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EIGHTH EDITION

Mathematical Ideas


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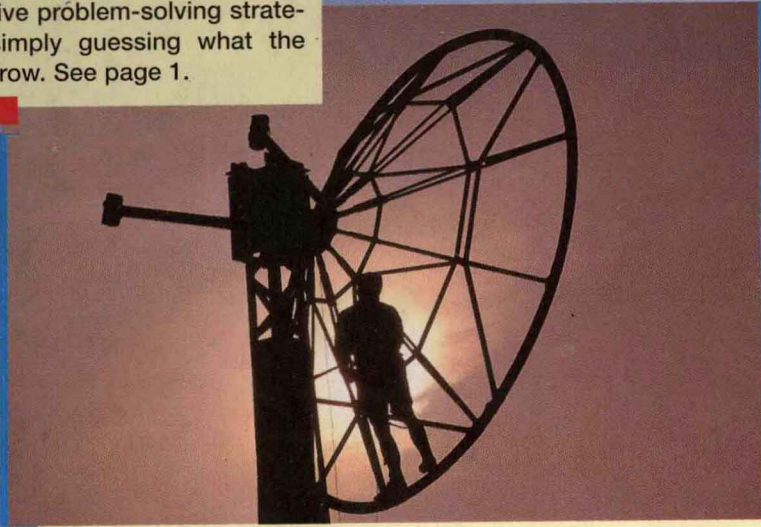
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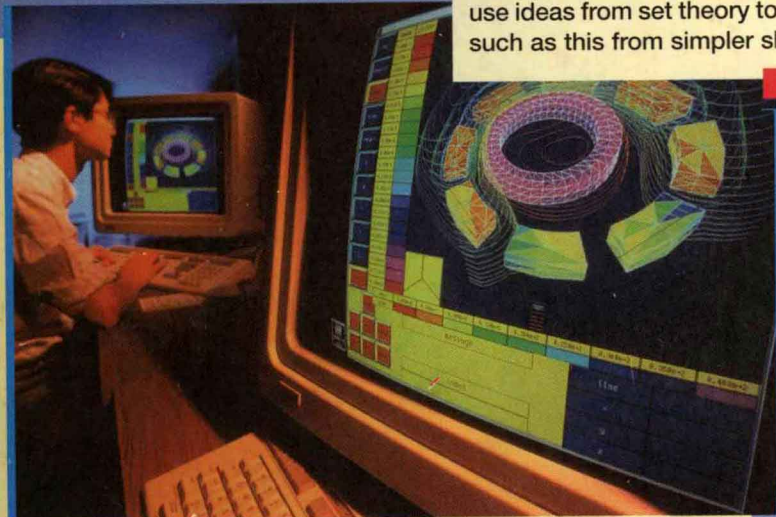
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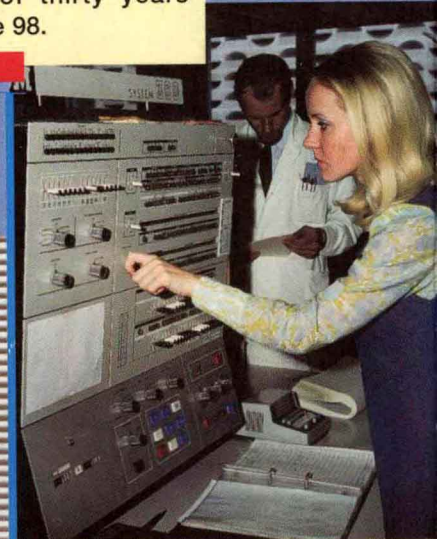
Meteorologists use a variety of technologies, like the heliostat shown here, to predict the weather. But technology without effective problem-solving strategies is no more than simply guessing what the weather will be like tomorrow. See page 1.



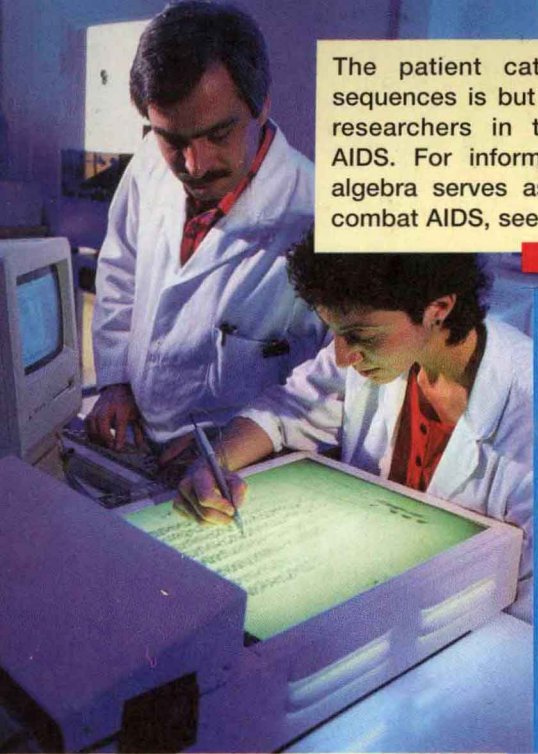
This photo from Cornell University shows a three-dimensional graphic of a rotary motor. Designers can use ideas from set theory to construct complex shapes such as this from simpler shapes. See page 51.



Computers in the 1960s contained hundreds of logic gates made from bulky transistors. Today, desktop computers hold thousands of logic gates on silicon chips and possess more computing power than mainframes of thirty years ago. See page 98.



The patient cataloging of DNA sequences is but one tool used by researchers in the fight against AIDS. For information about how algebra serves as another tool to combat AIDS, see page 318.



In order to avoid excessive exposure to ultraviolet radiation, sunbathers should be aware of the intensity of the ultraviolet rays at their favorite beach. Real numbers can be used to quantify the intensity of sunlight, which varies according to the latitude of the beach. See page 254.



International banking relies on the dependable and secure electronic transmission of financial information. The security of these transmissions relies on the mathematics of encryption systems. To learn more about one widely used method, see page 214.

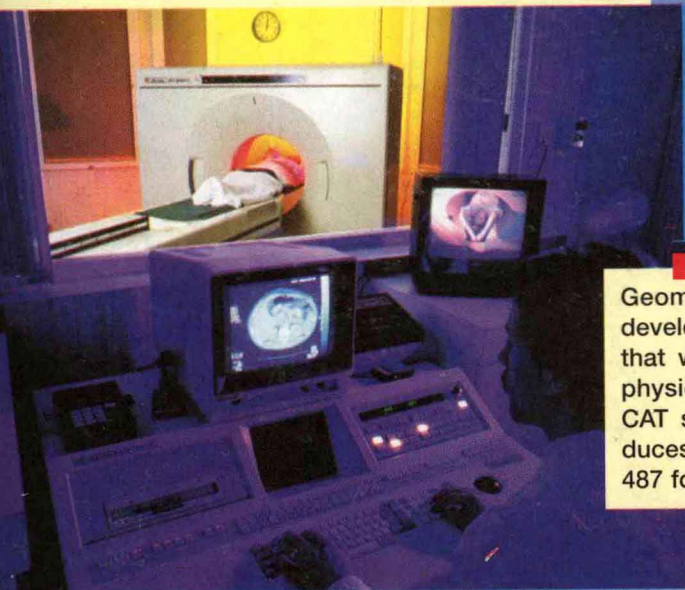


Bar code scanners not only translate the bar code to a sequence of numbers, they are also programmed to use properties of our numeration system to check that the code is valid. For another application of properties of our numeration system, see page 152.

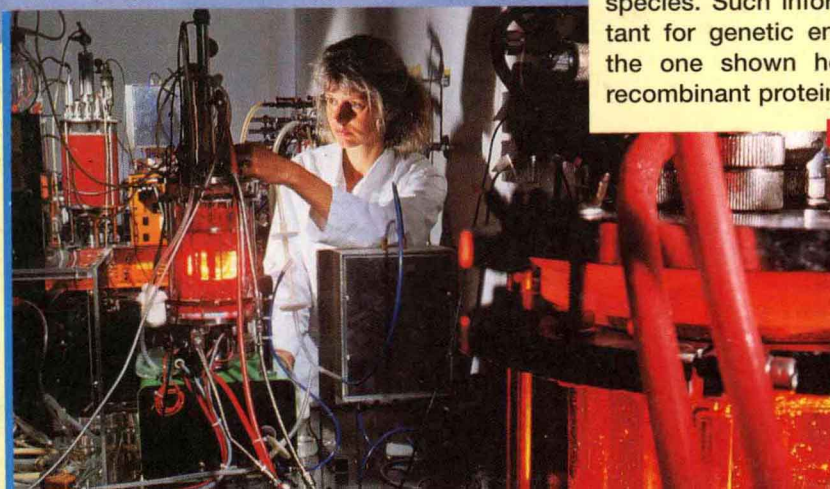




Rain forest deforestation, an example of which is shown here, is one factor contributing to the increase of atmospheric carbon dioxide. See page 400 to see how functions can be used to model the buildup of carbon dioxide in the atmosphere and the corresponding threat of global warming.

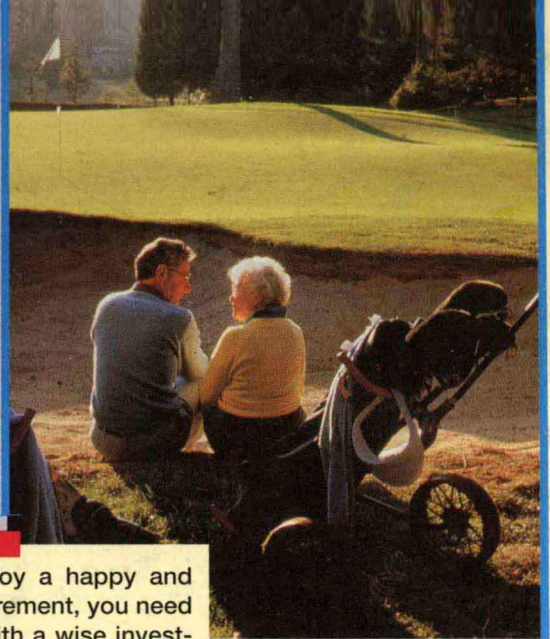
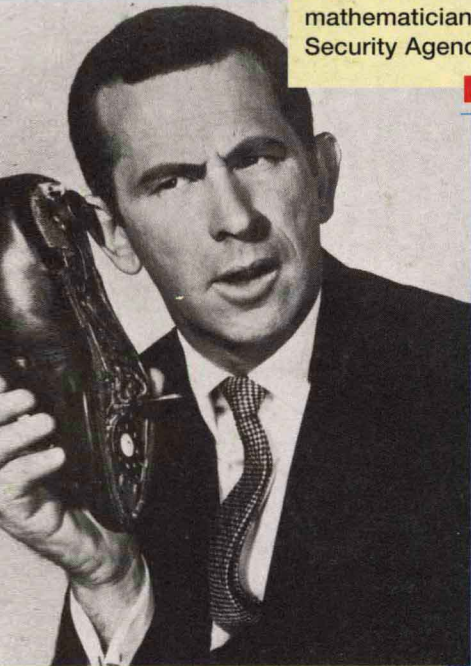


Geometry is used by researchers developing a new imaging technique that will produce dynamic images of physiological processes, unlike the CAT scanner shown here which produces only static images. See page 487 for more information.



Counting methods can be used to determine the number of possible DNA sequences there are for a given species. Such information is important for genetic engineers such as the one shown here investigating recombinant protein. See page 575.

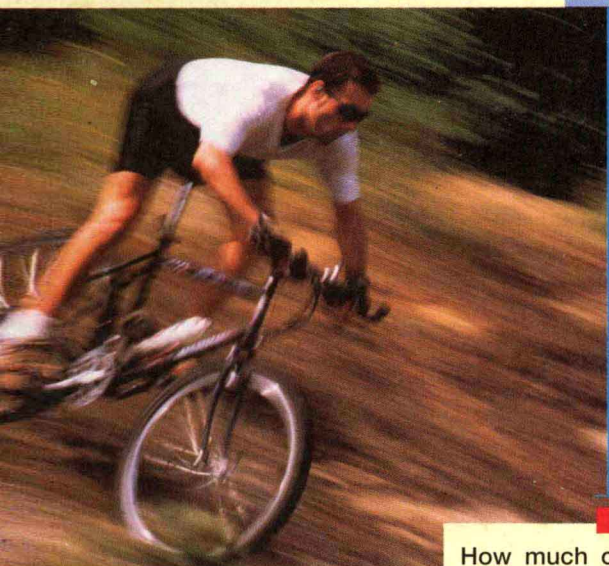
Maxwell Smart probably did not know much about the mathematics of cryptology, but you can learn something about it on page 805. Cryptology is an area in which the many mathematicians at the National Security Agency excel.



In order to enjoy a happy and prosperous retirement, you need to start early with a wise investment plan. The mathematics of consumer finance can help you choose the best plan for you. See page 746.

"It is now proved beyond doubt that smoking is one of the leading causes of statistics."

—Fletcher Knebel. See page 669 to see an example of statistics caused by smoking.



How much of a risk is this person taking? See page 619 to find out.



Preface

Nearly thirty years after its first appearance, *Mathematical Ideas* remains one of the most popular texts in college mathematics. This eighth edition, like its predecessors, has been designed with a variety of students in mind. It is well-suited for several types of courses, including mathematics for liberal arts students, survey courses in mathematics, finite mathematics, and mathematics for prospective and in-service elementary and middle school teachers. Ample topics are included for a two-term course, yet the variety of topics and flexibility of sequence make the text suitable for shorter courses as well.

In this edition we have attempted to retain many of the features of the seventh edition, and at the same time introduce new ones that will appeal to a new generation of instructors and students. Our main objectives continue to be comprehensive coverage of topics appropriate for a mathematics survey course, logical and flexible organization, clear exposition, an abundance of examples, and well-planned exercise sets with numerous applications.

We have worked diligently to incorporate into this edition the recommendations of the *Curriculum and Evaluation Standards for School Mathematics*, prepared by the National Council of Teachers of Mathematics (NCTM), and the *Standards for Introductory College Mathematics Before Calculus*, presented by the American Mathematical Association of Two-Year Colleges (AMATYC). For example, to address the issue of cooperative learning we have introduced the feature titled “Collaborative Investigation” in each chapter. The face of mathematics education is changing drastically due to the availability of low-cost graphing technology, and we have included many screens indicating how such technology can be used to support results found analytically in the text. (It is not essential, however, that the student have a graphing calculator to study from this book; the technology component is totally optional.)

The seventh edition included an abundance of exercises new to that edition. We have retained many of them, while also adding many new applications focusing on real-life data. These include graphs, tables, and charts when appropriate. Chapters 6, 7, and 8 are especially rich in these new exercises. In addition to the usual drill exercises, we include many that test conceptual understanding. In light of the current focus on writing across the curriculum, most sets include some exercises that require the student to answer by writing a few sentences.

The popular margin notes that have appeared in previous editions have been retained and updated when necessary. New ones have been added as well. We hope that users continue to enjoy them as much as we enjoy researching and composing them.

Some topics in the first six chapters require a basic knowledge of equation solving. Depending on the background of the students, the instructor may omit

topics that require this skill. On the other hand, the two algebra chapters (7 and 8) provide an excellent overview of basic algebra. Because of the flexibility of the text, they may be covered at almost any time.

Pedagogical Features

Several new or enhanced features, designed to assist students in the learning process and aid instructors in teaching, have been introduced and retained in this edition. We continue to use full color, and the art program remains one of the highlights of the book. Numerous carefully selected examples illustrate the concepts and skills being introduced. Illustrated margin notes present items of interest that explore the human dimension and historical context of mathematical ideas. “For Further Thought” boxes encourage students to share among themselves their reasoning processes to gain a deeper understanding of key mathematical concepts. Special paragraphs labeled “Problem Solving” relate the discussion of strategies to techniques that have been learned earlier or will be applied later. Most sections include a few challenging exercises that require students to extend the ideas presented in the section. These are identified in the *Instructor’s Annotated Exercises (IAE)* by the symbol ▲. To complement the drill and application exercises, most exercise sets include a few exercises requiring a deeper understanding of the concepts introduced. In addition, many exercises require students to respond by writing a few sentences. These conceptual and writing exercises are indicated in the *IAE* by the symbols ◎ and ✎.

Overview of the Chapters

Chapter 1 (Approach to Problem Solving) introduces the student to inductive reasoning, pattern recognition, and problem-solving techniques. Chapters 2 and 3 (Sets and Logic) give brief overviews of set theory and elementary logic; instructors wishing to do so may cover Chapter 3 before Chapter 2. Chapter 4 (Numeration and Mathematical Systems) covers various types of numeration systems, modulo systems, and groups. In Chapter 5 (Number Theory) we present an elementary introduction to topics such as prime and composite numbers and the Fibonacci Sequence. In Chapter 6 (The Real Number System), we introduce some of the basic concepts of arithmetic of the real numbers in their various forms. Chapters 7 and 8 (The Basic Concepts of Algebra and Functions, Graphs, and Systems of Equations and Inequalities) form the core of the algebra component of the text, providing either an introduction to or a review of basic freshman algebra. In Chapter 9 (Geometry), the student is introduced to the basic concepts of Euclidean geometry. Several enrichment sections are included as well. Chapter 10 (Counting Methods) focuses on elementary counting techniques, in preparation for the chapter to follow. Chapter 11 (Probability) covers the basics of probability, odds, and expected value. In Chapter 12 (Statistics), we give a brief introduction to the various topics of elementary statistics. Chapter 13 (Consumer Mathematics) focuses on mathematics of interest, mortgages, and investments. In Chapter 14 (Matrices and Their Applications) we show how matrices play an important role in both theoretical and applied mathematics.

New to this Edition

In this edition we have added the following new features.

CHAPTER OPENERS A chapter opener focusing on an application of the topic to be addressed in the chapter is now included. These openers were researched by Gary Rockswold of Mankato State University. The goal of these openers is to show the student that real-world applications of mathematics do indeed exist, and hopefully they will enable the student to appreciate the relevance of mathematics in our world.

GRAPHING CALCULATOR NOTES While the use of graphing calculators is not a requirement for using this text, we have included many typical screens that illustrate how this modern technology can be used to supplement and support the concepts presented in the text. We often address the *limitations* of technology as well, pointing out that the calculator can only assist us if we are aware of the concepts first.

COLLABORATIVE INVESTIGATIONS At the end of each chapter we have provided an activity for group discussion, based on the material covered in the chapter. This is in response to instructors who like to incorporate such activities in their classes.

INCREASED EMPHASIS ON GRAPHS, CHARTS, AND REAL DATA We have included many new applications, particularly in Chapters 6, 7, 8, 12, and 13, that employ data from real-life sources. The data is often provided in the form of a graph or a chart.

A CHAPTER ON CONSUMER MATHEMATICS Chapter 13 is a newly written chapter on consumer mathematics. Such a chapter had been included for several editions but was eliminated from the seventh edition. Due to overwhelming response from users, it is back, completely rewritten for the 1990s.

A CHAPTER ON MATRICES Again, a chapter that was eliminated from the seventh edition is back. Chapter 14 is a newly written introduction to the theory and application of matrices.

Course Outline Considerations

For the most part, the chapters in the text are independent and may be covered in the order chosen by the instructor, with a few exceptions. Chapter 6 contains some material dependent on ideas found in Chapter 5. Chapter 6 should be covered before Chapter 7 if student background so dictates. Chapters 7 and 8 should be considered a “package” and covered in sequential order. A thorough coverage of Chapter 11 depends on knowledge of Chapter 10 material, although probability can be covered without learning extensive counting methods by avoiding the more difficult exercises. The latter part of Chapter 12, on inferential statistics, depends on an understanding of probability (Chapter 11).

Supplements

This edition is accompanied by an extensive supplemental package with offerings for both the instructor and the student.

For the Instructor

INSTRUCTOR'S ANNOTATED EXERCISES This manual contains all of the exercises from the student text, with each answer printed in color next to the corresponding exercise. In addition, challenging exercises, conceptual exercises, writing exercises, and calculator exercises are identified with colored symbols so that instructors may assign these problems at their discretion. Each section of this manual is page-referenced to the corresponding exercise set in the student text.

INSTRUCTOR'S TEST MANUAL This manual includes four different test forms for each chapter paralleling the chapter tests in the text, plus an additional 125–150 test items for each chapter. It also contains teaching tips and suggested course outlines.

INSTRUCTOR'S SOLUTION MANUAL This manual contains solutions to all even-numbered exercises and a list of all conceptual, writing, and challenging exercises.

TEST GENERATOR/EDITOR FOR MATHEMATICS WITH QUIZMASTER This is a computerized test generator that lets instructors select test questions by objective or section, or use a ready-made test for each chapter. The software is algorithm driven so that regenerated number values maintain problem types and provide a large number of test items in both multiple-choice and open-response formats for one or more test forms. The Editor lets instructors modify existing questions or create their own including graphics and accurate math symbols. Tests created with the Test Generator can be used with **QuizMaster**, which records student scores as they take tests on a single computer network, and prints reports for students, classes, or courses. CLAST and TASP versions of this package are also available (IBM, DOS, and Macintosh).

For the Student

STUDY GUIDE AND SOLUTION MANUAL This for-sale manual contains solutions to all odd-numbered section, extension, and appendix exercises. Also, chapter summaries review key points in the text, provide extra examples, and enumerate major topic objectives.

GUIDE TO FLORIDA CLAST MATHEMATICAL COMPETENCY This special for-sale study guide for use in Florida offers help in preparing for the College Level Academic Skills Test (CLAST). It includes a review of arithmetic, a CLAST pretest, supplementary exercises, and a CLAST posttest.

COMPUTER-ASSISTED TUTORIALS The tutorials offer self-paced, interactive review in IBM and Macintosh formats. Solutions are given for all examples and exercises, as needed. Students wishing to purchase the tutorials should contact their bookstore.

GRAPH EXPLORER With this sophisticated software, available in IBM and Macintosh versions, students can graph rectangular, conic, polar, and parametric equations; zoom; transform functions; and experiment with families of equations quickly and easily. Students wishing to purchase this software should contact their bookstore.

GEOEXPLORER Available for IBM and Macintosh, this software package enables students to draw, measure, modify, and transform geometric shapes on the screen. Students wishing to purchase this software should contact their bookstore.

STATEXPLORER This software package for IBM and Macintosh computers helps students enhance their understanding of statistics by exploring a wide range of statistical representations including graphs, centers and spreads, and transformations. Students wishing to purchase this software should contact their bookstore.

VIDEOTAPES A number of videotapes relevant to the course material are available. See your local representative for details.

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Vern E. Heeren
E. John Hornsby, Jr.

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