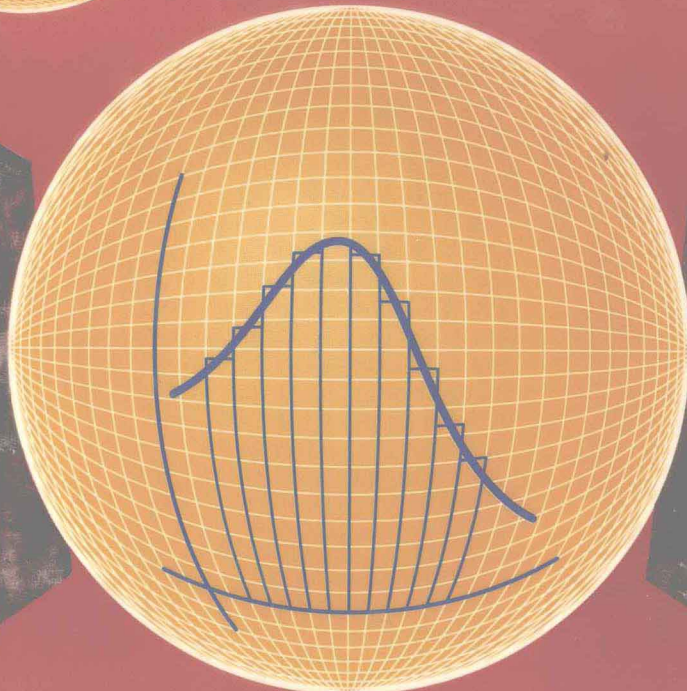
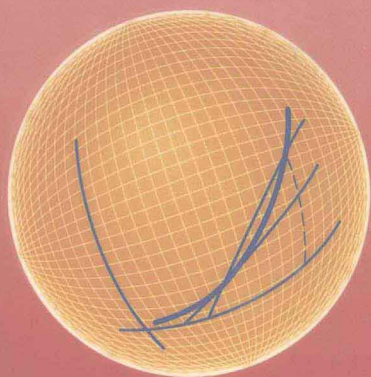


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BRIEF CALCULUS AND ITS APPLICATIONS

Fifth Edition



Goldstein • Lay • Schneider

ANNOTATED
INSTRUCTOR'S
EDITION

Brief Calculus and Its Applications

FIFTH EDITION

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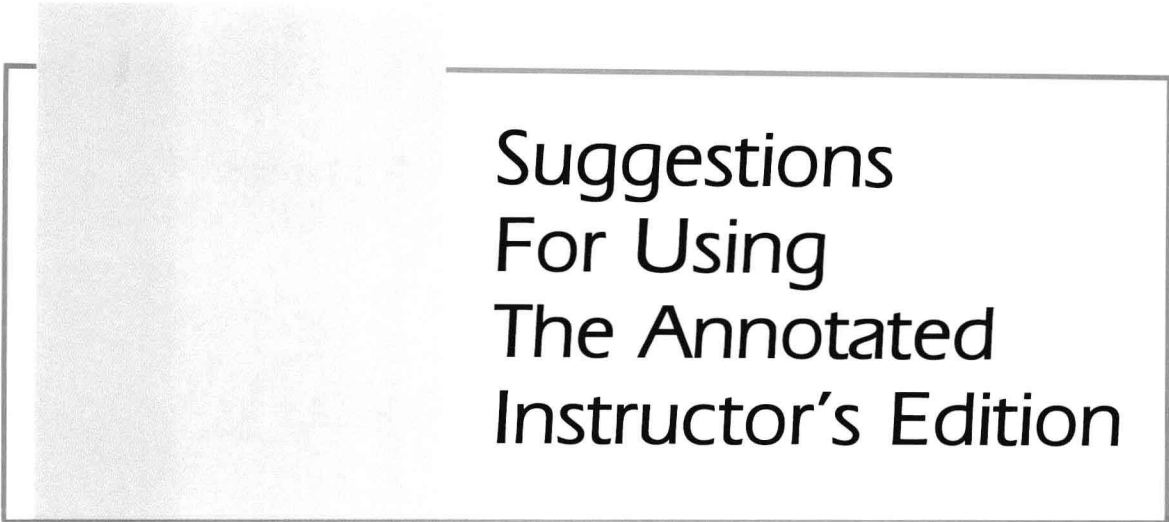
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Suggestions For Using The Annotated Instructor's Edition

Teaching calculus is challenging for all. This may be particularly true for part-time and/or new instructors. It is hoped that the annotations made in this text will assist the instructor, both old and new alike.

Some comments are designed to point out areas where students have exhibited particular problems. Others mention points to emphasize so as to make the material clearer to the students.

Still others suggest particular examples that may offer insight into the material.

These remarks are based on experience teaching the calculus and are not meant to be all-encompassing. It is hoped that instructors will review these annotations prior to presenting the material in class.

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Preface

We have been very pleased with the enthusiastic response of the fourth edition of *Brief Calculus and Its Applications* by teachers and students alike. The present work incorporates many of the suggestions they have put forward.

Although there are many changes, we have preserved the approach and the flavor. Our goals remain the same: to begin the calculus as soon as possible; to present calculus in an intuitive yet intellectually satisfying way; and to illustrate the many applications of calculus to the biological, social, and management sciences. We have tried to achieve these goals while paying close attention to students' real and potential problems in learning calculus. Our main concern, as always, is: Will it work for the students? Listed on the following pages are some of the features that illustrates various aspects of this student-oriented approach.

Applications We provide realistic applications that illustrate the uses of calculus in other disciplines. The reader may survey the variety of applications by turning to the Index of Applications on page xv. Wherever possible, we have attempted to use applications to motivate the mathematics.

Examples We have included many more worked examples than is customary (351). Furthermore, we have included computational details to enhance readability by students whose basic skills are weak.

Exercises There are more than 2000 exercises, comprising about one-quarter of the text—the most important part of the text in our opinion. The exercises at the ends of the sections are usually arranged in the order in which the text proceeds, so that the homework assignments may easily be made after only part of a section is discussed. Interesting applications and more challenging problems tend to be located near the ends of the exercise sets. Supplementary exercises at the end of each

chapter expand the other exercise sets and provide cumulative exercises that require skills from earlier chapters.

Practice Problems The practice problems introduced in the second edition have proved to be a popular and useful feature and are included in the present edition. The practice problems are carefully selected exercises that are located at the end of each section, just before the exercise set. Complete solutions are given following the exercise set. The practice problems often focus on points that are potentially confusing or are likely to be overlooked. We recommend that the reader seriously attempt the practice problems and study their solutions before moving on to the exercises. In effect, the practice problems constitute a built-in workbook.

Minimal Prerequisites In Chapter 0, we review those facts that the reader needs to study calculus. A few important topics, such as the laws of exponents, are reviewed again when they are used in a later chapter. A reader familiar with the content of Chapter 0 should begin with Chapter 1 and use Chapter 0 as a reference, whenever needed.

Numerical Methods With the common availability of microcomputers, numerical methods assume more significance than ever. We have included many discussions of numerical methods, including the differential in one variable (Section 1.7) and several variables (Section 7.5), numerical integration (Sections 6.5).

New in This Edition

Among the many changes in this edition, the following are the most significant.

1. *Additional Examples and Exercises.* The already ample stock of examples and exercises has been revised and expanded as the result of class testing. Among the new exercises are some that test understanding and other that challenge the better students.
2. *The Differential.* The discussion of the differential in one variable (Sec. 1.7) has been revised to parallel the two-variable discussion in Chapter 7. The discussion now includes the tradition Δ notation.
3. *The Definite Integral.* The definition of the definite integral has been significantly reworked. As previously, the chapter begins with antidifferentiation. However, it then proceeds to a discussion of Riemann sums and the approximation they provide to areas under curves. This is followed by the introduction of the definite integral and applications of integration. This arrangement of topics highlights the Fundamental Theorem for Calculus.
4. *Interval Notation.* In Chapter 0, we have introduced interval notation and used it to simplify and make more precise the statements of various theorems throughout the book.
5. *Limits.* Chapter 0 treats limits in the traditional order, namely before introducing the derivative.
6. *Techniques of Differentiation.* The product, quotient and chain rules are presented before curve sketching.

7. *Annotated Instructor's Edition.* This supplement features the full student text plus marginal notes for teachers including additional classroom examples, points to stress, and teaching tips. This *Annotated Instructor's Edition* also contains answers to all problems, and is not available for sale to students.
8. *Four-Color Format.* The text now incorporates the use of four colors to enhance its pedagogy and attractiveness.
9. *Instructor's Solutions Manual.* This new supplement contains worked solutions of all problems in the text.

This edition contains more material than can be covered in most one-semester courses. Optional sections are starred in the table of contents. In addition, the level of theoretical material may be adjusted to the needs of the students.

Answers to the odd-numbered exercises are included at the back of the book. Answers to the all exercises are contained in the *Annotated Instructor's Edition*. A Study Guide for students is available that contains detailed explanations and solutions of every sixth exercise. The Study Guide also includes helpful hints and strategies for studying that will help students improve their performance in the course.

We welcome any comments or suggestions you may have and hope that you enjoy using this text as much as we have enjoyed writing it.

Acknowledgements

While writing this book, we have received assistance from many persons. And our heartfelt thanks goes out to them all. Especially, we should like to thank the following reviewers, who took the time and energy to share their ideas, preferences, and often their enthusiasm, with us.

Reviewers of the first edition: Russell Lee, Allan Hancock College; Donald Hight, Kansas State College of Pittsburg; Ronald Rose, American River College; W. R. Wilson, Central Piedmont Community College; Bruce Swenson, Foothill College; Samuel Jasper, Ohio University; Carl David Minda, University of Cincinnati; H. Keith Stumpff, Central Missouri State University; Claude Schochet, Wayne State University; and James E. Huneycutt, North Carolina University.

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Larry J. Goldstein
David C. Lay
David I. Schneider

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Appendix Tables A1

Answers to Odd-Numbered Exercises A1E1

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