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An introduction to
Equity
derivatives

Theory and Practice

Second Edition

SÉBASTIEN BOSSU
PHILIPPE HENROTTE

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Philippe Henrotte



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An Introduction to Equity Derivatives

Foreword

Today, equity derivatives are used by hundreds of thousands of people around the world – not only sophisticated investors such as hedge funds, institutional investors, or investment banks, but also private investors. Their popularity is due to the wide array of applications they offer: directional strategies, risk hedging, volatility trading, structured products, to name a few.

This book will be an ideal partner for anyone discovering equity derivatives or who wants to learn more about them. The text is remarkably well structured and accessible, starting with basic concepts and slowly increasing the level of complexity. The problems and accompanying solutions add a lot of insight, and the two new chapters on volatility trading and exotic options are a must-read.

I was very pleased to be asked to write the foreword of this new edition from Sébastien Bossu under the authority of my colleague Philippe Henrotte at HEC Paris. Once again, Sébastien demonstrates his ability to combine his sleek and sharp academic style together with his first-rate practical experience. Along with his constant interaction with many market practitioners, he continues to successfully leverage off his ongoing personal research on some of the most topical pricing and modeling challenges faced by our always-evolving industry.

Olivier Bossard
Senior Managing Director
Head of Derivatives Trading EMEA at Macquarie Bank

Before joining Macquarie, Olivier Bossard has developed from scratch and led over a decade the exceptional growth of Lehman's Structured Products business in Europe. He has twenty years of experience as an exotic option trader, and has also been teaching Financial Engineering at HEC Paris Business School since 1998.

Preface

For this new edition of our 2005 title *Finance and Derivatives* we have considerably redrafted our text and focused our attention on equity derivatives which is our core area of expertise. There are two new chapters, numerous chapter additions, several new problems with solutions, more figures and illustrations, and more examples. As before our aim is to suit the needs of both professionals and aspiring professionals discovering the field. No prior knowledge in finance is assumed, the only required background is an undergraduate level in mathematics.

The chapters form a sequence of gradual difficulty which we grouped within three parts:

- **Part I: Building Blocks (Chapters 1 to 4)** covers the fundamental concepts used in quantitative finance: interest rates, the time value of money, bonds and yields, portfolio valuation, risk and return, diversification.
- **Part II: First Steps in Equity Derivatives (Chapters 5 to 8)** covers forward contracts, options and option strategies, the binomial model, the lognormal model, Monte-Carlo simulations, and dynamic hedging. This part only relies on discrete time concepts in order to remain widely accessible.
- **Part III: Advanced Models and Techniques (Chapters 9 to 12)** goes one level higher into continuous time finance and covers models for asset prices, stochastic processes and calculus, the Black-Scholes model, volatility trading, exotic derivatives, and advanced models.

Our approach was to focus on the fundamentals while covering a fair amount of practical applications. We endeavored to keep our text as concise and straightforward as possible, leaving non-essential concepts and technical proofs to problems of higher difficulty which are identified with an asterisk (*).

The 2007–2008 financial crisis highlighted the fact that derivatives were often poorly understood. We do not think that the solution is to ban them altogether: when you are in the passenger seat and have just escaped a fatal car crash after speeding, you typically don't get rid of the car. Rather, we believe that more information and training is needed in the field (along with better drivers), and we hope that this new edition will prove useful and insightful to a large audience.

Disclaimer

This is a book about finance intended for professionals and future professionals. We are not trying to sell you any security, or give you any investment advice. The views expressed here are solely ours and do not necessarily reflect those of any entity directly or indirectly related to us. We took great care in proof-reading this book, but we disclaim any responsibility for any remaining errors and any use to which the contents of this book are put.

Addendum:

A Path to Economic Renaissance

The following opinion piece only reflects the personal views of the author and does not engage any other contributor to this book.

This new publication provides me with the opportunity to comment on the current economic and cultural climate, which has changed markedly since the last edition. In particular, derivatives came into the spotlight and have been heavily criticized.

I want to emphasize that equity derivatives are not inherently harmful. When used competently, derivatives can reduce risk or, more precisely, they allow investors to select certain types of risks over others. While it is true that credit derivatives compounded losses early on in the recent economic crisis, they are not to be blamed for the culture of “real estate envy”, cheap money and ostentation which then prevailed.

The crisis is far from being fully resolved. There is a distressing gap between the pessimism in mainstream political and management discourse, and the reality in large banks and corporations where profits are close to record highs and executive pay is on the rise.

To paraphrase Ronald Reagan, there is a rising sentiment that our leaders are the problem, not the solution, as expressed by many popular movements such as ‘Occupy Wall Street’. Rather than reshuffling cards in favor of the next generation – a process known as ‘creative destruction’ in Schumpeter’s theory – we just seem to be doubling down on the people who failed.

It is urgent, in my opinion, to take actions to increase the circulation of wealth in the economy in order to restore confidence in economic growth and progress. A few years ago, in a joint op-ed article published by a respected French economics newspaper, I proposed to cut on income taxes, which would give a much-needed break to the middle class, and introduce in its place a small annual tax on individual net worth (i.e. assets minus liabilities.) Unfortunately this piece was not published in equivalent newspapers or magazines in the US and the UK, perhaps because it was then perceived as too unorthodox.

Meanwhile, I have been distressed by the flurry of extravagant proposals dominating the media space: salary caps, bans on speculation, bans on derivatives, taxes on financial transactions, to name a few. All these proposals would result in costly bureaucratic rigidities at a time when we need to foster entrepreneurship, mobility and innovation.

The desire to protect consumers is of course legitimate, but the best protection is often provided through transparency. For example, I have suggested that financial retailers clearly break down the price of investments they offer between the present values of their fees and wholesale costs. This would help consumers understand how much of their money is effectually spent on financial assets, and promote competition between providers.

Every financial investment, from buying a house to purchasing Treasury bonds or options, is speculative in nature. Some people manage to get rich very quickly through talent, vision and hard work, and that's admirable. Others manage to stay rich by promoting a culture of entitlement, status quo and cronyism, and we should resist against that.

I have no doubt we will get back on track as soon as the obvious choices are made. On the corporate side, expensive and redundant management layers must be cut in order to make room for new talent. On the political and economic side, we must promote a more equitable circulation of wealth. Above all, we must begin to select leaders not only because of their performance but also based on their ethics, bearing in mind the wisdom of ancient Greek philosophers who held that virtue cannot be taught: either you have it, or you don't.

Sébastien Bossu, February 2012

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Part I

BUILDING BLOCKS

