

Scattering of Electromagnetic Waves

Theories and Applications

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SCATTERING OF ELECTROMAGNETIC WAVES

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PREFACE

Electromagnetic wave scattering is an active, interdisciplinary area of research with myriad practical applications in fields ranging from atomic physics to medical imaging to geoscience and remote sensing. In particular, the subject of wave scattering by random discrete scatterers and rough surfaces presents great theoretical challenges due to the large degrees of freedom in these systems and the need to include multiple scattering effects accurately. In the past three decades, considerable theoretical progress has been made in elucidating and understanding the scattering processes involved in such problems. Diagrammatic techniques and effective medium theories remain essential for analytical studies; however, rapid advances in computer technology have opened new doors for researchers with the full power of Monte Carlo simulations in the numerical analysis of random media scattering. Numerical simulations allow us to solve the Maxwell equations exactly without the limitations of analytical approximations, whose regimes of validity are often difficult to assess. Thus it is our aim to present in these three volumes a balanced picture of both theoretical and numerical methods that are commonly used for tackling electromagnetic wave scattering problems. While our book places an emphasis on remote sensing applications, the materials covered here should be useful for students and researchers from a variety of backgrounds as in, for example, composite materials, photonic devices, optical thin films, lasers, optical tomography, and X-ray lithography. Introductory chapters and sections are also added so that the materials can be readily understood by graduate students. We hope that our book would help stimulate new ideas and innovative approaches to electromagnetic wave scattering in the years to come.

The increasingly important role of numerical simulations in solving electromagnetic wave scattering problems has motivated us to host a companion web site that contains computer codes on topics relevant to the book. These computer codes are written in the MATLAB programming language and are available for download from our web site at www.emwave.com. They are provided to serve two main purposes. The first is to supply our readers a hands-on laboratory for performing numerical experiments, through which the concepts in the book can be more dynamically relayed. The second is to give new researchers a set of basic tools with which they could quickly build on projects of their own. The fluid nature of the web site would also allow us to regularly update the contents and keep pace with new research developments.

The present volume covers the basic principles and applications of electromagnetic wave scattering and lays the groundwork for the study of more advanced topics in Volumes II and III. We start in Chapter 1 with exact and approximate solutions of wave scattering by a single particle of simple shape. Such problems can be solved exactly by expanding the fields in terms of scalar or vector waves in separable coordinates, depending on the geometry of the scatterer. When the size of the scatterer is small, Rayleigh scattering represents a simple and valid approximation. When scattering is weak, the Born approximation can be applied to the volume integral equation for the internal field. Approximate solutions are also useful when the scatterer lacks perfect symmetry as in the case of a finite cylinder. In Chapter 2, we discuss basic scattering theory. We introduce the Green's function for the wave equation and its various coordinate representations. From the vector Green's theorem, we derive the Huygens' Principle and the extinction theorem, which are especially useful for formulating surface integral equations in scattering problems. The reciprocity principle leads to useful symmetry relations in the scattering amplitudes and the Green's function, while energy conservation leads to the optical theorem. The T-matrix formulation with the extended boundary condition technique is a popular method that can be used to calculate scattering from an arbitrarily shaped object. We give explicit results for dielectric spheres and spheroids.

In Chapter 3, we begin the study of electromagnetic scattering by a random collection of scatterers. The concepts of fluctuating fields and ensemble averaging are of central importance in random media scattering. These and related ideas are explored in this chapter. The specific intensity is often used to describe energy transport through a random medium. The fully polarimetric description of the specific intensity is provided by the Stokes vector. As an application to passive remote sensing, we derive the emissivity of the four Stokes parameters using the fluctuation dissipation theorem. Basic radiative transfer (RT) theory elements including the extinction coefficient and scattering phase matrix are also introduced. In contrast to conventional RT theory, the phase functions are defined in terms of bistatic scattering cross sections. This allows for the development of the dense medium radiative transfer theory (DMRT) to be discussed in Volumes II and III. Many natural media, e.g., snow, vegetation, and ocean surfaces, can be effectively modeled in terms of simple random media. Chapter 4 is devoted to the statistical characterizations of such random discrete media and rough surfaces. Useful characterizations include the pair distribution function for volume scatterers and the power spectrum for rough surfaces.

In Chapter 5, we consider scattering and emission by plane-parallel layered media, which provide simple but very useful models for geophysical remote sensing. We solve this problem in two different ways: the coherent or wave approach, which is exact, versus the incoherent or radiative transfer approach. This gives us some insights into the approximations involved in RT theory. In Chapter 6, we discuss the single scattering approximation, where each particle is assumed to scatter independently. However, we take into account of the phase coherence in the addition of scattered fields. We demonstrate the existence of an interesting correlation effect in random media scattering known as the *memory effect*. As will be shown in Volumes II and III, this effect persists even when multiple scattering is included. Applications of single scattering to synthetic aperture radar (SAR) and random media scattering are also discussed.

In Chapters 7 and 8, we take a closer look at the radiative transfer equation and its solutions. The iterative method is useful when scattering is weak and provides physical correspondence with different orders of multiple scattering. When scattering is strong, the discrete ordinate eigenanalysis approach can be used to obtain numerically exact solutions. For scattering media with inhomogeneous profiles, the method of invariant imbedding can be applied. Diffusion approximation is useful when, upon multiple scattering, the intensities have been diffused almost uniformly in all directions. We illustrate these solution techniques with extensive examples from active and passive microwave remote sensing.

In Chapter 9, we discuss wave scattering by random rough surfaces. Despite much theoretical and numerical efforts, the two "classical" analytical approximations of small perturbation method and Kirchhoff approach are still the simplest and most widely used analytical methods for solving rough surface problems. Here they are illustrated using one-dimensional rough surfaces with Dirichlet and Neumann boundary conditions. Two-dimensional rough surface scattering problems are discussed extensively in Volumes II and III.

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Chapter 1

INTRODUCTION TO ELECTROMAGNETIC SCATTERING BY A SINGLE PARTICLE

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A major topic in this book is the study of propagation and scattering of waves by randomly distributed particles. We first consider scattering by a single particle. This chapter and the next discuss and derive the scattering characteristics of a single particle. Both exact and solutions are studied. Scattering by a single particle is an important subject in electromagnetics and optics. There exist several excellent textbooks on this subject [van de Hulst, 1957; Kerker, 1969; Bohren and Huffman, 1983]. We will treat those topics that are pertinent to later chapters of multiple scattering by random discrete scatterers.

1 Basic Scattering Parameters

1.1 Scattering Amplitudes and Cross Sections

Consider an electromagnetic plane wave impinging upon a particle which has permittivity $\epsilon_p(\bar{r})$ that is different from the background permittivity ϵ (Fig. 1.1.1). The finite support of $\epsilon_p(\bar{r}) - \epsilon$ is denoted as V .

The incident wave is in direction \hat{k}_i and has electric field in direction \hat{e}_i that is perpendicular to \hat{k}_i . The electric field of the incident wave is

$$\bar{E}_i = \hat{e}_i E_o e^{ik\hat{k}_i \cdot \bar{r}} \quad (1.1.1)$$

where

$$\bar{r} = x\hat{x} + y\hat{y} + z\hat{z} \quad (1.1.2)$$

is the position vector, and

$$k = \omega\sqrt{\mu\epsilon} = \frac{2\pi}{\lambda} \quad (1.1.3)$$

is the wavenumber. In (1.1.3), ω is the angular frequency, μ is the permeability, and λ is the wavelength. In (1.1.1), E_o is the amplitude of the electric field. The time harmonic dependence $\exp(-i\omega t)$ has been suppressed.

In the far field, the scattered field is that of a spherical wave with dependence e^{ikr}/r , where r is the distance from the particle. In general, the particle scatters waves in all directions. Let \bar{E}_s be the far field scattered field in direction of \hat{k}_s . Since Maxwell's equations are linear, we write

$$\bar{E}_s = \hat{e}_s f(\hat{k}_s, \hat{k}_i) E_o \frac{e^{ikr}}{r} \quad (1.1.4)$$

where \hat{e}_s is perpendicular to \hat{k}_s . The proportionality $f(\hat{k}_s, \hat{k}_i)$ is called the scattering amplitude from direction \hat{k}_i into direction \hat{k}_s .