College Algebra

a skills approach

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COLLEGE ALGEBRA
A Skills Approach

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PREFACE **

Purpose *College Algebra: A Skills Approach* has been written with two basic intentions. First, the authors have attempted to write a book that is readable for the student. The explanations, examples, and exercises have been developed with readability in mind, and the student will find them to be a sufficient aid to the learning of algebra.

Secondly, the book is written to be completed in a semester or quarter. Chapter 1 contains a rapid but complete review, which allows the course to proceed into topics that are too often not reached in a college algebra course.

This is not a formal text in the sense of modern algebra, but proofs are not ignored. As the text progresses, more and more formal proofs are given. The rationale being that the student is more able to understand and appreciate a proof after he or she has mastered the basic skills.

Prerequisites The manipulative skills of elementary and intermediate algebra are reviewed in the first chapter. If this chapter is more than a review, the student should take a developmental algebra course before proceeding to college algebra.

Organization The emphasis throughout the text is to involve the student in "doing" the mathematics through the use of illustrative examples and problem solving. By design, the explanations are concise and straight-forward. The "write-in" feature gives the student a ready review vehicle and also allows the instructor easy access to the students' work.

Each chapter is followed by a chapter review and chapter test. Answers to all problems (except proofs) are given in the text.

Instructional Techniques This text is designed to be used in conjunction with classroom lectures. The traditional lecture-assignment method can be used effectively. A method of short lecture, followed by a brief work time for each topic, can also be used. It is also possible to use the text in a situation where the student reads the text, works the problems, and consults the instructor. This method would be beneficial in situations where it is preferable for students to work at their own rate.

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CONTENTS

1 Basic Principles of Algebra

A study of college algebra requires certain skills from arithmetic and elementary algebra. In this text, some of these skills are assumed and others are briefly reviewed. Skills in arithmetic —adding, subtracting, multiplying and dividing whole numbers, decimals, and fractions—are assumed. Also the ability to operate with signed numbers is assumed.

This first chapter is a brief review of topics from elementary algebra with which the student should already be familiar.*

1.1 THE REAL NUMBERS

The numbers first encountered in elementary arithmetic are those used in counting, or the set of *counting numbers,*

$$\{1, 2, 3, 4, \ldots\}$$

This set is later extended to include zero and the negatives of the counting numbers giving us the set of *integers,*

$$\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

The set of *rational numbers*—those that can be expressed as a ratio of two integers—are generally referred to as *fractions*. This set includes the integers since, for example, 3 can be

*If the student finds this review insufficient he should carefully review an elementary algebra text such as Nanney and Cable, *Developing Skills in Algebra*. 2d ed. Boston: Allyn and Bacon, Inc., 1976.

expressed as $\frac{3}{1}, \frac{6}{2}$, etc. In fact, this set includes all number expressions which involve only the operations of addition, subtraction, multiplication, and division.

Next, a set of numbers called the *irrational numbers*, which cannot be expressed as the ratio of integers, is developed. This set includes such numbers as π, $\sqrt{5}$, $\sqrt[3]{7}$, etc.

The set of rational and irrational numbers together make up a set called the *real numbers.* Elementary algebra is a study of the real numbers and their properties.

The basic set of properties (axioms) of the real numbers can be used to justify all of the manipulations used in both arithmetic and elementary algebra. If this set of properties is accepted as being true, then all other properties can be proved as theorems. Even though an intuitive approach rather than one of rigorous proofs is used in this text, mention is often made of these basic properties. They are listed here for easy reference.

PROPERTIES OF THE REAL NUMBERS

1. The real numbers are *closed* under addition.
   If a and b are real numbers, then $(a + b)$ is also a real number.
2. Addition of real numbers is *commutative.*
   If a and b are real numbers, then $a + b = b + a$
3. Addition of real numbers is *associative.*
   If a, b and c are real numbers, then $(a + b) + c = a + (b + c)$
4. There is a real number which is the *additive identity.* This number is zero. For all real numbers a,
   $a + 0 = a$
5. Each real number has a *negative* (additive inverse).
   If a is a real number, then there is a real number $(-a)$ such that $a + (-a) = 0$
6. The real numbers are *closed* under multiplication.
   If a and b are real numbers, then ab is also a real number.
7. Multiplication of real numbers is *commutative.*
   If a and b are real numbers, then $ab = ba$.
8. Multiplication of real numbers is *associative.*
   If a, b and c are real numbers, then $(ab)c = a(bc)$
9. There is a real number that is the *multiplicative identity.* This number is 1. For all real numbers a,
   $a(1) = a$
10. Each nonzero real number has a *reciprocal* (multiplicative inverse).
    If a is a real number (not zero), then there is a real number $\frac{1}{a}$ such that $a\left(\frac{1}{a}\right) = 1.$
11. The real numbers obey the *distributive property* of multiplication over addition.
    If a, b and c are real numbers, then $a(b + c) = ab + ac.$

Any set of elements in which all eleven of the above properties are true is called a *field.* The field of real numbers is a subset of another field called the *complex numbers* which will be studied in Chapter 2.

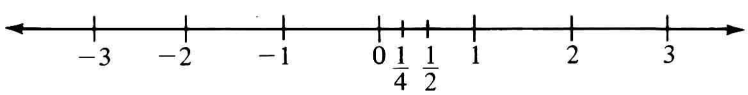
The field of real numbers has another property, not shared by any other field, called the *ordering property*.

ORDERING PROPERTY

There is a subset of the set of real numbers (i.e., a set of numbers belonging to the real numbers) called the *positive* numbers such that;

1. This set (the positive real numbers) is closed under the operations of addition and multiplication.
2. For every real number a, one and only one of the following statements is true:
   a. a is a member of the set of positive real numbers.
   b. a is the negative of a member of the set of positive real numbers.
   c. a is zero.

Since all real numbers are classified as positive, negative, or zero by the ordering property, this set can be represented on a number line. By using a straight line from plane geometry (a straight line has infinite length) and choosing a point to be represented by 0 and another point to be represented by 1, we can, with certain agreements, establish a correspondence between the real numbers and the points of the line. We first agree to place the positive numbers to the right of 0, and the negative numbers to the left of 0. We also agree that the length of the line segment from 0 to 1 will be used as a unit measurement between all real numbers that differ by 1 and that real numbers between 0 and 1 will be represented by proportional parts of this unit. Such a number line is represented below.



The number line, as represented above, is a very useful tool in the study of mathematics. One example of its use is in the study of inequalities. The symbol for inequality "<" can be defined in terms of the number line.

DEFINITION. If a and b are real numbers, $a < b$ means that a is to the left of b on the number line.

This same symbol is sometimes turned in the other direction ">." $a < b$ is usually read "a is less than b," and $b > a$ is usually read "b is greater than a." Of course, both expressions mean the same thing and could be read in either direction. A good way to remember the meaning of the symbol is to recognize that the pointed end is always in the direction of the smaller number (i.e., the number to the left of the other).

EXERCISE 1.1.1

Place the proper symbol, $<$ or $>$, for the question mark in each of the following to make a true statement.

1. $4 \; ? \; 6$
2. $9 \; ? \; 2$
3. $-3 \; ? \; 0$
4. $5 \; ? \; 0$
5. $-1 \; ? \; -5$
6. $3 \; ? \; -6$
7. $\frac{3}{2} \; ? \; 2$
8. $-\frac{10}{3} \; ? \; -3$
9. $-.5 \; ? \; -.6$
10. $\frac{1}{3} \; ? \; .33$

Distance, without regard to direction, is defined by the term *absolute value*. The absolute value of an expression is indicated by placing vertical bars before and after the expression. $|a - b|$ is read as "the absolute value of $a - b$." $|x|$ is read as "the absolute value of x." $|a - b|$ implies the distance from a to b or the distance from b to a, without regard to direction. A formal definition follows.

DEFINITION.

$$|x| = \begin{cases} x & \text{if } x \text{ is positive} \\ 0 & \text{if } x \text{ is zero} \\ -x & \text{if } x \text{ is negative} \end{cases}$$

An immediate result of this definition is that the absolute value of an expression is always nonnegative. If x is positive or zero, it is clear from the definition that $|x|$ is nonnegative. Also if x is negative then $|x| = -x$ (opposite of x) and is positive.

Absolute value is used in mathematics to measure the "nearness" of points.

EXERCISE 1.1.2

1. Evaluate the following;

a. $|9-3|$

b. $|5-6|$

c. $|3-3|$

d. $|10-7|$

e. $|7-10|$

f. $|8-(-5)|$

g. $\left|\frac{2}{5}-\frac{3}{8}\right|$

h. $\left|\frac{3}{7}-\frac{4}{9}\right|$

2. Justify each of the following by stating *one* of the field properties.

a. $4+(2+3)=(4+2)+3$

b. $5+0=5$

c. $(6)(1)=6$

d. $(4)\left(\frac{1}{4}\right)=1$

e. $5(2+3)=5(2)+5(3)$

f. $8 + (-8) = 0$

g. $(2 + 3) + 6 = (3 + 2) + 6$

h. $[(4)(2)](3) = (3)[(4)(2)]$

3. Determine which of the following sets are closed under addition and multiplication.
a. {Integers}
b. {Rational numbers}
c. {All positive real numbers}
d. {All negative real numbers}
e. {Even counting numbers}
f. {Odd counting numbers}
g. $\{0\}$
h. $\{-1, 0, 1\}$

4. The value of π to five decimal places is 3.14159. A less accurate value of $\frac{22}{7}$ is often used for π in elementary work. Find the decimal approximation of $\frac{22}{7}$ and then substitute $<$ or $>$ for the question mark in the following to make a true statement.

$$\frac{22}{7} \; ? \; \pi$$

5. Restate the three statements in part *b* of the ordering property by using the symbols $>$, $<$, and $=$.

6. a. What is the largest integer less than or equal to π?

b. What is the largest rational number less than or equal to π?

c. What is the largest real number less than or equal to π?

1.2 EXPONENTS AND RADICALS

DEFINITION: $x^n = \underbrace{(x)(x)(x)(x) \cdots (x)}_{n \text{ factors}}$, if n is a positive integer. In words: "x to the nth power, when n is a positive integer, implies that x is used as a factor n times."

The above definition gives rise to the following laws of exponents.

Law I $x^a \cdot x^b = x^{a+b}$

To multiply like bases, add the exponents.

EXAMPLE

$$x^3 \cdot x^5 = x^8$$

Law II $x^a \div x^b = x^{a-b} \quad x \neq 0$

To divide like nonzero bases, subtract the exponent of the divisor from the exponent of the dividend.

EXAMPLE

$$x^7 \div x^5 = x^2$$

Law III $(x^a)^b = x^{ab}$

To raise a power to a power, multiply exponents.

EXAMPLE

$$(x^3)^5 = x^{15}$$

Law IV $(xy)^a = x^a y^a$

The power of a product is the product of the powers.

EXAMPLE

$$(xy)^3 = x^3 y^3$$

Law V $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \quad y \neq 0$

The power of a quotient is the quotient of the powers.

EXAMPLE

$$\left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$$

Law II of exponents gives rise to the necessity of two further definitions to expand the meaning of exponents. As written in Law II above, $x^a \div x^b = x^{a-b}$, $x \neq 0$, actually would only apply when $a > b$, if we restrict ourselves to positive integers. To expand the concept of exponent let us consider the cases when $a = b$ and $a < b$.

If $a = b$ and $x \neq 0$, then

$$\frac{x^a}{x^a} = x^{a-a} = x^0$$

but

$$\frac{x^a}{x^a} = 1$$

since any nonzero quantity divided by itself is 1. This observation gives rise to the following definition.

DEFINITION: $x^0 = 1$ for all $x \neq 0$.

When $a < b$, $\dfrac{x^a}{x^b} = x^{a-b}$ would give rise to a negative exponent. For instance

$$\frac{x^3}{x^5} = x^{3-5} = x^{-2}$$

From the definition of exponent, however, and a basic principle of simplifying fractions we have

$$\frac{x^3}{x^5} = \frac{(x)(x)(x)}{(x)(x)(x)(x)(x)} = \frac{1}{x^2}$$

Hence the following definition.

DEFINITION: $x^{-a} = \dfrac{1}{x^a}$, a is a positive integer and $x \neq 0$.

Notice that the definitions are consistent with the laws of exponents, and therefore Laws I to V will still be valid when we allow exponents to be *any* integer.

EXERCISE 1.2.1

Simplify the following. Leave all answers with only positive exponents.

1. $(2x^2)^4$

2. $(3xy^2)^3$

3. $\left(\dfrac{2}{x^2y}\right)^3$

4. $[(x^2)^3]^4$

5. $\left(\dfrac{xy^2}{x^2y}\right)^2$

6. $\left(\dfrac{2x^3}{xy^2}\right)^3$

7. $[2(xy)^2]^3$

8. $\left(\dfrac{1}{3x^2}\right)^2$

9. $\dfrac{(x^2)^3}{(x^3)^2}$

10. $\dfrac{(-3x)^3}{9x}$

11. $\dfrac{-4x^2}{(2x)^3}$

12. $\dfrac{(2x^2y^3)^3}{(2x^3y^2)^2}$

13. $2x^2y(x^2y)^3$

14. $(-x^2y)^3(-2xy^3)^2$

15. $[2(xy)^2]^5[3(x^2y)^3]^2$

16. $\left(\dfrac{x}{y}\right)^3 \left(\dfrac{2x}{y}\right)^4$

17. $\left(\frac{-2}{x}\right)^3 \quad \left(\frac{x}{2}\right)^2$

18. $\left(\frac{x^2y}{3}\right) \quad \left(\frac{3}{xy^2}\right)^2$

19. $\left(\frac{-2x}{5y^2}\right)^3 \quad \left(\frac{5y}{4x^2}\right)^2$

20. $\left(\frac{x^3}{8}\right)^2 \quad \left(\frac{4}{x^2}\right)^3$

21. $x^{-3}y^{-5}$

22. $\left(\frac{a}{b}\right)^{-5}$

23. $\frac{1}{3^{-2}}$

24. $(ab)^{-1}$

25. $(a + b)^{-1}$

26. $a^{-1} + b^{-1}$

27. $x^{-2}y^4z^{-1}$

28. $(x^3)^{-2}$