GUSTAFSON • FRISK

# Beginning Algebra

THIRD EDITION

THIRD EDITION

# Beginning Algebra

**R. David Gustafson**
**Peter D. Frisk**

Rock Valley College

Brooks/Cole Publishing Company
Pacific Grove, California

**Brooks/Cole Publishing Company**
A Division of Wadsworth, Inc.

© 1992, 1988, 1985 by Wadsworth, Inc., Belmont, California 94002.
All rights reserved. No part of this book may be reproduced, stored in a retrieval system, or transcribed, in any form or by any means—electronic, mechanical, photocopying, recording, or otherwise—without the prior written permission of the publisher, Brooks/Cole Publishing Company, Pacific Grove, California 93950, a division of Wadsworth, Inc.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

**Library of Congress Cataloging-in-Publication Data**

Gustafson, R. David (Roy David), [date]
Beginning algebra / R. David Gustafson, Peter D. Frisk. — 3rd ed.
p. cm.
Includes index.
ISBN 0-534-16392-0
1. Algebra. I. Frisk, Peter D., [date]. II. Title.
QA152.2.G85 1991
512.9—dc20 91-3767
CIP

*Sponsoring Editor:* Paula-Christy Heighton
*Editorial Assistant:* Carol Ann Benedict
*Production Editor:* Ellen Brownstein
*Manuscript Editor:* David Hoyt
*Permissions Editor:* Carline Haga
*Interior and Cover Design:* E. Kelly Shoemaker
*Cover Photo:* Lee Hocker
*Art Coordinator:* Lisa Torri
*Interior Illustration:* Lori Heckelman
*Photo Editor:* Ruth Minerva
*Typesetting:* Syntax International
*Cover Printing:* Lehigh Press Lithographers
*Printing and Binding:* R. R. Donnelley & Sons, Crawfordsville

*Photo Credits:* **P. 1,** Comstock; **p. 68,** Rick Rusing/Tony Stone Worldwide; **p. 123,** Sylvain Grandadam/Tony Stone Worldwide; **p. 170,** Michael Thompson/Comstock; **p. 216,** Michael Stuckey/Comstock; **p. 271,** Andy Sacks/Tony Stone Worldwide; **p. 328,** David Joel/Tony Stone Worldwide; **p. 372,** John Telford/Index Stock Photography; **p. 415,** Rene Sheret/Tony Stone Worldwide.

To

Caitlin Mallory Barth

Nicholas Connor Barth

Prescott Alexander Heighton

Laurel Marie Heighton

Daniel Mark Voeltner

and the new generation of mathematics students

THIRD EDITION

# Beginning Algebra

此为试读,需要完整PDF请访问: www.ertongbook.com

# CONTENTS

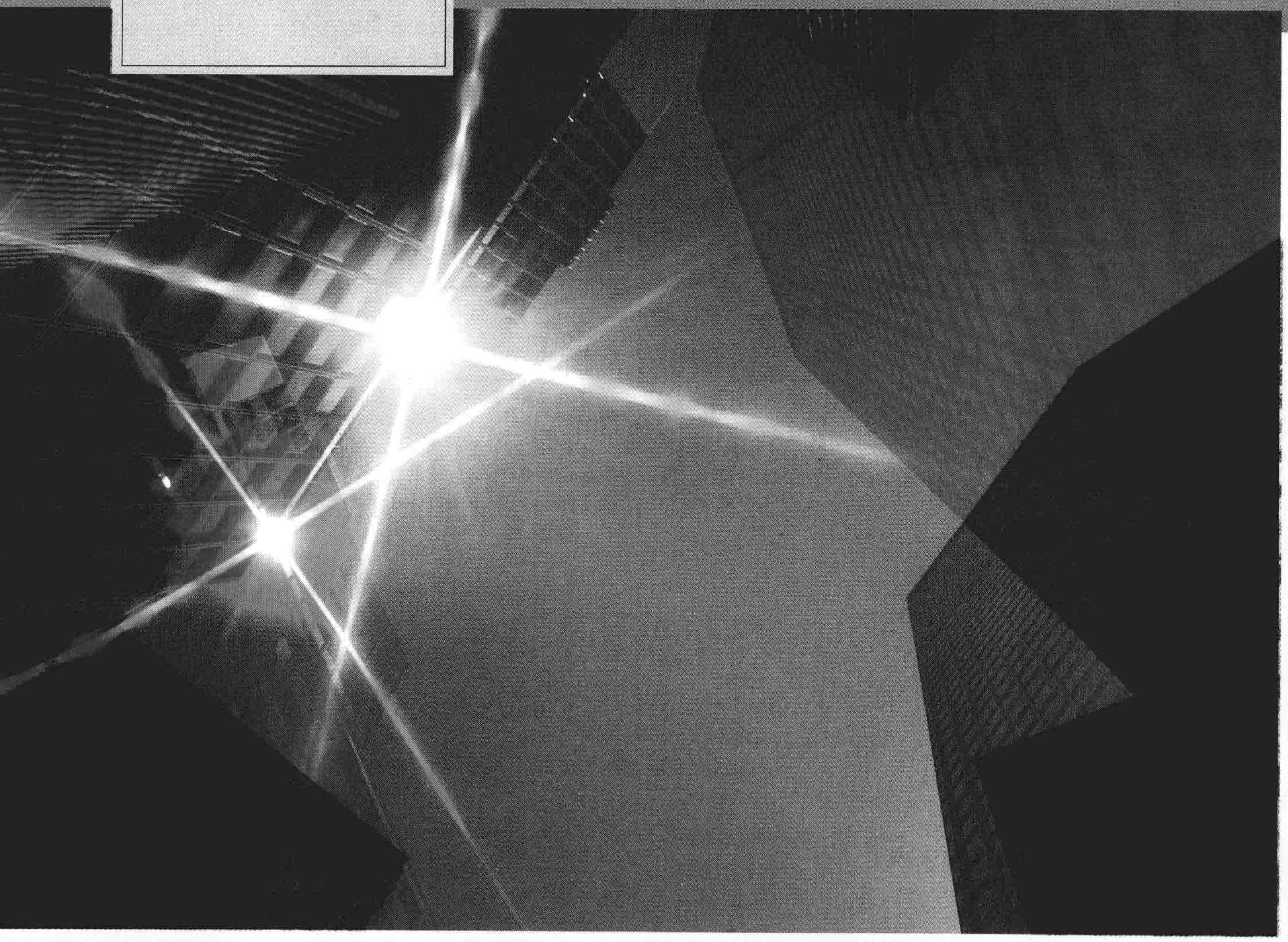
[redacted]

[redacted]

# 1



# Real Numbers and Their Basic Properties



Algebra is an extension of arithmetic. In algebra the operations of addition, subtraction, multiplication, and division are performed on both numbers and letters, with the understanding that the letters can be replaced by numbers. This idea is simple but powerful. It leads to solutions of problems that would be difficult or impossible for us to solve by using arithmetic alone.

Algebra is not new. It has been used for thousands of years in China, India, Persia, and Arabia. The origins of algebra are found in a papyrus written before 1000 B.C. by an Egyptian priest named Ahmes. This papyrus contains 80 algebra problems and their solutions. Because the Egyptians did not have a suitable system of notation, however, they were unable to develop algebra completely.

Further development of algebra had to wait until the ninth century and the rise of the Moslem civilization. In A.D. 830 one of the greatest mathematicians of Arabian history, al-Khowarazmi, wrote a book called *Ihm al-jabr wa'l muqabalah*. This imposing title was soon shortened to *al-Jabr*. The spelling has changed over the centuries, and we now know the subject as *algebra*. The French mathematician François Vieta (1540–1603) later simplified the study of algebra by developing the symbolic notation that we use today.

We begin the study of algebra by discussing the properties of numbers.

## 1.1 SETS OF NUMBERS AND THEIR GRAPHS

The most basic **set** of numbers is the set of **natural numbers** that we use for counting. These are the numbers 1, 2, 3, 4, 5, 6, 7, 8, and so on.

> **DEFINITION.** The **natural numbers** are the numbers
>
> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, . . .

The three dots, called the **ellipsis,** in the previous definition indicate that the list continues forever. There is no largest natural number.

The natural numbers that can be divided exactly by 2—that is, the division has a remainder of 0—are called the **even natural numbers.** They are the numbers

2, 4, 6, 8, 10, 12, . . .

The natural numbers that cannot be divided exactly by 2—that is, the division has a remainder of 1—are called the **odd natural numbers.** They are the numbers

1, 3, 5, 7, 9, 11, 13, . . .

Some natural numbers can be divided exactly only by 1 and the number itself. For example, the only two divisors of 3 are 1 and 3, and the only two divisors of 17 are 1 and 17. Natural numbers with exactly two divisors are called **prime numbers.**

> **DEFINITION.** A **prime number** is a natural number that is larger than 1 and is exactly divisible only by 1 and by itself.

TEACHING TIP:
Point out that if the last digit is even, the number is even, and if the last digit is odd, the number is odd.

TEACHING TIP:
Point out that 1 is not prime. If it were, then every prime would be the product of primes:

$7 = 7 \cdot 1$

此为试读,需要完整PDF请访问: www.ertongbook.com

Leonardo Fibonacci (late 12th and early 13th cent.) Fibonacci, an Italian mathematician, is also known as Leonardo da Pisa. In his work *Liber abaci,* he advocated the adoption of Arabic numerals, the numerals that we use today. He is best known for a sequence of numbers that bears his name. Can you find the pattern in this sequence?

1, 1, 2, 3, 5, 8, 13, . . .

The prime numbers are the numbers

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, . . .

The number 2 is the only even prime number. All of the other prime numbers are odd natural numbers.

The natural number 12 can be divided exactly by several numbers—1, 2, 3, 4, 6, and 12. Because 12 has more than two divisors, it is not a prime number. Rather, it is an example of a number called a **composite number.**

**DEFINITION.** A **composite number** is a natural number greater than 1 that is not a prime number.

The composite numbers are the numbers

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, . . .

The natural number 1 is neither a prime number nor a composite number.

The natural numbers together with 0 form the set of **whole numbers.** Thus, the whole numbers are the numbers

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, . . .

**TEACHING TIP:** Emphasize that the natural numbers, together with 0, are the whole numbers.

Braces are often used to enclose a list of the members of a set. Each pair of braces is read as "the set of."

The set of natural numbers is $\{1, 2, 3, 4, 5, 6, \ldots\}$.
The set of prime numbers is $\{2, 3, 5, 7, 11, 13, \ldots\}$.
The set of composite numbers is $\{4, 6, 8, 9, 10, 12, \ldots\}$.
The set of whole numbers is $\{0, 1, 2, 3, 4, 5, \ldots\}$.

Each member of a set is called an **element** of the set. The number 3, for example, is an element of the set of natural numbers, an element of the set of prime numbers, and an element of the set of whole numbers. However, 3 is *not* an element of the set of composite numbers.

**EXAMPLE 1** Classify the numbers **a.** 7, **b.** 6, and **c.** 0 into the categories of natural number, even natural number, odd natural number, prime number, composite number, and whole number.

***Solution***

**a.** The number 7 is a natural number, an odd natural number, a prime number, and a whole number.
**b.** The number 6 is a natural number, an even natural number, a composite number, and a whole number.
**c.** The number 0 is a whole number. ■

**CLASSROOM ACTIVITY:** Have students decide if 1007 and 1009 are prime. ($1007 = 19 \cdot 53$, 1009 is prime.)

## Graphing Sets of Numbers

Various sets of numbers can be pictured on the **number line.** To construct the number line shown in Figure 1-1, we pick some point on the line, label it 0, and call it the **origin.** We then pick some unit length, mark off points to the right of the origin, and label these points with the natural numbers, as shown in the figure.

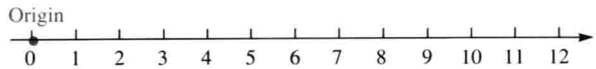



**FIGURE 1-1**

The point labeled 3 on the number line, for example, is 3 units to the right of the origin. The point labeled 6 is 6 units to the right of the origin.

Sets of numbers can be represented, or graphed, on the number line. For example, Figure 1-2 shows the graph of the even natural numbers from 2 to 8. The point corresponding to the number 4, for example, is called the **graph of 4.** The number 4 is called the **coordinate** of its corresponding point.

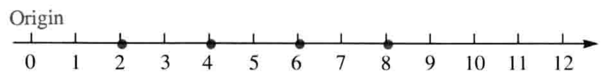



**FIGURE 1-2**

**EXAMPLE 2** Graph the set of prime numbers between 1 and 20 on the number line.

***Solution*** The prime numbers between 1 and 20 are 2, 3, 5, 7, 11, 13, 17, and 19. The graph of this set of numbers is shown in Figure 1-3.

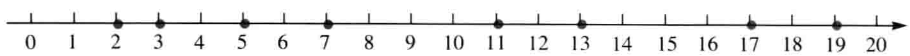



**FIGURE 1-3** ■

**WRITING ACTIVITY:**
Have students write a paragraph about Robert Recorde (1510–1560), who was the first person to use the equals sign.

One of the most common symbols in mathematics is the **equals sign,** written as =. It is used to indicate that two expressions represent the same number. Because 4 + 5 and 9 represent the same number, we can write

$4 + 5 = 9$ Read as "the sum of 4 and 5 is 9," or "4 plus 5 equals 9."

Likewise, we can write

$5 - 3 = 2$ Read as "the difference between 5 and 3 is 2," or "5 minus 3 equals 2."

$4 \cdot 5 = 20$ Read as "the product of 4 and 5 is 20," or "4 times 5 equals 20."

and

$30 \div 6 = 5$ Read as "the quotient obtained when 30 is divided by 6 is 5," or "30 divided by 6 equals 5."

Other symbols, called **inequality symbols,** are used to indicate that expressions are *not* equal.

TEACHING TIP:
You may wish to point out that
$7 \not< 4$ means $7 \geq 4$
and
$8 \not> 8$ means $8 \leq 8$

| Symbol | Read as |
|---|---|
| $\neq$ | "is not equal to" |
| $<$ | "is less than" |
| $>$ | "is greater than" |
| $\leq$ | "is less than or equal to" |
| $\geq$ | "is greater than or equal to" |

**EXAMPLE 3** **a.** $6 \neq 9$ is read as "6 is not equal to 9."

**b.** $8 < 10$ is read as "8 is less than 10."

**c.** $12 > 1$ is read as "12 is greater than 1."

**d.** $5 \leq 5$ is read as "5 is less than or equal to 5." (Because 5 is equal to 5, this is a true statement.)

**e.** $9 \geq 7$ is read as "9 is greater than or equal to 7." (Because 9 is greater than 7, this is a true statement). ■

TEACHING TIP:
Think of the < and > symbols as jaws that will bite the greater number:
$2 < 7$ and $7 > 2$

Statements of inequality can be written so that the inequality symbol points in the opposite direction. For example, the inequality statement

$$5 < 7$$

is read as "5 is less than 7," and the statement

$$7 > 5$$

is read as "7 is greater than 5." Both indicate that 5 is a smaller number than 7. Likewise,

$12 \geq 3$ Read as "12 is greater than or equal to 3."

and

$3 \leq 12$ Read as "3 is less than or equal to 12."

are equivalent statements.

TEACHING TIP:
Think of < as a crescendo, which gets louder or bigger.

If one point is to the *right* of a second point on the number line, its coordinate is the *greater*. For example, the point with coordinate 5 in Figure 1-3 lies to the right of the point with coordinate 2. Thus, $5 > 2$. If one point is to the *left* of another, its coordinate is the *smaller*. The point with coordinate 11, for example, is to the left of the point with coordinate 19. Thus, $11 < 19$.

## EXERCISE 1.1

*In Exercises 1–8, list the numbers in the set* {0, 1, 2, 4, 13, 15} *that satisfy the given condition.*

**1.** natural number 1, 2, 4, 13, 15

**2.** even natural number 2, 4

**3.** odd natural number 1, 13, 15

**4.** prime number 2, 13

**5.** composite number 4, 15

**6.** whole number 0, 1, 2, 4, 13, 15

**7.** even prime number 2

**8.** odd composite number 15

*In Exercises 9–16, list the numbers in the set* {0, 1, 2, 6, 11, 12} *that satisfy the given condition.*

**9.** whole number 0, 1, 2, 6, 11, 12

**10.** prime number 2, 11

**11.** even natural number 2, 6, 12

**12.** composite number 6, 12

**13.** odd prime number 11

**14.** even composite number 6, 12

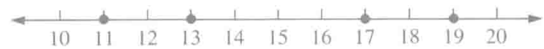
**15.** neither prime nor composite 0, 1

**16.** whole number but not a natural number 0

*In Exercises 17–26, graph each set of numbers on the number line.*

**17.** The natural numbers between 2 and 8.

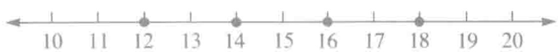

**18.** The prime numbers from 10 to 20.




**19.** The composite numbers from 20 to 30.


**20.** The whole numbers less than 6.


**21.** The even natural numbers greater than 10 but less than 20.




**22.** The even natural numbers that are also prime numbers.


**23.** The numbers that are whole numbers but not natural numbers.


**24.** The prime numbers between 20 and 30.


**25.** The natural numbers between 11 and 25 that are exactly divisible by 6.


**26.** The odd natural numbers between 14 and 28 that are exactly divisible by 3.


*In Exercises 27–40, place one of the symbols* =, <, *or* > *in the box to make a true statement.*

**27.** $5 \boxed{=} 3 + 2$

**28.** $9 \boxed{>} 7$

**29.** $25 \boxed{<} 32$

**30.** $2 + 3 \boxed{<} 17$

**31.** $5 + 7 \boxed{>} 10$

**32.** $3 + 3 \boxed{=} 9 - 3$

**33.** $3 + 2 + 5 \boxed{=} 5 + 2 + 3$

**34.** $8 - 5 \boxed{=} 5 - 2$

**35.** $3 + 9 \boxed{=} 20 - 8$

**36.** $19 - 3 \boxed{>} 8 + 6$

**37.** $4 \cdot 2 \boxed{=} 2 \cdot 4$

**38.** $7 \cdot 9 \boxed{>} 9 \cdot 6$

**39.** $8 \div 2 \boxed{<} 4 + 2$

**40.** $0 \div 7 \boxed{<} 1$

*In Exercises 41–46, write each statement as a mathematical expression.*

**41.** Seven is greater than three. $7 > 3$

**42.** Five is less than thirty-two. $5 < 32$

**43.** Seventeen is less than or equal to seventeen. $17 \leq 17$

**44.** Twenty-five is not equal to twenty-three. $25 \neq 23$

**45.** The result of adding three and four is equal to seven. $3 + 4 = 7$

**46.** Thirty-seven is greater than or equal to the result of multiplying three and four. $37 \geq 3 \cdot 4$

*In Exercises 47–58, rewrite each inequality statement as an equivalent inequality in which the inequality symbol points in the opposite direction.*

**47.** $3 \le 7$ $7 \ge 3$

**48.** $5 > 2$ $2 < 5$

**49.** $6 > 0$ $0 < 6$

**50.** $34 \le 40$ $40 \ge 34$

**51.** $3 + 8 > 8$
$8 < 3 + 8$

**52.** $8 - 3 < 8$
$8 > 8 - 3$

**53.** $6 - 2 < 10 - 4$
$10 - 4 > 6 - 2$

**54.** $8 \cdot 2 \ge 8 \cdot 1$
$8 \cdot 1 \le 8 \cdot 2$

**55.** $2 \cdot 3 < 3 \cdot 4$
$3 \cdot 4 > 2 \cdot 3$

**56.** $8 \div 2 \ge 9 \div 3$
$9 \div 3 \le 8 \div 2$

**57.** $\frac{12}{4} < \frac{24}{6}$
$\frac{24}{6} > \frac{12}{4}$

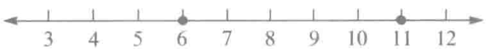
**58.** $\frac{2}{3} \le \frac{3}{4}$
$\frac{3}{4} \ge \frac{2}{3}$

*In Exercises 59–66, graph each pair of numbers on a number line. Indicate which number in the pair is the greater and which number lies to the right of the other number on the number line.*

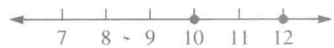
**59.** 3, 6 6 is the greater; 6 lies to the right of 3.


**60.** 4, 7 7 is the greater; 7 lies to the right of 4.


**61.** 11, 6 11 is the greater; 11 lies to the right of 6.

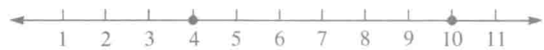



**62.** 12, 10 12 is the greater; 12 lies to the right of 10.

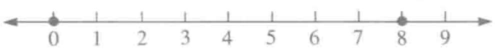



**63.** 0, 2 2 is the greater; 2 lies to the right of 0.

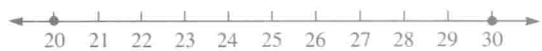

**64.** 4, 10 10 is the greater; 10 lies to the right of 4.




**65.** 8, 0 8 is the greater; 8 lies to the right of 0.




**66.** 20, 30 30 is the greater; 30 lies to the right of 20.




**67.** Explain why there is no greatest natural number.
By adding 1, we can always obtain a larger natural number.

**68.** Explain why 2 is the only even prime number.
Any even number greater than 2 is divisible by 2.

**69.** Explain why no natural numbers are both even and odd.
A natural number can either be divided by 2 or it can't.

**70.** Find the only natural number that is neither a prime number nor a composite number. 1

## REVIEW EXERCISES

*In Review Exercises 1–8, perform the indicated operations.*

**1.** Add: 132
45
73
250

**2.** Add: 261
79
31
371

**3.** Subtract: 321
173
148

**4.** Subtract: 532
437
95

**5.** Multiply: 437
38
16,606

**6.** Multiply: 529
42
22,218

**7.** Divide: $37\overline{)3885}$
105

**8.** Divide: $53\overline{)11607}$
219

*In Review Exercises 9–10, perform the indicated operations.*

**9.** $235 + 517 - 26$ 726

**10.** $135 + 22 - 156$ 1

**11.** Sally bought four paintings for \$350, \$900, \$820, and \$1000. One year later, she sold them for \$750, \$850, \$1250, and \$2700, respectively. Find her total profit. \$2480

**12.** An airplane is cruising at 29,000 feet. The pilot receives permission to descend 7000 feet and later to climb 11,000 feet. At what altitude is the plane then cruising? 33,000 ft

## 1.2 FRACTIONS

We continue the review of the number system by discussing **fractions.** In fractions such as

$$\frac{1}{2}, \quad \frac{3}{5}, \quad \frac{2}{17}, \quad \text{and} \quad \frac{37}{7}$$

the number above the fraction bar is called the **numerator,** and the number below is called the **denominator.**

WRITING ACTIVITY:
Have students write a definition of the word *fraction.*

A fraction can be used to indicate parts of a whole. In this case, the denominator indicates the total number of equal parts into which the whole is divided, and the numerator indicates the number of these equal parts that are being considered. For example, if a test contains 25 problems and a student completes 21 of them, the student has completed $\frac{21}{25}$ (twenty-one twenty-fifths) of the test. In Figure 1-4**a**, a rectangle has been divided into 5 equal parts and 3 of the parts are shaded. Thus, $\frac{3}{5}$ of the figure is shaded. In Figure 1-4**b**, $\frac{5}{7}$ of the rectangle is shaded.

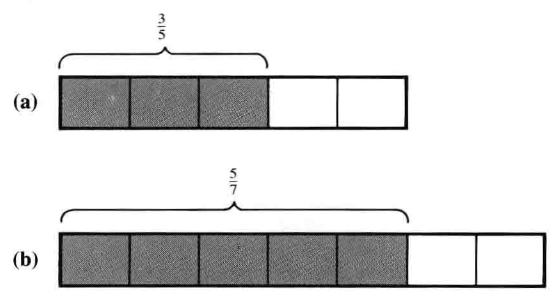



**FIGURE 1-4**

A fraction can also be interpreted as an indicated division. For example, the fraction $\frac{8}{2}$ indicates that 8 is to be divided by 2:

$$\frac{8}{2} = 8 \div 2 = 4$$

此为试读，需要完整PDF请访问：www.ertongbook.com