Applied and Numerical Harmonic Analysis

Advances in Mathematical Finance

Michael C. Fu
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Editors

Birkhäuser

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Applied and Numerical Harmonic Analysis

In honor of Dilip B. Madan on the occasion of his 60th birthday

ANHA Series Preface

The *Applied and Numerical Harmonic Analysis (ANHA)* book series aims to provide the engineering, mathematical, and scientific communities with significant developments in harmonic analysis, ranging from abstract harmonic analysis to basic applications. The title of the series reflects the importance of applications and numerical implementation, but richness and relevance of applications and implementation depend fundamentally on the structure and depth of theoretical underpinnings. Thus, from our point of view, the interleaving of theory and applications and their creative symbiotic evolution is axiomatic.

Harmonic analysis is a wellspring of ideas and applicability that has flourished, developed, and deepened over time within many disciplines and by means of creative cross-fertilization with diverse areas. The intricate and fundamental relationship between harmonic analysis and fields such as signal processing, partial differential equations (PDEs), and image processing is reflected in our state-of-the-art *ANHA* series.

Our vision of modern harmonic analysis includes mathematical areas such as wavelet theory, Banach algebras, classical Fourier analysis, time-frequency analysis, and fractal geometry, as well as the diverse topics that impinge on them.

For example, wavelet theory can be considered an appropriate tool to deal with some basic problems in digital signal processing, speech and image processing, geophysics, pattern recognition, biomedical engineering, and turbulence. These areas implement the latest technology from sampling methods on surfaces to fast algorithms and computer vision methods. The underlying mathematics of wavelet theory depends not only on classical Fourier analysis, but also on ideas from abstract harmonic analysis, including von Neumann algebras and the affine group. This leads to a study of the Heisenberg group and its relationship to Gabor systems, and of the metaplectic group for a meaningful interaction of signal decomposition methods. The unifying influence of wavelet theory in the aforementioned topics illustrates the justification for providing a means for centralizing and disseminating information from the broader, but still focused, area of harmonic analysis. This will be a key role of *ANHA*. We intend to publish with the scope and interaction that such a host of issues demands.

Along with our commitment to publish mathematically significant works at the frontiers of harmonic analysis, we have a comparably strong commitment to publish major advances in the following applicable topics in which harmonic analysis plays a substantial role:

Antenna theory	*Prediction theory*
Biomedical signal processing	*Radar applications*
Digital signal processing	*Sampling theory*
Fast algorithms	*Spectral estimation*
Gabor theory and applications	*Speech processing*
Image processing	*Time-frequency and*
Numerical partial differential equations	*time-scale analysis*
	Wavelet theory

The above point of view for the *ANHA* book series is inspired by the history of Fourier analysis itself, whose tentacles reach into so many fields.

In the last two centuries Fourier analysis has had a major impact on the development of mathematics, on the understanding of many engineering and scientific phenomena, and on the solution of some of the most important problems in mathematics and the sciences. Historically, Fourier series were developed in the analysis of some of the classical PDEs of mathematical physics; these series were used to solve such equations. In order to understand Fourier series and the kinds of solutions they could represent, some of the most basic notions of analysis were defined, e.g., the concept of "function." Since the coefficients of Fourier series are integrals, it is no surprise that Riemann integrals were conceived to deal with uniqueness properties of trigonometric series. Cantor's set theory was also developed because of such uniqueness questions.

A basic problem in Fourier analysis is to show how complicated phenomena, such as sound waves, can be described in terms of elementary harmonics. There are two aspects of this problem: first, to find, or even define properly, the harmonics or spectrum of a given phenomenon, e.g., the spectroscopy problem in optics; second, to determine which phenomena can be constructed from given classes of harmonics, as done, for example, by the mechanical synthesizers in tidal analysis.

Fourier analysis is also the natural setting for many other problems in engineering, mathematics, and the sciences. For example, Wiener's Tauberian theorem in Fourier analysis not only characterizes the behavior of the prime numbers, but also provides the proper notion of spectrum for phenomena such as white light; this latter process leads to the Fourier analysis associated with correlation functions in filtering and prediction problems, and these problems, in turn, deal naturally with Hardy spaces in the theory of complex variables.

Nowadays, some of the theory of PDEs has given way to the study of Fourier integral operators. Problems in antenna theory are studied in terms of unimodular trigonometric polynomials. Applications of Fourier analysis abound in signal processing, whether with the fast Fourier transform (FFT), or filter design, or the adaptive modeling inherent in time-frequency-scale methods such as wavelet theory. The coherent states of mathematical physics are translated and modulated Fourier transforms, and these are used, in conjunction with the uncertainty principle, for dealing with signal reconstruction in communications theory. We are back to the raison d'être of the *ANHA* series!

John J. Benedetto
Series Editor
University of Maryland
College Park

Preface

The "Mathematical Finance Conference in Honor of the 60th Birthday of Dilip B. Madan" was held at the Norbert Wiener Center of the University of Maryland, College Park, from September 29 – October 1, 2006, and this volume is a Festschrift in honor of Dilip that includes articles from most of the conference's speakers. Among his former students contributing to this volume are Ju-Yi Yen as one of the co-editors, along with Ali Hirsa and Xing Jin as co-authors of three of the articles.

Dilip Balkrishna Madan was born on December 12, 1946, in Washington, DC, but was raised in Bombay, India, and received his bachelor's degree in Commerce at the University of Bombay. He received two Ph.D.s at the University of Maryland, one in economics and the other in pure mathematics. What is all the more amazing is that prior to entering graduate school he had never had a formal university-level mathematics course! The first section of the book summarizes Dilip's career highlights, including distinguished awards and editorial appointments, followed by his list of publications.

The technical contributions in the book are divided into three parts. The first part deals with stochastic processes used in mathematical finance, primarily the Lévy processes most associated with Dilip, who has been a fervent advocate of this class of processes for addressing the well-known flaws of geometric Brownian motion for asset price modeling. The primary focus is on the Variance-Gamma (VG) process that Dilip and Eugene Seneta introduced to the finance community, and the lead article provides an historical review from the unique vantage point of Dilip's co-author, starting from the initiation of the collaboration at the University of Sydney. Techniques for simulating the Variance-Gamma process are surveyed in the article by Michael Fu, Dilip's longtime colleague at Maryland, moving from a review of basic Monte Carlo simulation for the VG process to more advanced topics in variation reduction and efficient estimation of the "Greeks" such as the option delta. The next two pieces by Marc Yor, a longtime close collaborator and the keynote speaker at the birthday conference, provide some mathematical properties and identities for gamma processes and beta and gamma random variables. The final article in the first part of the volume, written by frequent collaborator Robert Elliott and his co-author John van der Hoek, reviews the theory of fractional Brownian motion in the white noise framework and provides a new approach for deriving the associated Itô-type stochastic calculus formulas.

The second part of the volume treats various aspects of mathematical finance related to asset pricing and the valuation and hedging of derivatives. The article by Bob Jarrow, a longtime collaborator and colleague of Dilip in the mathematical finance community, provides a tutorial on zero volatility spreads and option adjusted spreads for fixed income securities – specifically bonds with embedded options – using the framework of the Heath-Jarrow-Morton model for the term structure of interest rates, and highlights the characteristics of zero volatility spreads capturing *both* embedded options and mispricings due to model or market errors, whereas option adjusted spreads measure only the mispricings. The phenomenon of market bubbles is addressed in the piece by Bob Jarrow, Phillip Protter, and Kazuhiro Shimbo, who provide new results on characterizing asset price bubbles in terms of their martingale properties under the standard no-arbitrage complete market framework. General equilibrium asset pricing models in incomplete markets that result from taxation and transaction costs are treated in the article by Xing Jin – who received his Ph.D. from Maryland's Business School co-supervised by Dilip – and Frank Milne – one of Dilip's early collaborators on the VG model. Recent work on applying Lévy processes to interest rate modeling, with a focus on real-world calibration issues, is reviewed in the article by Wolfgang Kluge and Ernst Eberlein, who nominated Dilip for the prestigious Humboldt Research Award in Mathematics. The next two articles, both co-authored by Ali Hirsa, who received his Ph.D. from the math department at Maryland co-supervised by Dilip, focus on derivatives pricing; the sole article in the volume on which Dilip is a co-author, with Massoud Heidari as the other co-author, prices swaptions using the fast Fourier transform under an affine term structure of interest rates incorporating stochastic volatility, whereas the article co-authored by Peter Carr – another of Dilip's most frequent collaborators – derives forward partial integro-differential equations for pricing knock-out call options when the underlying asset price follows a jump-diffusion model. The final article in the second part of the volume is by Hélyette Geman, Dilip's longtime collaborator from France who was responsible for introducing him to Marc Yor, and she treats energy commodity price modeling using real historical data, testing the hypothesis of mean reversion for oil and natural gas prices.

The third part of the volume includes several contributions in one of the most rapidly growing fields in mathematical finance and financial engineering: credit risk. A new class of reduced-form credit risk models that associates default events directly with market information processes driving cash flows is introduced in the piece by Dorje Brody, Lane Hughston, and Andrea Macrina. A generic one-factor Lévy model for pricing collateralized debt obligations that unifies a number of recently proposed one-factor models is presented in the article by Hansjörg Albrecher, Sophie Ladoucette, and Wim Schoutens. An intensity-based default model that prices credit derivatives using utility functions rather than arbitrage-free measures is proposed in the article by Ronnie Sircar and Thaleia Zariphopoulou. Also using the utility-based pricing approach is the final article in the volume by Marek Musiela and Thaleia Zariphopoulou, and they address the integrated portfolio management optimal investment problem in incomplete markets stemming from stochastic factors in the underlying risky securities.

Besides being a distinguished researcher, Dilip is a dear friend, an esteemed colleague, and a caring mentor and teacher. During his professional career, Dilip was one of the early pioneers in mathematical finance, so it is only fitting that the title of this Festschrift documents his past and continuing love for the field that he helped develop.

Michael Fu
Bob Jarrow
Ju-Yi Yen
Robert Elliott
December 2006

MATHEMATICAL FINANCE CONFERENCE in honor of the 60th birthday of Dilip B. Madan

KEYNOTE SPEAKER:
Marc Yor (Université Paris VI, France)

INVITED SPEAKERS:

Peter Carr (Bloomberg LP and Courant Institute)
Freddy Delbaen (ETH-Zurich, Switzerland)
Bruno Dupire (Bloomberg LP)
Ernst Eberlein (University of Freiburg, Germany)
Robert Elliott (University of Calgary, Canada)
Hélyette Geman (Université Paris IX Dauphine and ESSEC, France)
Ali Hirsa (Caspian Capital)
Lane Hughston (King's College London)
Robert Jarrow (Cornell University)
Ajay Khanna
Andreas Kyprianou (University of Bath, United Kingdom)
Frank Milne (Queen's University, Canada)
Marek Musiela (BNP Paribas, UK)
Philip Protter (Cornell University)
Wim Schoutens (Katholieke Universiteit Leuven, Belgium)
Eugene Seneta (University of Sydney, Australia)
Thaleia Zariphopoulou (University of Texas at Austin)

Organizers:
John J. Benedetto (University of Maryland)
Peter Carr (Bloomberg LP and Courant Institute)
Michael C. Fu (University of Maryland)
Ioannis Konstantinidis (University of Maryland)
Ju-Yi Yen (Vanderbilt University and University of Maryland)

Hosted by the Norbert Wiener Center

Sponsored in part by the Department of Finance,
Robert H. Smith School of Business,
University of Maryland

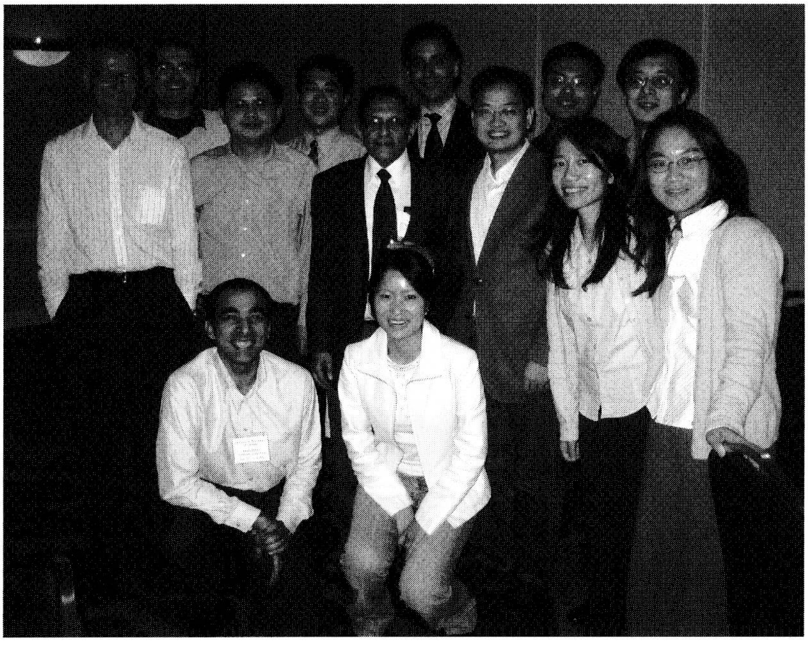
SEPTEMBER 29 - OCTOBER 1, 2006

www.norbertwiener.umd.edu/Madan/

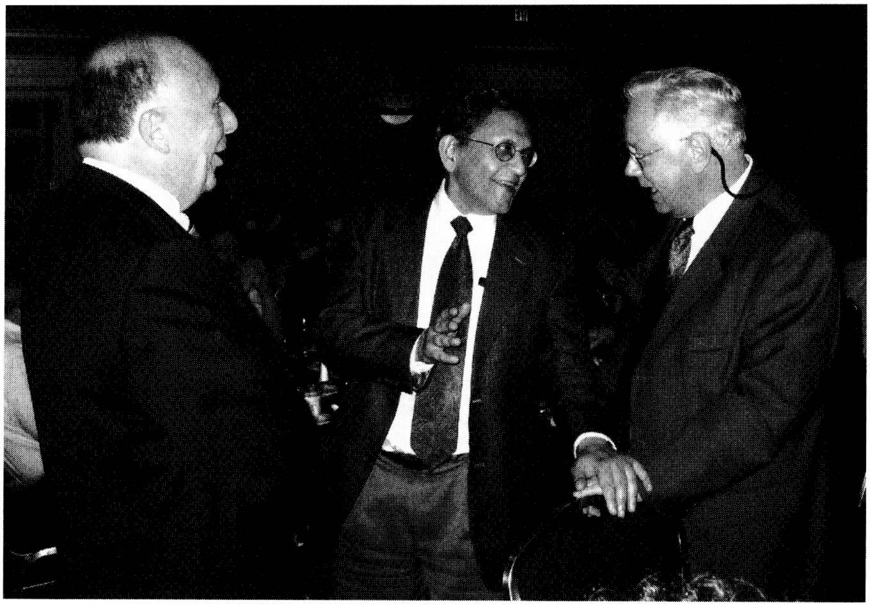
Conference poster (designed by Jonathan Sears).

Photo Highlights (September 29, 2006)

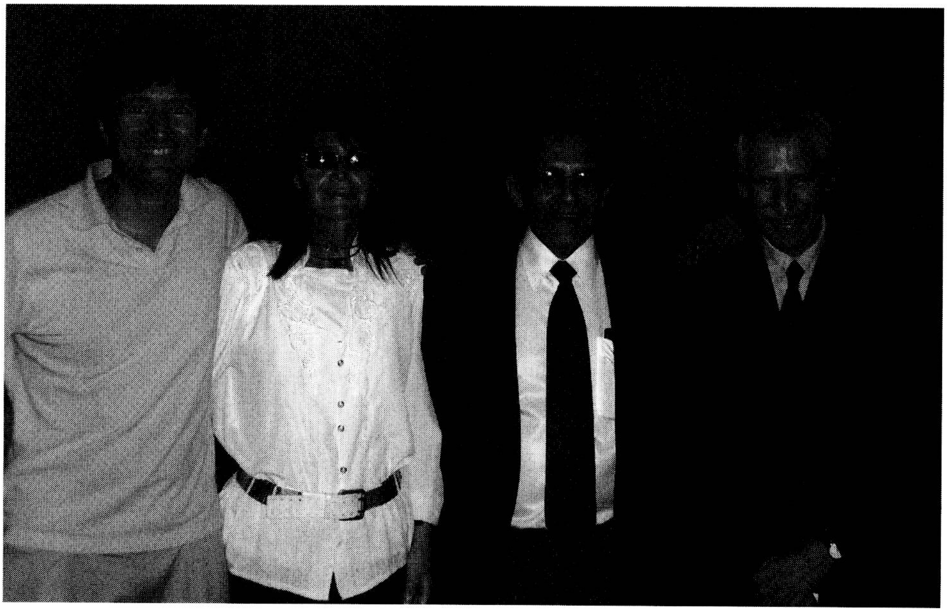
Dilip delivering his lecture.



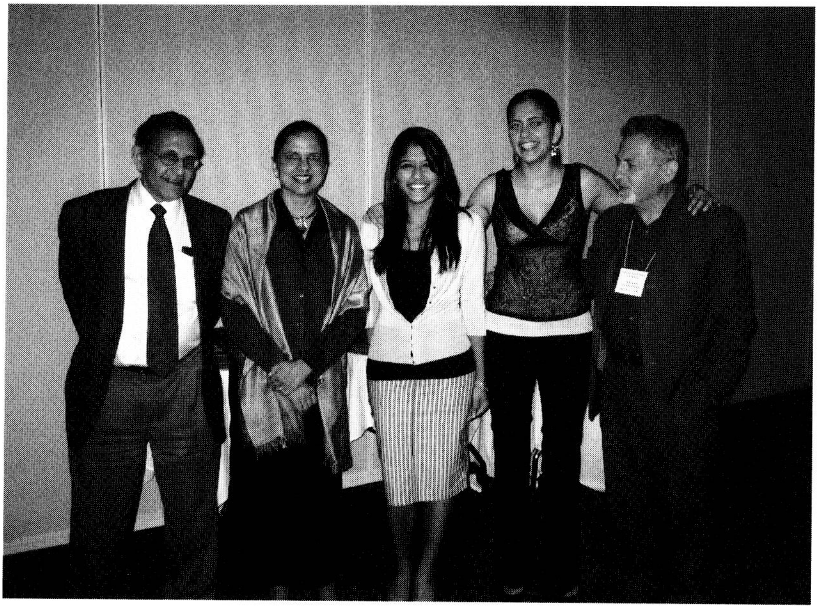
Dilip with many of his Ph.D. students.



Norbert Wiener Center director John Benedetto and Robert Elliott.



Left to right: CGMY (Carr, Geman, Madan, Yor).



VG inventors (Dilip and Eugene Seneta) with the Madan family.

Dilip's wife Vimla cutting the birthday cake.

Career Highlights and List of Publications

Dilip B. Madan

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Career Highlights

1971 Ph.D. Economics, University of Maryland
1975 Ph.D. Mathematics, University of Maryland

2006 recipient of Humboldt Research Award in Mathematics
President of Bachelier Finance Society 2002–2003
Managing Editor of *Mathematic Finance, Review of Derivatives Research*
Series Editor on Financial Mathematics for CRC, Chapman and Hall
Associate Editor for *Quantitative Finance, Journal of Credit Risk*

1971–1975: Assistant Professor of Economics, University of Maryland
1976–1979: Lecturer in Economic Statistics, University of Sydney
1980–1988: Senior Lecturer in Econometrics, University of Sydney
1981–1982: Acting Head, Department of Econometrics, Sydney
1989–1992: Assistant Professor of Finance, University of Maryland
1992–1997: Associate Professor of Finance, University of Maryland
1997–present: Professor of Finance, University of Maryland

Visiting Positions:
La Trobe University, Cambridge University (Isaac Newton Institute), Cornell University, University Paris,VI, University of Paris IX at Dauphine

Consulting:
Morgan Stanley, Bloomberg, Wachovia Securities, Caspian Capital, FDIC