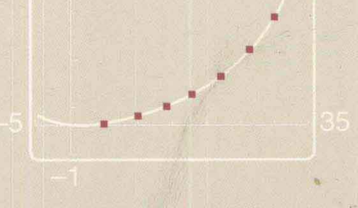
FIFTH EDITION

INTERMEDIATE

ALGEBRA

Applications, Graphs, and Models



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INTERMEDIATE ALGEBRA

Applications, Graphs, and Models

Fifth Edition

M. A. Munem
C. West
Macomb College

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Formulas—Algebra

SPECIAL PRODUCTS

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
4. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

FACTORING

1. $a^2 - b^2 = (a - b)(a + b)$
2. $a^2 \pm 2ab + b^2 = (a \pm b)^2$
3. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
4. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

EXPONENTS

1. $a^m \cdot a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $(ab)^n = a^n b^n$
4. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
5. $\frac{a^m}{a^n} = a^{m-n}$
6. $a^{-n} = \frac{1}{a^n}$
7. $a^0 = 1$

RADICALS

1. $\sqrt[n]{a^m} = a^{m/n}$
2. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$
3. $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$
4. $\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$
5. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

LOGARITHMS

1. $\log_b MN = \log_b M + \log_b N$
2. $\log_b \frac{M}{N} = \log_b M - \log_b N$
3. $\log_b M^P = P \log_b M$

INEQUALITY PROPERTIES

1. If $a < b$, then $a + c < b + c$
2. If $a < b$ and $c > 0$, then $ac < bc$
3. If $a < b$ and $c < 0$, then $ac > bc$

ABSOLUTE VALUE

1. $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$
2. If $|u| = a$ and $a > 0$, then $u = a$ or $u = -a$
3. If $|u| < a$ and $a > 0$, then $-a < u < a$.
4. If $|u| > a$ and $a > 0$, then $u < -a$ or $u > a$.

Preface

This is a major revision of the previous edition. The overriding goal of this edition is to focus on *conceptual understanding* and *applicability* of intermediate algebra. In pursuit of that goal, we have taken into account market surveys and worked closely with a talented team of reviewers, users of previous editions, and colleagues who class tested the manuscript to ensure that the book is sufficiently *flexible*. The content of this edition represents our vision of how intermediate algebra can be taught. It is flexible enough that topics can easily be added or deleted, or the order changed.

Guiding Principles

In keeping with modern reform, this edition introduces topics from four perspectives, *geometric through graphs*, *numeric through data analysis* based on real-world situations, *analytic*, and *verbal*. We have also included many examples and problems that promote *visual thinking*. The text is filled with signals to the reader. Important terms are first introduced intuitively and identified in *italics*, then shown in **boldface** when they are defined and used in theorems. Labels in the margins highlight notes to students and help them avoid common errors. Tables appear throughout that summarize the procedures and supporting examples are discussed in the book. Special attention has been paid to point out the connection between the algebraic and geometric interpretations of important concepts.

Organization

In this edition, we continue to incorporate the best aspects of reform in a meaningful yet easy-to-use manner. Here are some of the variety of organizational changes.

1. **Review Material**: The review material from beginning algebra has been placed separately in Chapter R in the beginning of the book.
2. **Functions**: Functions are introduced in Chapter 3, and they are used throughout the book. Increased emphasis has been placed on the use of functions that describe interesting real-world situations.
3. **Systems of Equations**: In this edition, systems of linear equations are introduced in Chapter 4 to provide students with problem-solving tools. This will give the student the opportunity to translate problem situations into systems of linear equations. It also provides a useful reinforcement to translating problems into linear equations in one variable discussed in Chapter 2.
4. **Graphing**: Graphing is introduced in Chapter 1 of this edition and is integrated throughout the book. Sketching graphs by hand and equation recognition are fundamental parts of learning mathematics at this level. The text uses graphing tools to supplement and extend this process. *Interpreting graphs*, *exploring new ideas from graphs*, and *extracting information from graphs* are of primary importance in this edition.

5. **Applications and Models**: One of the goals of this edition is to link the student's own experience to real-world situations. For this reason, throughout the book, we have placed emphasis on applied problems drawn from a variety of disciplines. We also believe that we offer an approach to mathematical modeling that will capture the student's attention. Based on our classroom experiences, we have selected certain types of problems and organized their solutions through a sequence of questions that help students through the difficult thought processes of developing mathematical models.
6. **Revised Examples**: This edition makes algebraic skills attainable by providing numerous illustrative examples that are presented one step at a time. All examples, whether they are drill or applied, are revised and many new examples are added. They have a title so that students can see the purpose of each example. Each section includes solving applied problems from various disciplines, thus showing students the current application of these ideas. We focus on adding examples at the end of each section that are technology oriented.
7. **Revised Problem Sets**: All problem sets have been reorganized and rewritten. They have been checked and rechecked for accuracy by professors who class-tested this edition of the book. The exercises in each problem set have been grouped into three categories:
   (a) The first category is designed to provide the practice needed to *master the concepts* in the book. They are generally modeled after the examples and follow the order of presentation of the book.
   (b) The second category provides a broad range of applications and models that require students to *apply the concepts*. Special emphasis has been put on the design of these problems.
   (c) The third category challenges students to *develop and extend the concepts*. These problems encourage *critical thinking*, and provide an excellent opportunity to stimulate class discussions that foster *collaborative learning* among students.

   The problems in the last two categories offer the opportunity for writing assignments such as a report or a class project. Also new to this edition are many modeling data problems that ask students to find and interpret mathematical models from the real-life data.
8. **Use of Technology**: We use the generic term *grapher* to refer to *graphing calculators* and *computer software*. We provide examples and problems at the end of each section where technology explorations are used. This will enable students to visualize, discover, and explore the model from graphical and numerical viewpoints. We recommend the use of a grapher—not thoughtless key stroking—and stress that its use is not a substitute for understanding the concepts involved. Quite the contrary, we believe that one usually needs to understand the concepts well in order to verify and apply the results of the technology. Although the material in this book can be covered without the use of a grapher, it is likely that many students and instructors will want to make use of these devices. To assist these students and instructors, grapher's activities are clearly identified by the symbol **G** and can be omitted without the loss of continuity, if desired. Instructors and students are expected to use their own judgment to determine what specific calculator or software to use as well as where technology is useful.
9. **New Design:** The new design is more open and readable. Each section begins with a list of specific objectives. These objectives are restated in the margin at their point of use, to provide organizational breakdown of the section. The use of color is intended to highlight the important parts of graphs and important statements. All art has been redrawn for this edition.

10. **Review Problems and Chapter Tests**: As with previous editions, each chapter concludes with a review problem set. Also, a chapter test follows the comprehensive collection of chapter review problems. Chapter tests focus on the problems in the chapter so that students can determine if they are ready to take an actual class test.

Supplements

The fifth edition is accompanied by the following supplements package.

- **For the Instructor**: An instructor's test manual includes six different test forms for each chapter of the textbook. These tests have been written at graded levels of difficulty, with three of the tests for each chapter made up of multiple choice questions, and three of standard problem-solving questions. Answers to all test questions are provided.
- **For the Student**: A student solutions guide offers worked-out solutions to every other odd-numbered problem in the book. Chapter objectives are also included.

Acknowledgments

In preparing this edition, we have drawn from our own experience in teaching intermediate algebra as well as the feedback provided by our friends, colleagues, and students. We offer them all our grateful thanks. At the risk of omitting some, they include the following:

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M. A. Munem
C. West

DISTANCE AND MIDPOINT FORMULAS

1. The distance between (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

2. The midpoint of the line segment with endpoints (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

QUADRATIC FORMULA

The solutions to $ax^2 + bx + c = 0$, with $a \neq 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

SLOPE FORMULA

Slope (m) of the line segment (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\text{change of } y}{\text{change of } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

EQUATIONS OF LINES

1. slope-intercept form

$$y = mx + b$$

2. point-slope form

$$y - y_1 = m(x - x_1)$$

3. General (or standard) form

$$Ax + By = C$$

4. Horizontal line

$$y = k$$

5. Vertical line

$$x = h$$

FUNCTIONS

1. Linear function

$$f(x) = mx + b$$

2. Quadratic function

$$f(x) = ax^2 + bx + c$$

3. Cubic function

$$f(x) = x^3$$

4. Exponential function

$$f(x) = b^x,\ b \neq 1,\ b > 0$$

5. Radical function

$$f(x) = \sqrt[n]{p(x)}, \quad p(x) \geq 0$$

6. Logarithmic function

$$f(x) = log_b\, x, \quad x > 0, \quad b > 0,\ b \neq 1$$

CONIC SECTIONS

1. Circle with center at the origin

$$x^2 + y^2 = r^2$$

2. Circle with center at (h,k)

$$(x - h)^2 + (y - k)^2 = r^2$$

3. Parabola

upward	$y - k = 4c(x - h)^2$
downward	$y - k = -4c(x - h)^2$
right	$x - h = 4c(y - k)^2$
left	$x - h = -4c(y - k)^2$

4. Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

5. Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Contents

3 Linear Graphs and Functions

4 Systems of Equations and Inequa

5 Exponents, Radicals, and Compl

6 Quadratic Equations and Functior

7 Exponential and Logarithmic Func

8 Topics in Algebra

9 Sequences, Series and the Binom

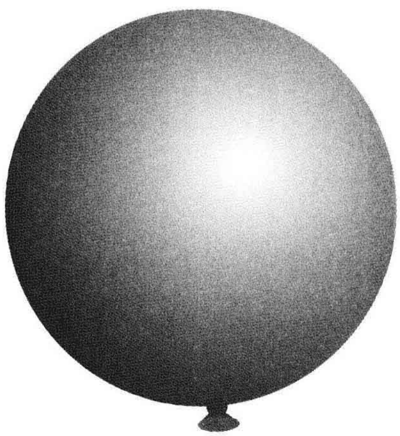
Chapter R

REVIEW OF BASIC CONCEPTS

In our daily lives, we often have to evaluate algebraic expressions or formulas by substituting numbers for the letters involved. Suppose that, a spherical balloon is being inflated with helium. Its volume V is given by the formula

$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the balloon. Find the volume (in cubic feet) of the balloon at the instant when its radius is 60 inches. Example 3 on page 29 solves this problem.



The study of Intermediate Algebra depends
ginning Algebra course. This review materi
lar topics that are important, and to establi
the key word in this introduction.

OBJECTIVES

1. Introduce Notation
2. Use Exponents
3. Use Order of Operations

R1 Notation and the Order of

Introducing Notation

Algebra begins with a systematic study of
cation, and division operations which serve
lations. In order to achieve generality, lette
and y are used to represent numbers. A lett
ber is called a **variable.** The four arithmeti
bols and words in Table 1, where a and b re

TABLE 1

Operation	Operation in Symb
Addition	$a + b$
Subtraction	$a - b$
Multiplication	$ab, a \cdot b, a(b), (a)b$
Division	$a/b, a \div b, \frac{a}{b}, b \neq$

We can use the relationship between divisi
vision involving zero. Suppose we try to di
a number represented by the letter c. That i

$$c = 5 \div ($$

this means that $5 = c \cdot 0$. This is impossit
say that $\frac{5}{0}$ is undefined. However, $0 \div 5 = 0$
In general,

Note that here $\frac{0}{0}$ is undefined.

$\frac{a}{0}$ **where $a \neq 0$ is undefi**

EXAMPLE 1 Translating Words into Symbols and Vice Versa

(a) Write the following statement in symb

The sum of 4 and twice y

(b) Express $4 + 3y > 10$ in words.

Solution (a) $4 + 2y > 5$

(b) The sum of 4 and the product of 3 and

Table 2 shows six additional symbols that a
expressions.

TABLE 2

Comparison in Symbols	
$a = b$	
$a \neq b$	
$a \leq b$	
$a < b$	
$a \geq b$	
$a > b$	
$a > 0$	
$a < 0$	
$a \geq 0$	

Using Exponents

Algebraic notation is designed to clarify id
lowing us to write expressions compactl
product $3 \cdot 3$ can be written as 3^2, read "
can be written as 3^4, read "3 to the fourth
an **exponential form**, whereas $3 \cdot 3 \cdot 3 \cdot 3$
form of 3^4 and each number is called a f
we say that the **value** of 3^4 is 81. The use o
tation for products. Thus,

$$x \cdot x = x^2, \qquad x \cdot x \cdot x = x^3,$$

In general, if n is a positive integer, the

$$x^n = \overbrace{x \cdot x \cdot x}^{n \text{ fac}}$$

In using the **exponential notation** x^n, we r
ponent or **power**. It should be noted that x

EXAMPLE 2 Using Exponential Notation

(a) Find the value of the expression 5^4.
(b) Write the expression $a \cdot b \cdot b \cdot b \cdot b \cdot c \cdot$

Solution (a) $5^4 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot = 625$

Using the Order of Operations

Consider the problem of evaluating the exp

$$2 + 3 \cdot 5$$

If we add 2 and 3 first, then multiply by 5,
3 and 5 first, then add 2, we get 17. In orde
ues for the same expression, the following s
operations should be performed.

The Order of Operations

1. **Grouping Symbols** Perform operatio
ing symbols include **parentheses** ( ), **br**
bars —. Work from the innermost grou
2. **Powers** Find the value of any powers
3. **Multiply (Divide)** Perform all multiplic
4. **Addition (Subtraction)** Perform all a
to right.

A fraction bar is a grouping symbol which groups the *numerator* and the *denominator*.

EXAMPLE 3 Using the Order of Operations

Find the value of each expression.

(a) $3 \cdot 2^3 + 10/5 - 3^2$

(b) $5[4^3 + 3(6^2 - 3 \cdot 2)]$

(c) $\dfrac{5^2 + 2 \cdot 5}{3 + 10 \div 5}$

Solution

(a) $3 \cdot 2^3 + 10/5 - 3^2$ Given

$= 3 \cdot 8 + 10/5 - 9$ Powers

$= 24 + 2 - 9 = 17$ Multiplicatio

(b) $5[4^3 + 3(6^2 - 3 \cdot 2)]$ Given

$= 5[64 + 3(36 - 6)]$ Powers

$= 5[64 + 3(30)]$ Innermost pa

$= 5[64 + 90]$ Multiplicatio

$= 5[154] = 770$ Add and mul

(c) $\dfrac{5^2 + 2 \cdot 5}{3 + 10 \div 5}$ Use of a frac

$= (25 + 10) \div (3 + 2)$ Simplify nun

$= 35 \div 5 = 7$ Divide

An expression formed from any combi
using the four operations in Table 1 as well
is called an **algebraic expression**. If an al
connected by plus or minus signs, each of t
ceding it is called a **term**. For example, $4x$
sion with the following terms:

$$4x^2, \quad 3x, \quad \text{an}$$

EXAMPLE 4 Evaluating an Expression for Specific Values

Evaluate $\frac{5x + y^2}{3}$ for $x = 12$ and $y = -3$.

Solution Using parentheses to substitute 12 for x an

$$\frac{5x + y^2}{3} = \frac{5(12) + (-3)^2}{3}$$

$= (60 + 9) / 3$

$= 69/3 = 23$ Fraction bar as

EXAMPLE 5 Translating a Situation into Symbols

The cruise control of a car is set at y miles pe
plied to drop the speed by 30 miles per hour.

(a) Write an algebraic expression that descri
(b) Find the speed while passing the truck i
 (i) 55 miles per hour
 (ii) 65 miles per hour.

Solution (a) y The initial spe
$y - 30$ The speed is d
$2(y - 30)$ The speed is d

(b) (i) If the initial speed is 55 mi/hr, t
$2(y - 30) = 2(55 - 30) = 2(25) =$
 (ii) If the initial speed is 65 mi/hr, t
$2(y - 30) = 2(65 - 30) = 2(35) =$

PROBLEM SET R1

In Problems 1–4, write each statement in symbols.

1. (a) The sum of 4 and 5 equals 9.
 (b) The sum of 5 and 4 equals 9.
 (c) The sum of 4 and 5 equals the sum of 5 and 4.
2. (a) The difference of 12 and 2 equals 10.
 (b) The difference of 2 and 12 does not equal 10.
 (c) The difference of 12 and 2 does not equal the difference of 2 and 12.
3. The product of x and 5 is greater than the quotient of x and 5.
4. The difference of twice x and 7 is less than or equal to 21.

In Problems 5–8, express each algebraic statement in words.

5. (a) $5x = 20$
 (b) $5x - 4 = 20$
6. (a) $\frac{x}{3} = 7$
 (b) $\frac{x}{3} + 2 = 7$
7. (a) $x + 2 < 10$
 (b) $10 > x + 2$
8. (a) $3x \le 15$
 (b) $15 \ge 3x$

In Problems 9 and 10, write each expression in expanded form and find its value.

In Problems 13–20, ap
value of each expressio

13. (a) $6 + 12 - 2$ -
 (b) $6 + (12 - (2$
15. (a) $3(2 + 3(7 -$
 (b) $3 \cdot 2 + 9(7$
17. $3^4 - 2^4$
19. $(9 + 6)^2/5$

In Problems 21 and 22,

21. (a) $\frac{24}{6} + 2$
 (b) $\frac{24}{6 + 2}$

In Problems 23–28, eva
given values for the var

23. $7 + 7x + 7y$ for x
25. (a) $10(x - y)$ –
 (b) $6(x - y)$ for

In Problems 29–32, write each sentence in symbols.

29. (a) The product of 5 and the sum of 3 and 4 equals 35.
 (b) The sum of the product of 5 and 3 and the product of 5 and 4 equals 35.
30. (a) The product of 3 and the difference of 6 and 4 equals 6.
 (b) The difference of the product of 3 and 6 and the product of 3 and 4 equals 6.
31. (a) One number is 10 times 20 more than another.
 (b) One number is 20 more than 10 times another.
32. (a) One number is the square of the sum of two other numbers.
 (b) One number is the sum of the squares of two other numbers.

In Problems 33 and 34, express each algebraic statement in words.

33. (a) $3y - 1 \neq 5$
 (b) $3(y - 1) = 5$

34. (a) $\frac{y}{4} + 3 = 7$
 (b) $\frac{y+3}{4} \neq 7$

In Problems 35–38, insert parentheses so that each statement is true.

35. (a) $4 + 5 \cdot 2 + 3 = 21$
 (b) $4 + 5 \cdot 2 + 3 = 45$
 (c) $4 + 5 \cdot 2 + 3 = 29$
36. (a) $12/3 + 1 + 2 = 7$
 (b) $12/3 + 1 + 2 = 5$
 (c) $12/3 + 1 + 2 = 2$
37. (a) $4 + 3 \cdot 8 - 1 + 6 = 31$
 (b) $4 + 3 \cdot 8 - 1 + 6 = 33$
 (c) $4 + 3 \cdot 8 - 1 + 6 = 7$
38. (a) $4 \cdot 6 + 1 - 5 + 4 \cdot 3 = 1$
 (b) $4 \cdot 6 + 1 - 5 + 4 \cdot 3 = 35$
 (c) $4 \cdot 6 + 1 - 5 + 4 \cdot 3 = 20$

In Problems 39–44, find the value of each expression, if possible. (Hint: Recall that division by zero is undefined.)

39. (a) $\frac{7(4-4)}{3(5+3)}$
 (b) $\frac{3(5+3)}{7(4-4)}$

40. (a) $\frac{9 + 3 \cdot 2}{24 - 8 \cdot 3}$
 (b) $\frac{24 - 8 \cdot 3}{9 + 3 \cdot 2}$

41. $\frac{25^2 + 18 \div 2 \cdot 3 + 8 \cdot 5}{2 \cdot 2}$

42. $\frac{3^2(3+2) + 3(3+2)^2}{5 \cdot 2 - 2(3-1)}$

43. $\frac{(5+7)^2 - (3 \cdot 4)^2}{5^2 + 7^2 - 3 \cdot 4^2}$

44. $\frac{5^2 + 7^2 - 3 \cdot 4^2}{(5+7)^2 - (3 \cdot 4)^2}$

In Problems 45 and 46, write the calculator key strokes necessary to find the value of each expression on your calculator.

45. (a) $3 + 4/5$
 (b) $(3 + 4)/5$
 (c) $4 + \frac{5(3-2)}{10}$

46. (a) $\frac{1}{3+4}$
 (b) $\frac{1}{3} + \frac{1}{4}$
 (c) $(4 + 17)(18 - 3)$

47. **House Value:** A house that originally costs \$95,000 doubled in value over several years. Due to a recession, it then decreased in value by \$$y$. Write an expression for the value of the house after the recession. Find its value if $y = \$5000$ and $y = \$7000$.

48. **Sunset:** A week ago the sun set at 6:55 PM. Each day for the next five days the sun sets x minutes earlier. Write an expression for the time the sun sets on the sixth day. At what time would it set if $x = 4$ minutes?

49. **Depreciation:** A computer for an office costs \$$w$ and will be used for five years at which time it will have a salvage value of \$300. If the company uses straight line depreciation (the difference of cost and salvage value divided by useful life), write an expression for this depreciation amount. If the computer costs \$4100, what is the depreciation amount?

OBJECTIVES

1. Introduce Sets
2. Create a Number Line
3. Categorize Numbers in Sets
4. Introduce the Properties of Real Numbers

R2 Properties of Real Numbers

In this section, we review sets of numbers and their properties.

Introducing the Notation of Sets

A **set** may be thought of as a collection of objects or numbers. An object in a set is called an **element** of the set. Capital letters, such as A, B, C, and D, are often used as labels for sets. Braces, { }, are used to enclose the elements of a set which are separated by commas. Thus, we write

$$A = \{1, 2, 3, 4, 5\}$$

to represent the set A whose elements are 1, 2, 3, 4, 5. The symbol $\in$ is used to mean that an element belongs to a set. The notation

$$3 \in A$$