COMPUTATIONAL MODELING OF MULTIPHASE GEOMATERIALS

FUSAO OKA
SAYURI KIMOTO

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Preface

Over the last three decades, studies on constitutive models and numerical analysis methods have been well developed. Nowadays, numerical methods play a very important role in geotechnical engineering and in a related activity called computational geotechnics. This book deals with the constitutive modeling of multiphase geomaterials and numerical methods for predicting the behavior of geomaterials such as soil and rock. The book provides fundamental knowledge of continuum mechanics, constitutive modeling, numerical methods for multiphase geomaterials, and their applications. In addition, the monograph includes recent advances in this area, namely, the constitutive modeling of soils for rate-dependent behavior, strain localization, the multiphase theory, and their applications in the context of large deformations. The presentation is self-contained. Much attention has been paid to viscoplasticity, water–soil coupling, and strain localization.

Chapter 1 presents the fundamental concept and results in continuum mechanics, such as motion, deformation, and stress, which are necessary for understanding the following chapters. This chapter helps readers make a self-consistent study of the contents of this book.

Chapter 2 deals with the governing equations for multiphase geomaterials based on the theory of porous media, such as water-saturated and air–water–soil multiphase soils including soil–water characteristic curves. This chapter is essential for the study of computational geomechanics.

Chapter 3 starts with the elastic constitutive model and reviews the fundamental constitutive models including plasticity and viscoplasticity. For the plasticity theory, the stability concept in the sense of Lyapunov is discussed. At the end of the chapter, cyclic plasticity and viscoplasticity models are presented with kinematical hardening rules.

In Chapter 4, failure criteria and the Cam-clay model are reviewed. For the failure criteria, many well-known criteria have been proposed in this chapter, from Coulomb's criterion to Matsuoka–Nakai's criterion. Then, the Cam-clay model is reviewed since the model includes a description of the basic properties of soil behavior such as dilatancy and the critical state concept.

Chapter 5 is devoted to the rate- and time-dependent behavior and modeling of soils. At first, typical rate- and time-dependent behaviors of soils are reviewed based on the experimental measurements. Several rate-dependent models are discussed and elastoviscoplastic models based on the Cam-clay model and Perzyna's viscoplasticity theory are presented. Adachi and Oka's model is first described and then an elastoviscoplastic model considering structural degradation is introduced. The chapter ends with the calibration of these models using the experimental results.

In Chapter 6, the virtual work theorem is presented and then the finite element method for two-phase materials is described for quasi-static and dynamic problems within the framework of the infinitesimal strain theory.

Chapter 7 deals with a typical multiphase phenomenon of soils; namely, the consolidation problem. In particular, the effects of sample thickness on consolidation, using Aboshi's well-known data, and the anomalous behavior of pore water development in the clay foundation beneath the embankment, during loading and after the end of construction embankment, are numerically analyzed.

Chapter 8 starts with a review of the study on the strain localization behavior of soils. Several issues related to the strain localization are then discussed for rate-independent and rate-dependent models. Finally, a numerical analysis of the strain localization of water-saturated clay is presented for triaxial tests and practical problems.

In Chapter 9, a liquefaction analysis method is presented with a cyclic elastoplastic model using the two-phase theory presented in Chapter 2 for water-saturated soils. Applications of the liquefaction behavior to a man-made island during an earthquake and of the soil–pile–structure interaction are shown.

Chapter 10 deals with recent advances in geomechanics. It includes the temperature-dependent behavior of soils such as consolidation due to the change in temperature, and the numerical analysis of air–water–soil coupled problems; namely, the deformation–seepage flow coupled analysis of an unsaturated river embankment is presented.

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Chapter 1

Fundamentals in continuum mechanics

In this book, we use vectors and tensors in components, and the direct notations for these vectors and tensors are given without further explanation. A dot denotes a contraction of the inner indices, for example, $a_i b_i \equiv a \cdot b$ so that $A_{ij} B_{ij} \equiv A{:}B$.

1.1 MOTION

The position of the material point $X_i (i = 1,2,3)$ of a body at time t is expressed by

$$x_i = \hat{x}_i(X_j, t) \tag{1.1}$$

Material point X_i can be given by the position of x_i at a time $t = 0$. Equation (1.1) expresses the motion of the material point of the body. The rectangular Cartesian coordinates used in this book are described by $(o, \tilde{e}_1, \tilde{e}_2, \tilde{e}_3)$ with origin o and unit base vector $\tilde{e}_i$.

There are two methods for describing the motion of a particle. One is the material description, in which the motion is expressed by material point X_i, and the other is the spatial description, in which the motion is expressed by spatial coordinates x_i. The material description is called the Lagrangian description and the spatial description is called the Eulerian description.

The velocity vector of a particle is given by

$$v_i = \frac{\partial x_i(X_j, t)}{\partial t} \tag{1.2}$$

In the material description, the acceleration of a particle in a body is expressed by

$$a_i = \frac{\partial v_i(X_j, t)}{\partial t} \tag{1.3}$$

In the spatial description, on the other hand, the acceleration of a particle is given by

$$a_i = \frac{\partial v_i(x_j,t)}{\partial t} + v_k \frac{\partial v_i(x_j,t)}{\partial x_k} \tag{1.4}$$

1.2 STRAIN AND STRAIN RATE

1.2.1 Strain tensor

Strain is the change in shape or the change in volume of a body during the application of force to the body. We need an objective measure of strain that can be derived through changes in the variation of the line element.

Let us consider the motion of the body shown in Figure 1.1. Material points P and Q have moved to points P′ and Q′ after the deformation. Points Q and Q′ are the points located in the vicinity of points P and P′.

Distance, dS, between points P and Q, is given by

$$dS^2 = dX_a dX_a \tag{1.5}$$

and the distance between points P′ and Q′ after the deformation, ds, is given by

$$ds^2 = dx_b dx_b \tag{1.6}$$

where the summation convention is used for $a,b = 1,2,3$.

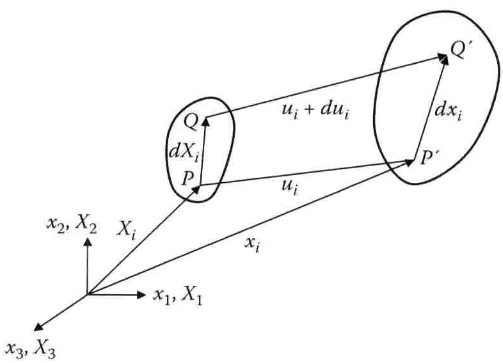

Figure 1.1 Motion.