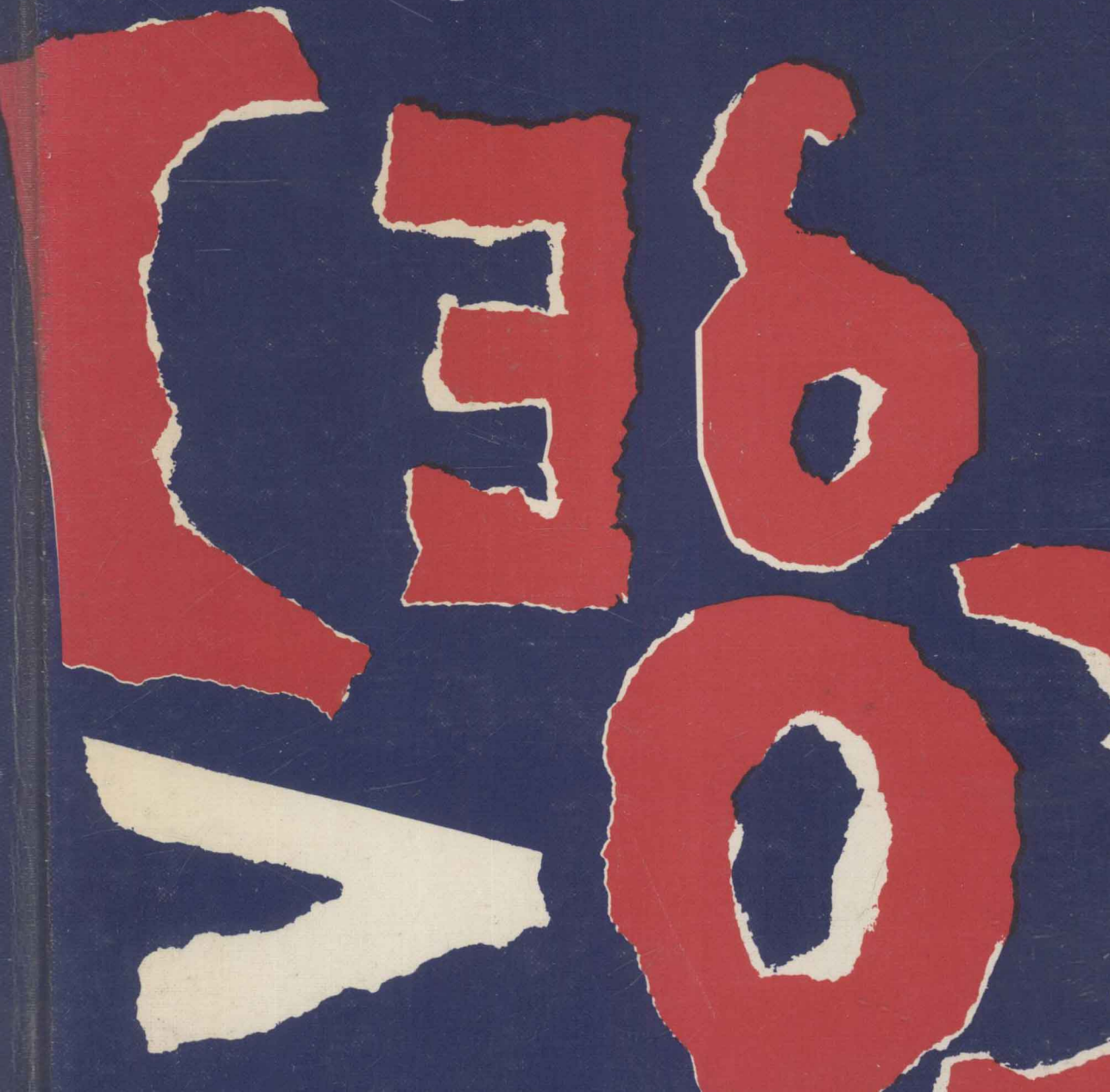
INTRODUCTION TO CALCULUS 1 AND 2



introduction to calculus 1 and 2

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Introduction to Calculus 1
A. B. Willcox · R. Creighton Buck · H. G. Jacob · D. W. Bailey

Introduction to Calculus 2
A. B. Willcox · R. Creighton Buck · H. G. Jacob · D. W. Bailey

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To Our Wives

preface

Now that the book is written we come face-to-face with the inevitable question: Why did we do it? Every man who invests a part of his life in a pile of printed pages must answer that question, sooner or later, in his own mind. But if the fruit of his labor is a calculus book—*another* calculus book—an author has a special responsibility to answer the question in public. With that responsibility in mind, we offer several remarks . . .

. . . *about this series.* The recommendations for a *General Curriculum in Mathematics for Colleges* prepared by the Committee on the Undergraduate Program in Mathematics (CUPM) call for a basic sequence of five one-semester courses for the first two years of college. The sequence carries the student through beginning courses in calculus, probability and statistics, linear algebra, and multivariable calculus. It is not a sequence which most students can be expected to cover in two years—it is designed to accept students at a variety of levels of preparation and to provide a number of different tracks for students of a variety of interests and talents. *Introduction to Calculus 1 and 2* are the first and second books in a series designed to help bring these recommendations into the classroom.

In writing the books of this series, the authors have allowed their own experience to color freely the broad outlines of the committee. We have tried very hard to capture the spirit of the recommendations and hope that we have succeeded, at least in part. However, we also hope that in creating a concrete model we have improved upon the ideal. Most of all, we hope that our efforts will significantly help teachers to impart clarity and motivation to their courses which will challenge and excite their students.

. . . *about this book.* There are several features of this book which are novel. Aside from the overall adherence to the CUPM outline, the major focus is on readability and careful motivation. Our program for introducing continuity and limits is not the traditional one. Continuity comes first, via a highly geometrical approach. Since this is the first significant idea of the calculus encountered by

the student, considerable attention is devoted to its evolution from the most informal and intuitive image suggested by the words "continuous curve." Next, the idea of a function being limiting, or having a limit, at a point is defined as a sort of almost-continuity. This seems to us to be the natural progression from a simple intuitive idea to a more sophisticated variation. The discussion is informal—no ϵ's or δ's appear until the first chapter of *Introduction to Calculus 2*. The ideas of continuity and limits, which could hardly be more basic to the subject, must appeal to the intuition of the student if he is to take them at all seriously. We believe in rigor at this level only when it contributes to clarity, and we believe that an insistence on complete logical detail can often inhibit rather than enhance understanding. If the basic ideas of derivative and integrals can be transmitted by an intuitive approach, so can the ideas of continuity and limits. We have tried to do all these things.

The content of *Introduction to Calculus 1* (*C 1*) includes the necessary ingredients for a one-semester course covering both differential and integral calculus. Combining derivatives and integrals in a one-semester course along with the necessary basic techniques and applications to give these ideas flesh required some pruning. If you find that some of the topics which you consider absolutely essential for a first course are missing from *C 1*, you will probably find them in *Introduction to Calculus 2* (*C 2*), designed for the second semester. In *C 2* we return to each of the main ideas originally presented in *C 1*, sharpening techniques, extending concepts, and giving additional applications. This deeper examination of the basic concepts leads naturally into the discussions of series, elementary differential geometry of plane curves, and first order differential equations which conclude the second volume.

Thus, after the first semester the student should be ready to begin work in such subjects as probability, statistics, and physics, which are enhanced by some knowledge of the calculus. After two semesters he will have acquired a strong acquaintance with one-variable calculus and will be ready to branch out into higher dimensions.

In our writing we have had in mind the student who has had some introduction to the elementary functions and analytic geometry. This is in keeping with a trend to treat these subjects as precalculus ones which should properly be taught in high school and which in the ideal situation should already be familiar to the student when he begins his study of the calculus. However, reality dictates moderation of this view. Even those who espouse it zealously admit that its fulfillment lies in the future. Thus, our treatment of the elementary functions, although brief, is complete, and there is an appendix on analytic geometry, inequalities, and absolute value which can be used either for review or as a mini-text.

. . . *for the teacher.* This book is written for your student, not for you! It is designed to be read by your student, not to be a reference treatise. With this purpose in mind, we have tried to talk to the reader in much the same way that we have talked to our students in the classroom, attempting always to lead him on and to challenge him, but being careful not to outrun him. While frankly appealing to the reader's intuition, we have tried to keep our statements of mathematical definition and fact quite correct and expressed in the current idiom. There are many places for you to augment our presentation or to digress from it. We invite you also to disagree with it and alter it where you will. Treat the book as a companion in the classroom—one which has a point of view which need not always be your own but which can instruct by generating discussion and provoking thought. Calculus ex cathedra has not been our aim. The authors, who know the thrill of seeing a spark ignited in a student's eye, simply hope to help you bring the calculus to life for your students in your own way.

It has become almost a tradition in textbook writing to close a preface with some remarks to the teacher who anticipates difficulty in covering all of the offered material in the time available. We shall honor that tradition. However, the teacher is warned that this book is not a buffet from which meals can be selected for every appetite. We have striven for a connected narrative treating only those topics which most teachers consider essential for a first course. To shorten the treatment it is probably better to sample most topics lightly than to leave out many.

Exclusive of the sections numbered n.0, $n = 1, 2, \ldots$, which are introductory and may be read rapidly, there are forty-eight sections in *C 1* and thirty-nine sections in *C 2*. Some adjustments will have to be made if these two parts are to conform roughly to a two-semester course. In colleges on a quarter system the two-part division will have to be ignored entirely. Although we expect, as a general rule, that a teacher will want to assign one section per class meeting, it is clear that some compression will be needed to fit most academic years.

These adjustments may be accomplished in many ways. Some of the shorter sections may be assigned in pairs or triples, with students asked to read certain ones lightly and rapidly. It may be necessary—and it is entirely reasonable—to skip over some of the longer proofs, omitting most of the details. A few sections may be omitted entirely without leaving serious gaps. There are so many variations to fit special interests and time restrictions that we are reluctant to write out any menus. As an example of what might be done to provide a course covering most of the material presented in two semesters of thirty-seven lectures each, we offer the following two-semester diet. A possible plan for breaking this into three quarters is given.

First Semester

1. 2. 3.	Analytic Geometry—from the Appendix	*C 1*
4. 1.0, 1.1 5. 1.2, 1.3 6. 1.4, 1.5 7. 1.6 8. 1.7	Functions, Continuity, and Limits	*C 1*
9. 2.0, 2.1 10. 2.2 11. 2.3	Derivatives	*C 1*
12. 3.0, 3.1 13. 3.2 14. 3.3, 3.4 15. 3.5	Finding Derivatives	*C 1*
16. 4.0, 4.1 17. 4.2, 4.3 18. 4.4 19. 4.5 20. 4.6	The Mean Value Theorem and Its Applications	*C 1*
21. 5.0, 5.1 22. 5.2 23. 5.3 24. 5.4 25. 5.5	Applications of Derivatives	*C 1*

End of First Quarter

26. 6.0, 6.1 27. 6.2, 6.3 28. 6.4 29. 6.6	The Definite Integral	*C 1*
30. 7.0, 7.1 31. 7.2 32. 7.3, 7.4	The Logarithm and Exponential Functions	*C 1*
33. 8.0, 8.1 34. 8.2 35. 8.4	Techniques of Integration	*C 1*
36. 9.0, 9.1 37. 9.2	Applications of Definite Integrals	*C 1*

Second Semester

1. 1.0, 1.1 2. 1.2 3. 1.3	Limits Revisited	*C 2*
4. 2.0, 2.1, 2.2 5. 2.3 6. 2.4 7. 2.5	Derivatives Revisited	*C 2*
8. 8.5 9. 9.3 10. 9.4	Further Integration Techniques and Applications	*C 1*
11. 3.0, 3.1 12. 3.2 13. 3.3 14. 3.4 15. 3.5 16. 3.8	Integrals Revisited	*C 2*

End of Second Quarter

17. 4.0, 4.1 18. 4.2 19. 4.3	Limits and Approximations	*C 2*
20. 5.0, 5.1 21. 5.2 22. 5.3 23. 5.4 24. 5.5 25. 5.6 26. 5.7	Infinite Series	*C 2*
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. . . *about the help we have received.* It would be impossible to list the names of all those who have influenced this book. Almost all of our teachers, colleagues, and students have been involved in shaping our ideas and attitudes. The writing project might never have been born if two of us had not been active participants in some of the deliberations of CUPM, and we value highly this opportunity for interaction with teachers representing the broadest spectrum of educational and pedagogical experience and thinking. However, several persons contributed so tangibly to the books that we must thank them individually and publicly. Mrs. Gloria Harris gave generously of her time typing draft after draft and learned more than she ever hoped to know about mathematicians and the foibles of their pens. Miss Veronica McLoud of Houghton Mifflin Company contributed not only clarity to our prose but also accuracy to our mathematical presentation through her efficient editing. Without the help of these two women the books would still be lying in a dog-eared pile in a desk drawer. It was only through the enthusiasm and encouragement of Mr. Paul Kelly, college mathematics editor for Houghton Mifflin Company, that this writing project was born, and he has served as our efficient supporting team manager throughout. Finally, we are most grateful to Mr. Tom Hudson, a student at Amherst College, for the graphs in the *Solutions to Selected Exercises 1.*

A Note on Organization The book is divided into chapters, each of which is divided into sections. A notation "Section a.b" refers to Section b in Chapter a. Within a section, theorems, definitions, examples, and figures are numbered independently in serial order. Thus "Theorem a.b.c" refers to Theorem c in Section b of Chapter a. Lemmas and corollaries are considered theorems in the numbering system. A set of exercises follows each section. A reference "Exercise a.b.c" is to Exercise c in the set of exercises following Section a.b.

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