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Calculus Essentials

FOR DUMMIES®

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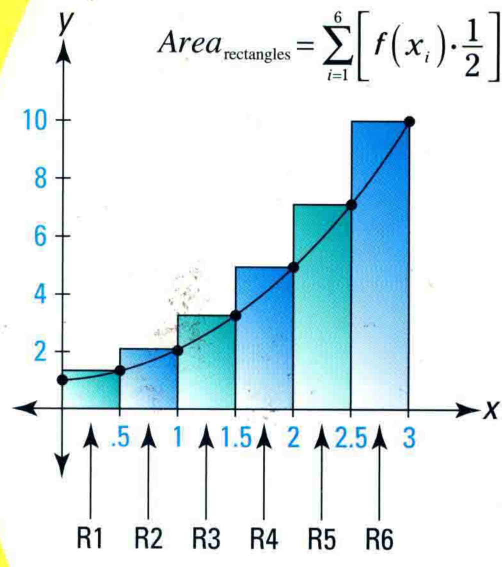
- **Exactly what you need to know to conquer calculus**
- **The "must-know" formulas and equations**
- **Core calculus topics in quick, focused lessons**



Mark Ryan

Founder and owner of The Math Center, author of Calculus For Dummies *and* Calculus Workbook For Dummies

Calculus Essentials FOR DUMMIES®

by Mark Ryan

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About the Author

Mark Ryan, a graduate of Brown University and the University of Wisconsin Law School, has been teaching math since 1989. He runs the Math Center in Winnetka, Illinois (`www.themathcenter.com`), where he teaches high school math courses including an introduction to calculus and a workshop for parents based on a program he developed, *The 10 Habits of Highly Successful Math Students*. He also does extensive one-to-one tutoring for all levels of mathematics and for standardized test preparation. In high school he twice scored a perfect 800 on the math portion of the SAT, and he not only knows mathematics, he has a gift for explaining it in plain English. He practiced law for four years before deciding he should do something he enjoys and use his natural talent for mathematics. Ryan is a member of the Authors Guild and the National Council of Teachers of Mathematics.

Calculus Essentials For Dummies is Ryan's sixth book. *Everyday Math for Everyday Life* was published in 2002, *Calculus For Dummies* (Wiley) in 2003, *Calculus Workbook For Dummies* (Wiley) in 2005, *Geometry Workbook For Dummies* (Wiley) in 2007, and *Geometry For Dummies,* 2nd Ed. (Wiley) in 2008. His math books have sold over a quarter of a million copies.

Publisher's Acknowledgments

We're proud of this book; please send us your comments at http://dummies.custhelp.com. For other comments, please contact our Customer Care Department within the U.S. at 877-762-2974, outside the U.S. at 317-572-3993, or fax 317-572-4002.

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Contents at a Glance

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Introduction

The mere thought of having to take a required calculus course is enough to make legions of students break out in a cold sweat. Others who have no intention of ever studying the subject have this notion that calculus is impossibly difficult unless you happen to be a direct descendant of Einstein.

Well, I'm here to tell you that you *can* master calculus. It's not nearly as tough as its mystique would lead you to think. Much of calculus is really just very advanced algebra, geometry, and trig. It builds upon and is a logical extension of those subjects. If you can do algebra, geometry, and trig, you can do calculus. Read this jargon-free book, get a handle on calculus, and join the happy few who can proudly say, "Calculus? Oh, sure, I know calculus. It's no big deal."

About This Book

Calculus Essentials For Dummies is intended for three groups of readers: students taking their first calculus course, students who need to brush up on their calculus to prepare for other studies, and adults of all ages who'd like a good introduction to the subject. For those who'd like a fuller treatment of the subject, check out *Calculus For Dummies*.

If you're enrolled in a calculus course and you find your textbook less than crystal clear, *Calculus Essentials For Dummies* is the book for you. It covers the two most important topics in the first year of calculus: differentiation and integration.

If you've had elementary calculus, but it's been a couple of years and you want to review the concepts to prepare for, say, some graduate program, *Calculus Essentials For Dummies* will give you a quick, no-nonsense refresher course.

Nonstudent readers will find the book's exposition clear and accessible. *Calculus Essentials For Dummies* takes calculus out of the ivory tower and brings it down to earth.

This is a user-friendly math book. Whenever possible, I explain the calculus concepts by showing you connections between the calculus ideas and easier ideas from algebra and geometry. I then show you how the calculus concepts work in concrete examples. Only later do I give you the fancy calculus formulas. All explanations are in plain English, not math-speak.

Conventions Used in This Book

The following conventions keep the text consistent and oh-so-easy to follow.

- Variables are in *italics*.
- Calculus terms are italicized and defined when they first appear in the text.
- In the step-by-step problem-solving methods, the general action you need to take is in **bold**, followed by the specifics of the particular problem.

Foolish Assumptions

Call me crazy, but I assume . . .

- You know at least the basics of algebra, geometry, and trig.

  If you're rusty, you might want to brush up a bit on these pre-calculus topics. Actually, if you're not currently taking a calculus course, and you're reading this book just to satisfy a general curiosity about calculus, you can get a good conceptual picture of the subject without the nitty-gritty details of algebra, geometry, and trig. But you won't, in that case, be able to follow all the problem solutions. In short, without the pre-calculus stuff, you can see the calculus forest, but not the trees. If you are enrolled in a calculus course, you've got no choice — you've got to know the trees.

- You're willing to do some w_ _ _.

  No, not the dreaded w-word! Yes, that's w-o-r-k, work. I've tried to make this material as accessible as possible, but it is calculus after all. You can't learn calculus by just listening to a tape in your car or taking a pill — not yet anyway.

Icons Used in This Book

Keep your eyes on the icons:

Next to this icon are the essential calculus rules, definitions, and formulas you should definitely know.

These are things you need to know from algebra, geometry, or trig, or things you should recall from earlier in the book.

The bull's-eye icon appears next to things that will make your life easier. Take note.

This icon highlights common calculus mistakes. Take heed.

In contrast to the Critical Calculus Concepts, you generally don't need to memorize the fancy-pants formulas next to this icon unless your calc teacher insists.

Where to Go from Here

Why, Chapter 1, of course, if you want to start at the beginning. If you already have some background in calculus or just need a refresher course in one area or another, then feel free to skip around. Use the table of contents and index to find what you're looking for. If all goes well, in a half a year or so, you'll be able to check calculus off your bucket list:

___ Run a marathon

___ Go skydiving

___ Write a book

_x_ Learn calculus

___ Swim the English Channel

___ Cure cancer

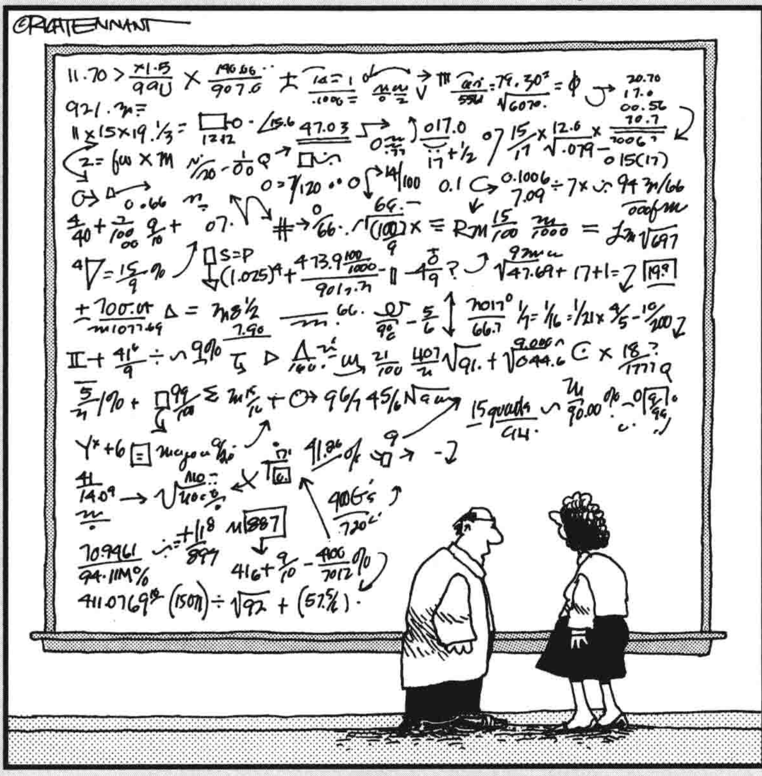
___ Write a symphony

___ Pull an inverted 360° at the X-Games

For the rest of your list, you're on your own.

The 5th Wave

By Rich Tennant



"What exactly are we saying here?"

Chapter 1

Calculus: No Big Deal

In This Chapter

- Calculus — it's just souped-up regular math
- Zooming in is the key
- Delving into the derivative: It's a rate or a slope
- Investigating the integral — addition for experts

In this chapter, I answer the question "What is calculus?" in plain English and give you real-world examples of how it's used. Then I introduce you to the two big ideas in calculus: differentiation and integration. Finally, I explain why calculus works. After reading this chapter, you *will* understand what calculus is all about.

So What Is Calculus Already?

Calculus is basically just very advanced algebra and geometry. In one sense, it's not even a new subject — it takes the ordinary rules of algebra and geometry and tweaks them so that they can be used on more complicated problems. (The rub, of course, is that darn *other* sense in which it *is* a new and more difficult subject.)

Look at Figure 1-1. On the left is a man pushing a crate up a straight incline. On the right, the man is pushing the same crate up a curving incline. The problem, in both cases, is to determine the amount of energy required to push the crate to the top. You can do the problem on the left with regular math. For the one on the right, you need calculus (if you don't know the physics shortcuts).

For the straight incline, the man pushes with an *unchanging* force, and the crate goes up the incline at an *unchanging* speed. With some simple physics formulas and regular math (including algebra and trig), you can compute how many calories of energy are required to push the crate up the incline. Note that the amount of energy expended each second remains the same.

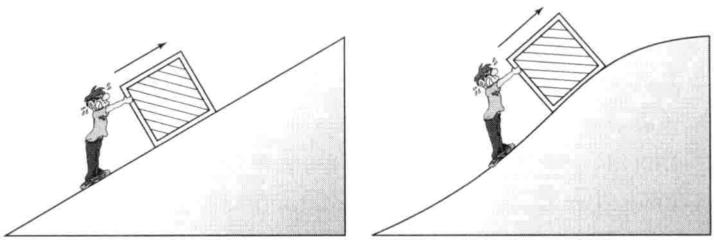


Figure 1-1: The difference between regular math and calculus: In a word, it's the curve.

For the curving incline, on the other hand, things are constantly *changing*. The steepness of the incline is *changing* — and it's not like it's one steepness for the first 3 feet and then a different steepness for the next 3 — it's *constantly changing*. And the man pushes with a *constantly changing* force — the steeper the incline, the harder the push. As a result, the amount of energy expended is also *changing*, not just every second or thousandth of a second, but *constantly*, from one moment to the next. That's what makes it a calculus problem. It should come as no surprise to you, then, that calculus is called "the mathematics of change." Calculus takes the regular rules of math and applies them to fluid, evolving problems.

For the curving incline problem, the physics formulas remain the same, and the algebra and trig you use stay the same. The difference is that — in contrast to the straight incline problem, which you can sort of do in a single shot — you've got to break up the curving incline problem into small chunks and do each chunk separately. Figure 1-2 shows a small portion of the curving incline blown up to several times its size.

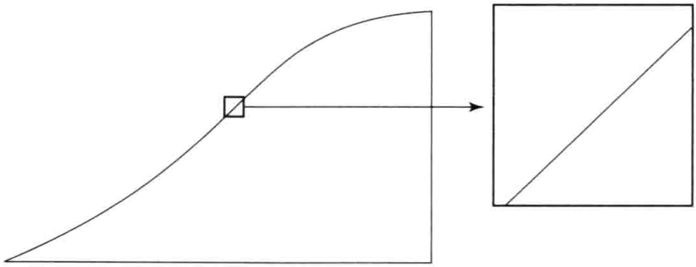


Figure 1-2: Zooming in on the curve — voilà, it's straight (almost).

When you zoom in far enough, the small length of the curving incline becomes practically straight. Then you can solve that small chunk just like the straight incline problem. Each small chunk can be solved the same way, and then you just add up all the chunks.

That's calculus in a nutshell. It takes a problem that can't be done with regular math because things are constantly changing — the changing quantities show up on a graph as curves — it zooms in on the curve till it becomes straight, and then it finishes off the problem with regular math.

What makes calculus such a fantastic achievement is that it does what seems impossible: it zooms in *infinitely*. As a matter of fact, everything in calculus involves infinity in one way or another, because if something is constantly changing, it's changing infinitely often from each infinitesimal moment to the next.

Real-World Examples of Calculus

So, with regular math you can do the straight incline problem; with calculus you can do the curving incline problem. With regular math you can determine the length of a buried cable that runs diagonally from one corner of a park to the other (remember the Pythagorean Theorem?). With calculus you can determine the length of a cable hung between two towers that has the shape of a *catenary* (which is different, by the way, from a simple circular arc or a parabola). Knowing the exact length is of obvious importance to a power company planning hundreds of miles of new electric cable.