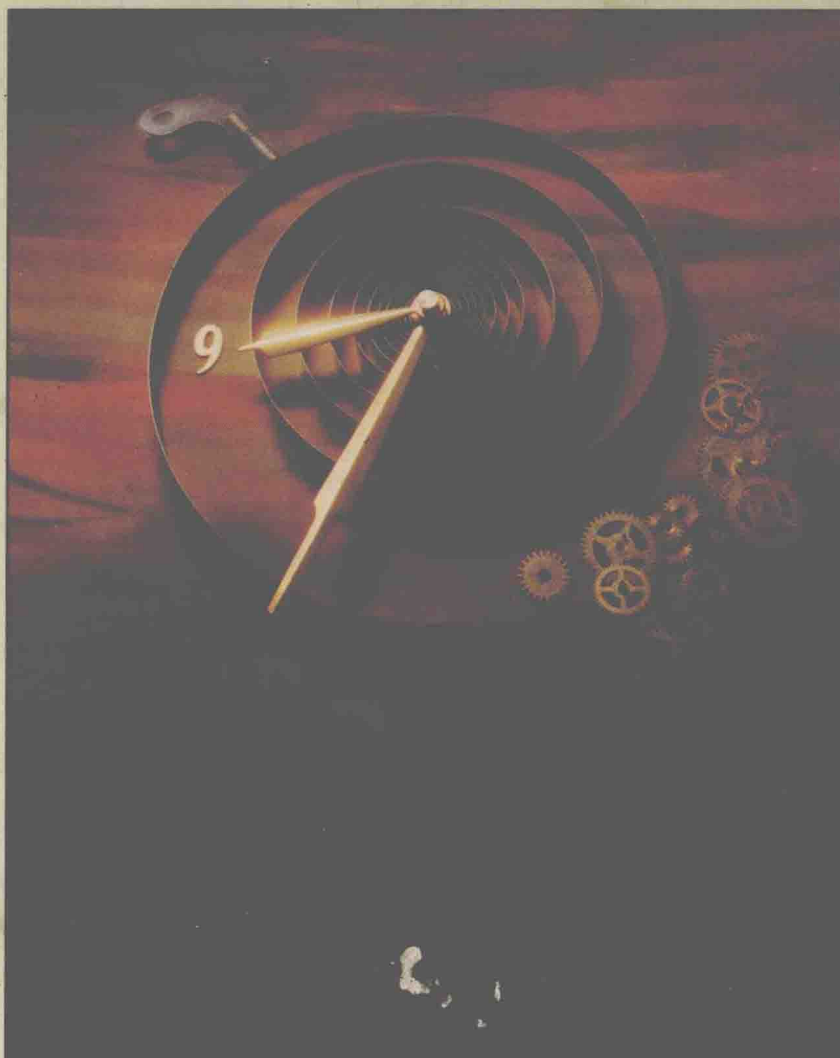
PRECALCULUS

ENHANCED WITH GRAPHING UTILITIES



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Precalculus

Enhanced with Graphing Utilities

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For Our Students . . .
Past and Present

Preface

To the Instructor

Times certainly are changing. This text represents two generations of mathematics learning and teaching in the Sullivan family.

As the father, I have been teaching at an urban public university for over 30 years. Many things have changed during those years. . . . Students today have many varied needs—some students have very little mathematical background and a fear of the subject, while others are extremely motivated and have a strong educational background. Many are trying to balance family, work and school. All of these added pressures enter into the classroom. To be able to reach these students and excite them about the subject I love is truly wonderful. Technology, too, is changing. For me, as with many of my colleagues, the move has been from slide rules to arithmetic/scientific calculators to graphing calculators and computer algebra systems. As a result, our careers have been exciting, challenging, and, most importantly, filled with a wide range of learning experiences.

As the son, I grew up with teaching and learning all around me. Having completed dual advanced degrees in economics and mathematics has impacted the way I think about college courses . . . applications and skills blend together. Unlike my father, technology was in every aspect of my college career. Now as a mathematics teacher at a community college, I have the opportunity to see the strengths of the traditional foundations of mathematics combined with the exploratory nature of technology and the power of real world applications. For many of my students, the visual nature of mathematics through technology compels them to become more active in the classroom. Being able to use real world data, do experiments, and work in groups has made learning interesting again to them.

Welcome to Precalculus: Enhanced with Graphing Utilities. We hope you enjoy using it as much as we do.

Michael Sullivan

Michael Sullivan, III

Our Philosophy on Technology

MAA, AMATYC, and NCTM all recommend the appropriate use of technology in the classroom and the mathematics laboratory. Appropriate is the difficult word to define. Our text assumes immediate access to graphing utilities by both students and instructors. We fully utilize the graphers ability to promote visualization, exploration, foreshadowing of concepts, data analysis, curve fitting and most of all, excitement in the classroom. We have not, however, thrown out all hand work. By blending together technology and traditional methods of problem solving, students are empowered with knowledge and ability that they will use throughout their lives.

How we integrate technology through the text is subtle—no big boxes, no icons. You will see both T1-82 screens and traditional line art where appropriate. Many examples are solved in two ways: with a graphing solution and with an algebraic solution. Many exercises are multi-tasked, beginning with questions that require the use of algebra or trigonometry and ending with questions that require not just a graph, but also an analysis of the graph. We encourage students to distinguish between solving a given problem using the full power of the technology vs. recognizing when the technology is limited or the simple nature of the problem makes a solution by hand the better choice.

One very effective use of a graphing utility is to develop the ability of students to recognize patterns. With a graphing utility, students can quickly and effectively start to recognize patterns; they can actually 'see' the mathematics, resulting in a better conceptual understanding of the concept at hand. Throughout the text, we have students graph related functions on the same screen, asking them leading questions to help them recognize patterns.

Another strong reason to use the technologies available is to move away from contrived and 'simple' numbers. Students can work with real information involving 'messy' numbers in a way that traditionally they could not do. Without taking away from the conceptual understanding of the objective at hand, students can analyze and interpret real data. Using this premise, we have included problems using real world data, as well as CBL exercises where students can actively collect their own data and manipulate it.

Technology can also be used to introduce ideas and solve problems that go beyond the traditional limitations of precalculus. For example, many concepts and problems typically covered in a calculus class can be investigated and solved using graphing utilities. Determining where a function is increasing and decreasing and where its local maxima and minima occur are topics that can now be introduced, discussed, and solved in Precalculus, thanks to the technology. Curve fitting is another topic that can be introduced, discussed, and solved, enabling students to relate algebra skills with data analysis.

These are a few of the uses made of technology in *Precalculus: Enhanced with Graphing Utilities.* A graphing utility is required for the student to use this text. If you or your school would like to use graphing utilities, but not require them, *Precalculus,* 4th Edition, by Michael Sullivan, which provides an optional use of graphing utilities, is also available. To receive an examination copy, contact your Prentice Hall representative.

About This Book

Content The content of this book is not different than that of a traditional Precalculus—but the emphasis and the approach is! For example, we approach the idea of Horizontal Shifts by first exploring the graphs of several functions using a graphing utility and then drawing a conclusion about what is happening. Also, many Examples and Exercises found in this text are ones that cannot be handled using only traditional methods.

Organization With the use of fully integrated technology, the order of presentation of certain topics shifts.

To emphasize the focus of the book, Chapter 1 begins with graphing. In recognition of the fact that some students may not be fully prepared to start here, an Appendix has been carefully designed to provide necessary review. The sections of the Appendix that apply are referenced at the beginning of a Chapter and also within the body of the chapter.

Chapter 1 treats the graphs of certain key equations, including the line and the circle. A section on Linear Curve Fitting demonstrates the power of a graphing utility to solve applied problems in this area.

Chapter 2 develops the concept of a function, the graphs of functions, and properties of functions. Chapter 3 solves equations and inequalities utilizing both graphing and algebraic techniques.

Chapter 4 discusses Polynomial and Rational Functions, with an emphasis on analyzing their graphs. Chapter 5, Exponential and Logarithmic Functions, also places emphasis on the analysis of graphs, but, in addition, contains an entire section devoted to Nonlinear Curve Fitting.

In Chapter 6, Trigonometric Functions, the trigonometric functions are introduced using the unit circle approach. Later it is shown that right triangle trigonometry is a special case. The graphs of the trigonometric functions are then discussed. The chapter concludes with the inverse trigonometric functions.

Chapter 7, Analytic Trigonometry, deals with trigonometric identities and trigonometric equations. Chapter 8, Applications of Trigonometric Functions, provides applications of trigonometry to solving triangles, both right and oblique. Also included is a discussion of sinusoidal graphs, harmonic motion, and damped vibrations.

Chapter 9, Polar Coordinates; Vectors, introduces polar coordinates, with an emphasis on graphing. Also included are vectors and the dot product. Chapter 10, Analytic Geometry, discusses the conics, including rotation of axes and polar forms. Parametric equations are also included.

Chapter 11, Systems of Equations and Inequalities, discusses both graphing and algebraic approaches. Many examples and exercises found here can only be solved using technology. Chapter 12, Sequences; Induction; Counting; Probability, is mostly traditional.

Examples Recognizing that many students learn through working problems by hand as well as through the use of a graphing utility, we have provided both traditional and technological examples of solving problems. Examples are worked out in appropriate detail, starting with simple, reasonable problems and working gradually up to more challenging ones. There are no magic steps and we encourage and often show the final step of checking the result. Many of the examples involve applications that will be seen in calculus or in other disciplines. At the end of many examples there are "Now Work" suggestions which refer students to an odd numbered problem in section exercises which is similar to the worked out example shown. This allows students immediate feedback on whether they understand the concept clearly before moving on.

Exercises The exercises in the text are mostly of three types: visual—where students are asked to draw conclusions about a graph; technological—where the students will fully utilize the power of their graphing utility; and open-ended—where critical thinking, writing, research or collaborative effort is required. The text contains over 5500 tested and true exercises, over 900 are applied problems. Exercise sets begin with problems designed to build confidence, continue with problems which relate to worked out examples in the text and conclude with problems that are more challenging. Many of the problems, especially those at the beginning are visual in character—such as showing a graph and asking for conclusions.

Answers are given in the back of the text for all the odd-numbered problems. Fully worked out, step by step solutions for the odd-numbered problems are found in the **Student's Solutions Manual,** while the even solutions are equally worked out in the **Instructor's Solutions Manual.**

Illustrations The design uses color effectively and functionally to help the student identify definitions, theorems, formulas and procedures. Included in the Preface to the Student is an overview of how these colorized elements can be helpful when studying. Many illustrations have been included to provide a dynamic realism to selected examples and exercises. All the graphing utility and line art has been computer generated for consistency and accuracy. For the purpose of clarity, all line art utilizes two colors. With over 1600 pieces of art throughout the text we aim to help the student to visualize mathematics and show how to use it to solve mathematical problems.

Applications Every opportunity has been taken to present understandable, realistic applications consistent with the abilities of the student, drawing from such sources as tax rate tables, the Guinness Book of World Records, Government publications and newspaper articles. For added interest, some of the applied exercises have been adapted from textbooks the students may be using in other courses (such as economics, chemistry, physics, etc.)

Communication and Problem Solving The recommendations of the NCTM, AMATYC and MAA support the inclusion of writing, verbalizing, research and critical thinking in mathematics. Throughout the text there are many ways we encourage these activities. In the exercises, these kind of problems are designated by a . All of these problems not only ask the student to write, discuss or to do active problem solving, but they also drive forward the concepts of the section.

Collaborative Projects In each chapter, a full page "Mission Possible" has been devoted to collaborative learning. These multi-tasked projects, written by Hester

Lewellen, one of the co-authors of the University of Chicago High School Mathematics Project, will help your students work together to solve some unusual problems. Some of the projects require utilizing the graphing utility but all require critical thinking and communication. Suggested solutions to the collaborative projects are found in the Instructor's Solutions Manual.

Calculus While some of your students may not be going on to calculus after this course, many will. To encourage students and to let them preview calculus, we have included many examples and exercises that foreshadow topics found in calculus.

CBL Projects In key places in the text, CBL (Calculator Based Laboratory) experiments are given in the exercises. Each demonstrates real world applications of the topics just covered. Students are asked to perform experiments and collect and analyze the data obtained.

Instructor's Supplementary Aids

Instructor's Solutions Manual Contains complete step-by-step worked out solutions to all the even numbered exercises in the textbook. Also included are strategies for using the collaborative learning projects found in each chapter. Transparency masters which duplicate important illustrations in the text can be found as well.

Video Review A new videotape series which has been created to accompany the Sullivan texts include a half hour review of the most important topics in each chapter. Each segment uses both traditional and technological ways of solving mathematical problems. Entertaining and educational, these videos provide an alternative process which can add to your students' success in this course. Also included is a graphing calculator video tutorial which walks one through the most common uses of the various calculators. Written by mathematics teachers, these videos concentrate on those topics that 'get them every time'.

Written Test Item File Features six tests per chapter plus four forms of a final examination, prepared and ready to be photocopied. Of these tests, three are multiple choice and three are free response.

Prentice Hall Custom Test (Computerized Testing Generator - Mac and Windows)
PH Custom Test is a fully networkable, easy to use test generator. Instructors may select questions by objective, section, chapter or use a ready-made test for each chapter. As the questions are algorithmically generated, an instructor can create up to 99 versions of each question while keeping the problem type and objective constant. PH Custom Test allows for on-line testing and offers a gradebook feature that not only organizes test grades but can be used for any other classroom grades or information. Instructors can download questions into word processing programs or create their own problems and insert them into PH Custom Test. Graphics and mathematical symbols are integrated into the program and other graphics from programs such as Mathematica, Maple, Derive or Matlab can be imported into the program.

Student's Supplementary Aids

Student's Solutions Manual Contains complete step-by-step worked out solutions to all the odd numbered exercises in the textbook. This is terrific for getting instant feedback for your students.

Prentice Hall Tutorial Program (Software tutorial both Mac and Windows) This computerized tutorial program is based on the highly successful Prentice Hall testing program. Utilizing the same algorithms programmed for the test generator, students are able to pretest their abilities, receive a diagnostic recommendation for further study, work through a step-by-step tutorial—complete with a graphing utility built into the program, and tutorial tests. The software is fully networkable and can be used with the gradebook feature in the Prentice Hall Custom Test.

Visual Precalculus A software package for IBM compatible computers which consists of two parts. Part One contains routines to graph and evaluate functions, graph conic sections, investigate series, carry out synthetic division and illuminate important concepts with animation. Part Two contains routines to solve triangles, graph systems of linear equations and inequalities, evaluate matrix expressions, apply Gaussian elimination to reduce or invert matrices and graphically solve linear programming problems. Those routines will provide additional insights to the material covered within the text.

X(PLORE) A powerful (yet inexpensive) fully programmable symbolic and numeric mathematical processor for IBM and Macintosh computers. This program will allow your students to evaluate expressions, graph curves, solve equations and use matrices. This software package may also be used for calculus or differential equations.

New York Times Supplement A free newspaper from Prentice Hall and the New York Times which includes interesting and current articles on mathematics in the world around us. Great for getting students to talk and write about mathematics. This supplement is created new each year.

For any of the above supplements, please contact your Prentice Hall representative.

Acknowledgments

Textbooks are written by an author, but evolve from an idea into final form through the efforts of many people. Special thanks to Don Dellen, who first suggested this book and the other books in this series.

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Preface

To the Student

As you begin your study of precalculus, you might feel overwhelmed by the number of theorems, definitions, procedures and equations that confront you. You may even wonder whether you can learn all this material in a single course. For many of you, this may be your last mathematics course, while for others, just the first in a series of many. Don't worry—either way, this text was written with you in mind.

This text was designed to help you—the student, master the terminology and basic concepts of precalculus. These aims have helped to shape every aspect of the book. Many learning aids are built into the format of the text to make your study of this material easier and more rewarding. This book is meant to be a "machine for learning," one that can help you to focus your efforts and get the most from the time and energy you invest.

This book requires that you have access to a graphing utility: a graphing calculator or a computer software package that has a graphing component. Be sure you have some familiarity with the device you are using before the course begins.

Here are some hints we give our students at the beginning of the course:

1. Take advantage of the feature PREPARING FOR THIS CHAPTER. At the beginning of each chapter, we have prepared a list of topics to review. Be sure to take the time to do this. It will help you proceed quicker and more confidently through the chapter.
2. Read the material in the book before the lecture. Knowing what to expect and what is in the book, you can take fewer notes and spend more time listening and understanding the lecture.
3. After each lecture, rewrite your notes as you re-read the book, jotting down any additional facts that seem helpful. Be sure to do the Now Work Problem x as you proceed through a section. After completing a section, be sure to do the assigned problems. Answers to the Odd ones are in the back of the book.
4. If you are confused about something, visit your instructor during office hours immediately, before you fall behind. Bring your attempted solutions to problems with you to show your instructor where you are having trouble.
5. To prepare for an exam, review your notes. Then proceed through the Chapter Review. It contains a capsule summary of all the important material of the chapter. If you are uncertain of any concept, go back into the chapter and study it further. Be sure to do the Review Exercises for practice.

Remember the two "golden rules" of precalculus:

1. DON'T GET BEHIND! The course moves too fast, and it's hard to catch up.
2. WORK LOTS OF PROBLEMS. Everyone needs to practice, and problems show where you need more work. If you can't solve the homework problems without help, you won't be able to do them on exams.

We encourage you to examine the following overview for some hints on how to use this text.

Best Wishes!

Michael Sullivan
Michael Sullivan, III

OVERVIEW

Chapter 2

PREPARING FOR THIS CHAPTER

Before getting started on this chapter, review the following concepts;

Topics from Algebra and Geometry (Appendix, Section 1)
Graphs of certain equations (Example 2, p. 15;
Example 4, p. 17; Example 5, p. 18; Example 16, p. 30)
Tests for symmetry of an equation (p. 28)
Procedure for finding intercepts of an equation (p. 22)

FUNCTIONS AND THEIR GRAPHS

Preview Getting from an Island to Town

An island is 2 miles from the nearest point P on a straight shoreline. A town is 12 miles down the shore from P.

(a) If a person can row a boat at an average speed of 3 miles per hour and the same person can walk 5 miles per hour, express the time T it takes to go from the island to town as a function of the distance x from P to where the person lands the boat.

(b) What is the domain of T?

(c) How long will it take to travel from the island to town if the person lands the boat 4 miles from P?

(d) How long will it take if the person lands the boat 8 miles from P?

(e) Use a graphing utility to graph the function T = T(x).

(f) Use the TRACE function to see how the time T varies as x changes from 0 to 12.

(g) What value of x results in the least time?
[Example 9 in Section 2.1]

Each chapter begins with a list of concepts to review. Refresh your memory by turning to the pages listed.

A section by section outline is also provided. This is a good way to organize your notes for studying.

Highlighting an application from the chapter, you are given a "Preview" of a coming use of the mathematics introduced in this chapter.

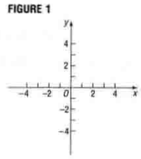
The idea of using a system of rectangular coordinates dates back to ancient times, when such a system was used for surveying and city planning. Apollonius of Perga, in 200 BC, used a form of rectangular coordinates in his work on conics, although this use does not stand out as clearly as it does in modern treatments. Sporadic use of rectangular coordinates continued until the 1600's. By that time, algebra had developed sufficiently so that René Descartes (1596–1650) and Pierre de Fermat (1601–1665) could take the crucial step, which was the use of rectangular coordinates to translate geometry problems into algebra problems, and vice versa. This step was supremely important for two reasons. First, it allowed both geometers and algebraists to gain critical new insights into their subjects, which previously had been regarded as separate but now were seen to be connected in many important ways. Second, the insights gained made possible the development of calculus, which greatly enlarged the number of areas in which mathematics could be applied and made possible a much deeper understanding of these areas. With the advent of technology, in particular, graphing utilities, we are now able not only to visualize the dual roles of algebra and geometry, but we are also able to solve many problems that before this technology required advanced methods.

1.1 Rectangular Coordinates; Graphing Utilities

We locate a point on the real number line by assigning it a single real number, called the *coordinate of the point*. For work in a two-dimensional plane, we locate points by using two numbers.

We begin with two real number lines located in the same plane: one horizontal and the other vertical. We call the horizontal line the **x-axis,** the vertical line the **y-axis,** and the point of intersection the **origin *O*.** We assign coordinates to every point on these number lines as shown in Figure 1, using a convenient scale. In mathematics, we usually use the same scale on each axis; in applications, a different scale is often used on each axis.

FIGURE 1

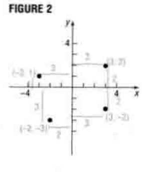


The origin O has a value of 0 on both the x-axis and the y-axis. We follow the usual convention that points on the x-axis to the right of O are associated with positive real numbers, and those to the left of O are associated with negative real numbers. Those on the y-axis above O are associated with positive real numbers, and those below O are associated with negative real numbers. In Figure 1, the x-axis and y-axis are labeled as x and y, respectively, and we have used an arrow at the end of each axis to denote the positive direction.

The coordinate system described here is called a **rectangular,** or **Cartesian* coordinate system.** The plane formed by the x-axis and y-axis is sometimes called the **xy-plane,** and the x-axis and y-axis are referred to as the **coordinate axes.**

Any point P in the xy-plane can then be located by using an **ordered pair** (x, y) of real numbers. Let x denote the signed distance of P from the y-axis (*signed* in the sense that, if P is to the right of the y-axis, then $x > 0$, and if P is to the left of the y-axis, then $x < 0$); and let y denote the signed distance of P from the x-axis. The ordered pair (x, y), also called the **coordinates** of P, then gives us enough information to locate the point P in the plane.

FIGURE 2



For example, to locate the point whose coordinates are $(-3, 1)$, go 3 units along the x-axis to the left of O and then go straight up 1 unit. We **plot** this point by placing a dot at this location. See Figure 2, in which the points with coordinates $(-3, 1)$, $(-2, -3)$, $(3, -2)$, and $(3, 2)$ are plotted.

The origin has coordinates $(0, 0)$. Any point on the x-axis has coordinates of the form $(x, 0)$, and any point on the y-axis has coordinates of the form $(0, y)$.

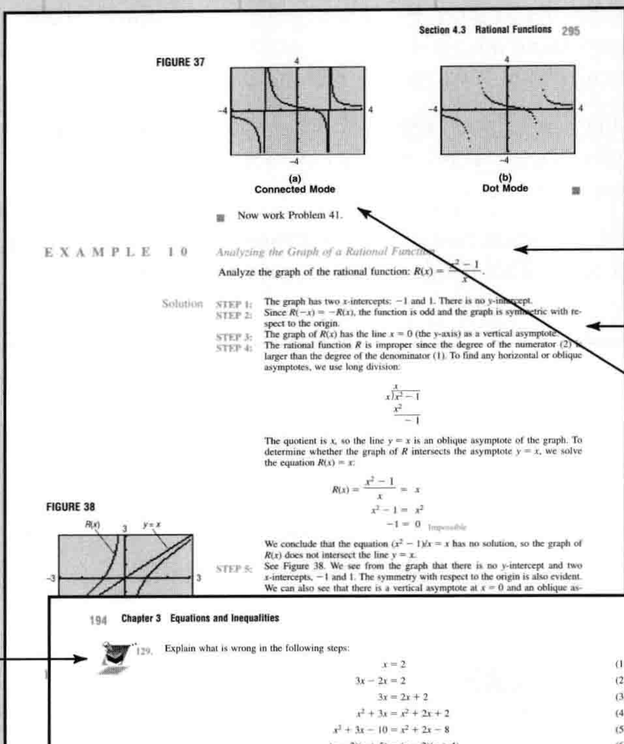
If (x, y) are the coordinates of a point P, then x is called the **x-coordinate,** or **abscissa,** of P and y is the **y-coordinate,** or **ordinate,** of P. We identify the

*Named after René Descartes (1596–1650), a French mathematician, philosopher, and theologian.

2

Most chapters open with a brief historical discussion of "where this material came from". It is helpful to understand how others created and used these ideas to solve their everyday problems.

New terms appear in boldface type where they are defined.



Section 4.3 Rational Functions 295

FIGURE 37

(a) Connected Mode

(b) Dot Mode

■ Now work Problem 41.

EXAMPLE 10 *Analyzing the Graph of a Rational Function*

Analyze the graph of the rational function: $R(x) = \frac{x^2 - 1}{x}$.

Solution STEP 1: The graph has two x-intercepts: -1 and 1. There is no y-intercept.
STEP 2: Since $R(-x) = -R(x)$, the function is odd and the graph is symmetric with respect to the origin.
STEP 3: The graph of $R(x)$ has the line $x = 0$ (the y-axis) as a vertical asymptote.
STEP 4: The rational function R is improper since the degree of the numerator (2) is larger than the degree of the denominator (1). To find any horizontal or oblique asymptotes, we use long division:

The quotient is x, so the line $y = x$ is an oblique asymptote of the graph. To determine whether the graph of R intersects the asymptote $y = x$, we solve the equation $R(x) = x$:

$$R(x) = \frac{x^2 - 1}{x} = x$$
$$x^2 - 1 = x^2$$
$$-1 = 0 \quad \text{Impossible}$$

FIGURE 38

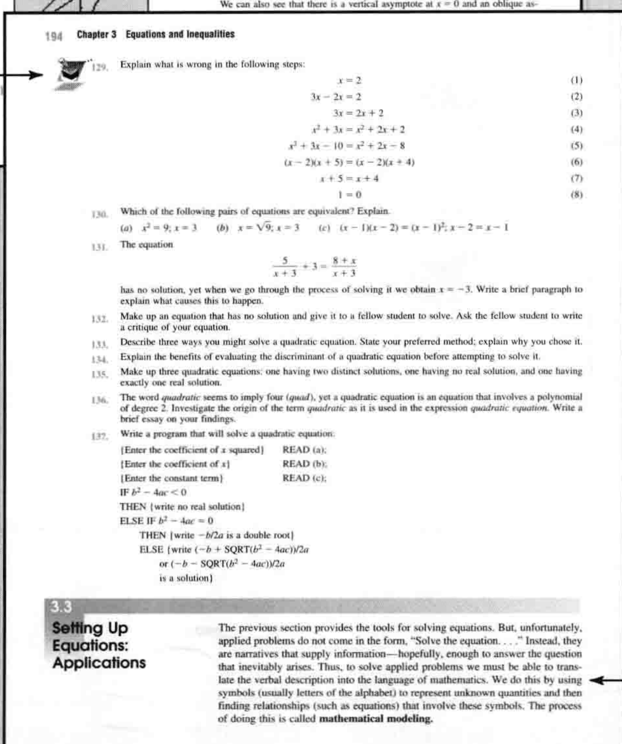
We conclude that the equation $(x^2 - 1)/x = x$ has no solution, so the graph of $R(x)$ does not intersect the line $y = x$.

STEP 5: See Figure 38. We see from the graph that there is no y-intercept and two x-intercepts, -1 and 1. The symmetry with respect to the origin is also evident. We can also see that there is a vertical asymptote at $x = 0$ and an oblique as-

Important Procedures and STEPS are noted in the left column and are separated from the body of the text by two horizontal color rules. You will need to know these procedures and steps to do your homework problems and prepare for exams.

Examples are easy to locate and are titled to tell you what concept they highlight. Most examples work the solution first with the graphing utility and then algebraically.

The Now Work Problem xx feature asks you to do a particular problem before you go on in the section. This is to ensure that you have mastered the material just presented before going on. By following the practice of doing these Now Work Problems, you will gain confidence and save time.



194 Chapter 3 Equations and Inequalities

129. Explain what is wrong in the following steps:

$x = 2$ (1)
$3x - 2x = 2$ (2)
$3x = 2x + 2$ (3)
$x^2 + 3x = x^2 + 2x + 2$ (4)
$x^2 + 3x - 10 = x^2 + 2x - 8$ (5)
$(x - 2)(x + 5) = (x - 2)(x + 4)$ (6)
$x + 5 = x + 4$ (7)
$1 = 0$ (8)

130. Which of the following pairs of equations are equivalent? Explain.
(a) $x^2 = 9; x = 3$ (b) $x = \sqrt{9}; x = 3$ (c) $(x - 1)(x - 2) = (x - 1)^2; x - 2 = x - 1$

131. The equation

$$\frac{5}{x+3} + 3 = \frac{8+x}{x+3}$$

has no solution, yet when we go through the process of solving it we obtain $x = -3$. Write a brief paragraph to explain what causes this to happen.

132. Make up an equation that has no solution and give it to a fellow student to solve. Ask the fellow student to write a critique of your equation.

133. Describe three ways you might solve a quadratic equation. State your preferred method; explain why you chose it.

134. Explain the benefits of evaluating the discriminant of a quadratic equation before attempting to solve it.

135. Make up three quadratic equations: one having two distinct solutions, one having no real solution, and one having exactly one real solution.

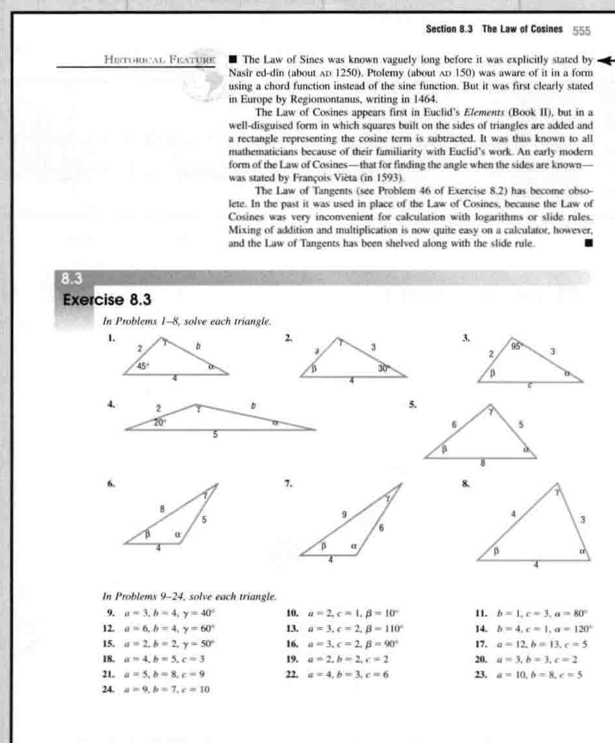
136. The word *quadratic* seems to imply four (*quad*), yet a quadratic equation is an equation that involves a polynomial of degree 2. Investigate the origin of the term *quadratic* as it is used in the expression *quadratic equation*. Write a brief essay on your findings.

137. Write a program that will solve a quadratic equation:

```
{Enter the coefficient of x squared}   READ (a);
{Enter the coefficient of x}           READ (b);
{Enter the constant term}              READ (c);
IF b^2 - 4ac < 0
THEN {write no real solution}
ELSE IF b^2 - 4ac = 0
    THEN {write -b/2a is a double root}
    ELSE {write (-b + SQRT(b^2 - 4ac))/2a
        or (-b - SQRT(b^2 - 4ac))/2a
        is a solution}
```

3.3 Setting Up Equations: Applications

The previous section provides the tools for solving equations. But, unfortunately, applied problems do not come in the form, "Solve the equation. . . ." Instead, they are narratives that supply information—hopefully, enough to answer the question that inevitably arises. Thus, to solve applied problems we must be able to translate the verbal description into the language of mathematics. We do this by using symbols (usually letters of the alphabet) to represent unknown quantities and then finding relationships (such as equations) that involve these symbols. The process of doing this is called **mathematical modeling.**



Section 8.3 The Law of Cosines 555

HISTORICAL FEATURE ■ The Law of Sines was known vaguely long before it was explicitly stated by Nasîr ed-dîn (about AD 1250). Ptolemy (about AD 150) was aware of it in a form using a chord function instead of the sine function. But it was first clearly stated in Europe by Regiomontanus, writing in 1464.

The Law of Cosines appears first in Euclid's *Elements* (Book II), but in a well-disguised form in which squares built on the sides of triangles are added and a rectangle representing the cosine term is subtracted. It was thus known to all mathematicians because of their familiarity with Euclid's work. An early modern form of the Law of Cosines—that for finding the angle when the sides are known—was stated by François Viète (in 1593).

The Law of Tangents (see Problem 46 of Exercise 8.2) has become obsolete. In the past it was used in place of the Law of Cosines, because the Law of Cosines was very inconvenient for calculation with logarithms or slide rules. Mixing of addition and multiplication is now quite easy on a calculator, however, and the Law of Tangents has been shelved along with the slide rule. ■

8.3 Exercise 8.3

In Problems 1–8, solve each triangle.

1. 2. 3. 4. 5. 6. 7. 8.

In Problems 9–24, solve each triangle.

9. $a = 3, b = 4, \gamma = 40°$ 10. $a = 2, c = 1, \beta = 10°$ 11. $b = 1, c = 3, \alpha = 80°$
12. $a = 6, b = 4, \gamma = 60°$ 13. $a = 3, c = 2, \beta = 110°$ 14. $b = 4, c = 1, \alpha = 120°$
15. $a = 2, b = 2, \gamma = 50°$ 16. $a = 3, c = 2, \beta = 90°$ 17. $a = 12, b = 13, c = 5$
18. $a = 4, b = 5, c = 3$ 19. $a = 2, b = 2, c = 2$ 20. $a = 3, b = 3, c = 2$
21. $a = 5, b = 8, c = 9$ 22. $a = 4, b = 3, c = 6$ 23. $a = 10, b = 8, c = 5$
24. $a = 9, b = 7, c = 10$

The "pencil and book" icon is used to indicate open-ended questions for discussion, writing, group or research projects.

How to solve word problems is explained in a step-by-step manner using mathematical modeling.

Historical Features place the mathematics you are learning in a historical context. By learning how others have used similar concepts you will understand how they may be used in your own life.

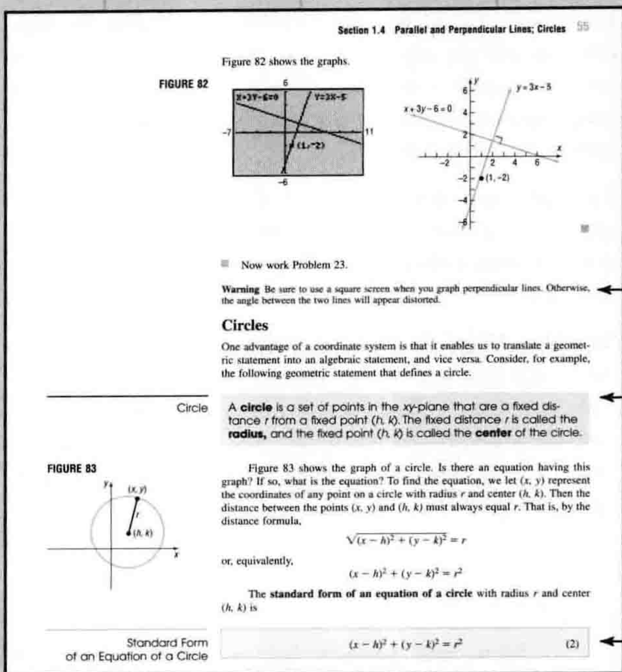


Figure 82 shows the graphs.

FIGURE 82

■ Now work Problem 23.

Warning Be sure to use a square screen when you graph perpendicular lines. Otherwise, the angle between the two lines will appear distorted.

Circles

One advantage of a coordinate system is that it enables us to translate a geometric statement into an algebraic statement, and vice versa. Consider, for example, the following geometric statement that defines a circle.

Circle

A **circle** is a set of points in the xy-plane that are a fixed distance r from a fixed point (h, k). The fixed distance r is called the **radius**, and the fixed point (h, k) is called the **center** of the circle.

FIGURE 83

Figure 83 shows the graph of a circle. Is there an equation having this graph? If so, what is the equation? To find the equation, we let (x, y) represent the coordinates of any point on a circle with radius r and center (h, k). Then the distance between the points (x, y) and (h, k) must always equal r. That is, by the distance formula,

$$\sqrt{(x-h)^2+(y-k)^2}=r$$

or, equivalently,

$$(x-h)^2+(y-k)^2=r^2$$

The **standard form of an equation of a circle** with radius r and center (h, k) is

Standard Form of an Equation of a Circle

$$(x-h)^2+(y-k)^2=r^2 \qquad (2)$$

Hints or warnings are offered where appropriate. Sometimes there are short cuts or pitfalls that students should know about—we've included them.

Major definitions appear in large type enclosed within a color screen. These are important vocabulary items for you to know.

All important formulas are enclosed by a box and shown in color. This is done to alert you to important concepts.

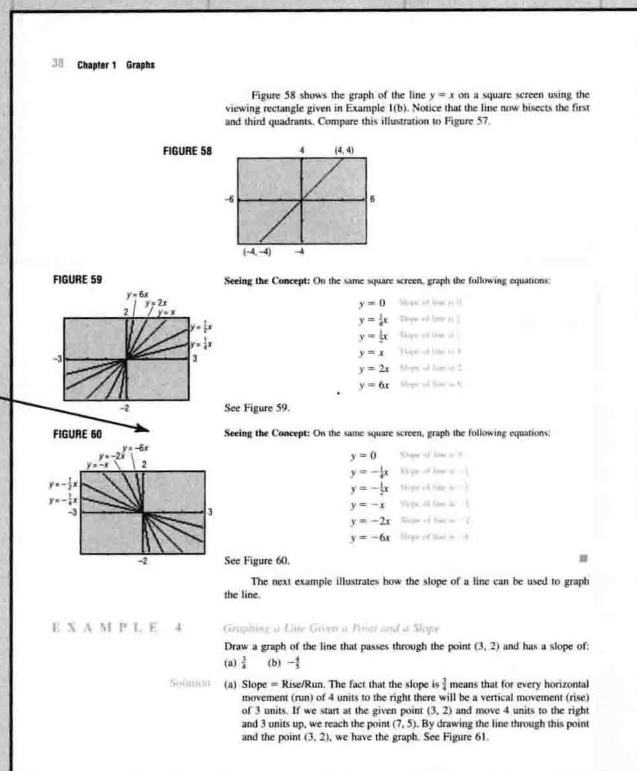


Figure 58 shows the graph of the line $y = x$ on a square screen using the viewing rectangle given in Example 1(b). Notice that the line now bisects the first and third quadrants. Compare this illustration to Figure 57.

FIGURE 58

FIGURE 59

Seeing the Concept: On the same square screen, graph the following equations:

$y = 0$
$y = \frac{1}{4}x$
$y = \frac{1}{2}x$
$y = x$
$y = 2x$
$y = 6x$

See Figure 59.

FIGURE 60

Seeing the Concept: On the same square screen, graph the following equations:

$y = 0$
$y = -\frac{1}{4}x$
$y = -\frac{1}{2}x$
$y = -x$
$y = -2x$
$y = -6x$

See Figure 60. ■

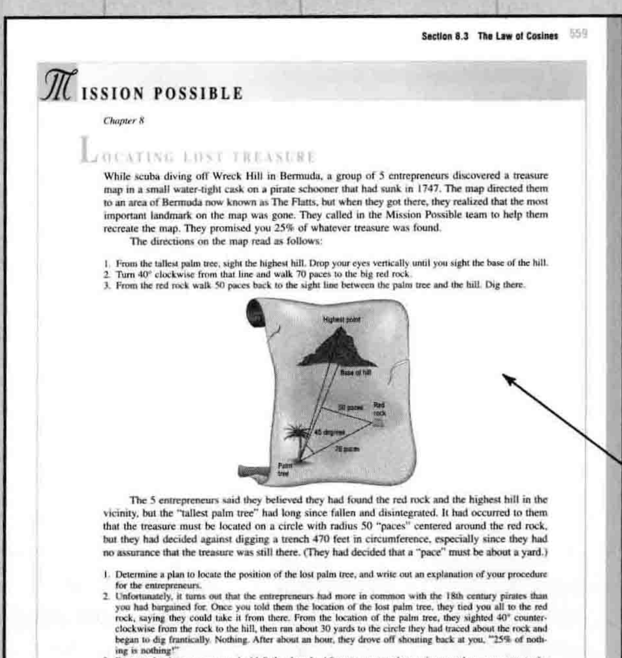
The next example illustrates how the slope of a line can be used to graph the line.

EXAMPLE 4 *Graphing a Line Given a Point and a Slope*

Draw a graph of the line that passes through the point (3, 2) and has a slope of:
(a) $\frac{3}{4}$ (b) $-\frac{4}{5}$

Solution (a) Slope = Rise/Run. The fact that the slope is $\frac{3}{4}$ means that for every horizontal movement (run) of 4 units to the right there will be a vertical movement (rise) of 3 units. If we start at the given point (3, 2) and move 4 units to the right and 3 units up, we reach the point (7, 5). By drawing the line through this point and the point (3, 2), we have the graph. See Figure 61.

"Seeing the Concept." Use your graphing utility to recognize patterns in the algebra.



MISSION POSSIBLE

Chapter 8

LOCATING LOST TREASURE

While scuba diving off Wreck Hill in Bermuda, a group of 5 entrepreneurs discovered a treasure map in a small water-tight cask on a pirate schooner that had sunk in 1747. The map directed them to an area of Bermuda now known as The Flatts, but when they got there, they realized that the most important landmark on the map was gone. They called in the Mission Possible team to help them recreate the map. They promised you 25% of whatever treasure was found.

The directions on the map read as follows:

1. From the tallest palm tree, sight the highest hill. Drop your eyes vertically until you sight the base of the hill.
2. Turn 40° clockwise from that line and walk 70 paces to the big red rock.
3. From the red rock walk 50 paces back to the sight line between the palm tree and the hill. Dig there.

The 5 entrepreneurs said they believed they had found the red rock and the highest hill in the vicinity, but the "tallest palm tree" had long since fallen and disintegrated. It had occurred to them that the treasure must be located on a circle with radius 50 "paces" centered around the red rock, but they had decided against digging a trench 470 feet in circumference, especially since they had no assurance that the treasure was still there. (They had decided that a "pace" must be about a yard.)

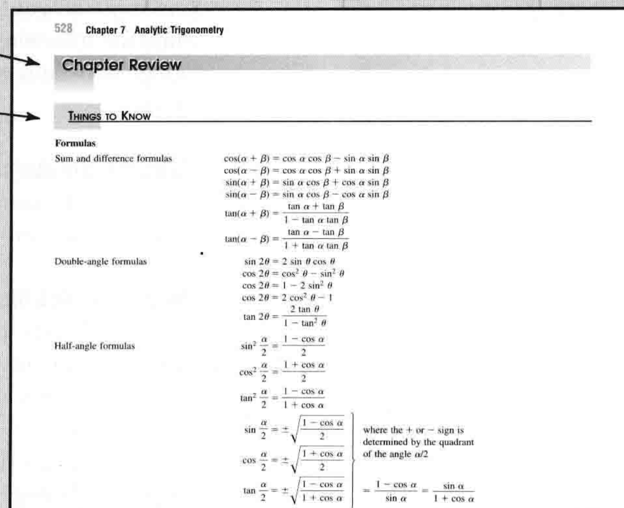
1. Determine a plan to locate the position of the lost palm tree, and write out an explanation of your procedure for the entrepreneurs.
2. Unfortunately, it turns out that the entrepreneurs had more in common with the 18th century pirates than you had bargained for. Once you told them the location of the lost palm tree, they tied you all to the red rock, saying they could take it from there. From the location of the palm tree, they sighted 40° counterclockwise from the rock to the hill, then ran about 30 yards to the circle they had traced about the rock and began to dig frantically. Nothing. After about an hour, they drove off shouting back at you, "25% of nothing is nothing!"
3. Fortunately, the entrepreneurs had left the shovels. After you managed to untie yourselves, you went to the correct location and found the treasure. Where was it? How far from the palm tree? Explain.
4. People who scuba dive for sunken treasure have certain legal obligations. What are they? Should you share the treasure with a lawyer, just to make sure you get to keep the rest?

Mission Possible: In the "real world" colleagues often collaborate to solve more difficult problems—or problems that may have more than one answer. Every chapter includes a "Mission Possible" for you and your classmates to work together. All of these projects require verbal communication or written answers. Good communication skills are very important to becoming successful—no matter what your future holds.

The Chapter Review is for your use in checking your understanding of the chapter materials.

"Things to Know" is the best place to start. Check your understanding of the concepts listed there.

"Important Functions" highlight the functions introduced and used throughout the chapter.



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Chapter Review

THINGS TO KNOW

Formulas

Sum and difference formulas

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Double-angle formulas

$\sin 2\theta=2\sin\theta\cos\theta$

$\cos 2\theta=\cos^2\theta-\sin^2\theta$

$\cos 2\theta=1-2\sin^2\theta$

$\cos 2\theta=2\cos^2\theta-1$

$\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}$

Half-angle formulas

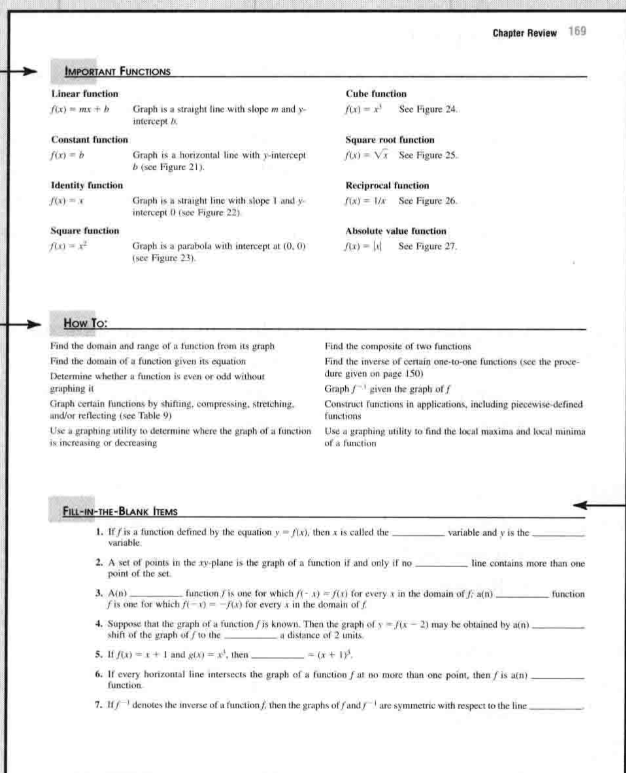
$\sin^2\dfrac{\alpha}{2}=\dfrac{1-\cos\alpha}{2}$

$\cos^2\dfrac{\alpha}{2}=\dfrac{1+\cos\alpha}{2}$

$\tan^2\dfrac{\alpha}{2}=\dfrac{1-\cos\alpha}{1+\cos\alpha}$

$\sin\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1-\cos\alpha}{2}}$, $\cos\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1+\cos\alpha}{2}}$, $\tan\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$ where the + or − sign is determined by the quadrant of the angle $\alpha/2$

$=\dfrac{1-\cos\alpha}{\sin\alpha}=\dfrac{\sin\alpha}{1+\cos\alpha}$



Chapter Review 169

IMPORTANT FUNCTIONS

Linear function
$f(x)=mx+b$ Graph is a straight line with slope m and y-intercept b.

Constant function
$f(x)=b$ Graph is a horizontal line with y-intercept b (see Figure 21).

Identity function
$f(x)=x$ Graph is a straight line with slope 1 and y-intercept 0 (see Figure 22).

Square function
$f(x)=x^2$ Graph is a parabola with intercept at (0, 0) (see Figure 23).

Cube function
$f(x)=x^3$ See Figure 24.

Square root function
$f(x)=\sqrt{x}$ See Figure 25.

Reciprocal function
$f(x)=1/x$ See Figure 26.

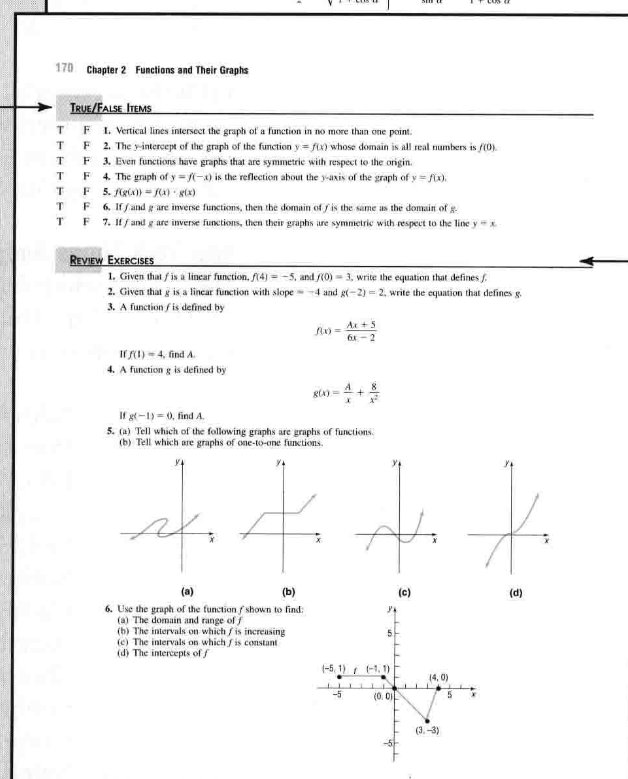
Absolute value function
$f(x)=|x|$ See Figure 27.

HOW TO:

- Find the domain and range of a function from its graph
- Find the domain of a function given its equation
- Determine whether a function is even or odd without graphing it
- Graph certain functions by shifting, compressing, stretching, and/or reflecting (see Table 9)
- Use a graphing utility to determine where the graph of a function is increasing or decreasing
- Find the composite of two functions
- Find the inverse of certain one-to-one functions (see the procedure given on page 150)
- Graph f^{-1} given the graph of f
- Construct functions in applications, including piecewise-defined functions
- Use a graphing utility to find the local maxima and local minima of a function

FILL-IN-THE-BLANK ITEMS

1. If f is a function defined by the equation $y=f(x)$, then x is called the ________ variable and y is the ________ variable.
2. A set of points in the xy-plane is the graph of a function if and only if no ________ line contains more than one point of the set.
3. A(n) ________ function f is one for which $f(-x)=f(x)$ for every x in the domain of f; a(n) ________ function f is one for which $f(-x)=-f(x)$ for every x in the domain of f.
4. Suppose that the graph of a function f is known. Then the graph of $y=f(x-2)$ may be obtained by a(n) ________ shift of the graph of f to the ________ a distance of 2 units.
5. If $f(x)=x+1$ and $g(x)=x^3$, then ________ $=(x+1)^3$.
6. If every horizontal line intersects the graph of a function f at no more than one point, then f is a(n) ________ function.
7. If f^{-1} denotes the inverse of a function f, then the graphs of f and f^{-1} are symmetric with respect to the line ________.



170 Chapter 2 Functions and Their Graphs

TRUE/FALSE ITEMS

T F 1. Vertical lines intersect the graph of a function in no more than one point.
T F 2. The y-intercept of the graph of the function $y=f(x)$ whose domain is all real numbers is $f(0)$.
T F 3. Even functions have graphs that are symmetric with respect to the origin.
T F 4. The graph of $y=f(-x)$ is the reflection about the y-axis of the graph of $y=f(x)$.
T F 5. $f(g(x))=f(x)\cdot g(x)$
T F 6. If f and g are inverse functions, then the domain of f is the same as the domain of g.
T F 7. If f and g are inverse functions, then their graphs are symmetric with respect to the line $y=x$.

REVIEW EXERCISES

1. Given that f is a linear function, $f(4)=-5$, and $f(0)=3$, write the equation that defines f.
2. Given that g is a linear function with slope $=-4$ and $g(-2)=2$, write the equation that defines g.
3. A function f is defined by $$f(x)=\frac{Ax+5}{6x-2}$$ If $f(1)=4$, find A.
4. A function g is defined by $$g(x)=\frac{A}{x}+\frac{8}{x^2}$$ If $g(-1)=0$, find A.
5. (a) Tell which of the following graphs are graphs of functions. (b) Tell which are graphs of one-to-one functions.

(a) (b) (c) (d)

6. Use the graph of the function f shown to find: (a) The domain and range of f (b) The intervals on which f is increasing (c) The intervals on which f is constant (d) The intercepts of f

Demonstrate to yourself that you know "How to" deal with the concepts listed.

"Fill in the Blanks" will determine your comfort with vocabulary.

"True/False" is a stickler for knowing definitions!

The "Review Exercises" provide a comprehensive supply of exercises using all the concepts contained in the chapter.