Lawrence G. Gilligan
Robert B. Nenno

Finite Mathematics

With Applications to Life

Second Edition

FINITE MATHEMATICS

WITH APPLICATIONS TO LIFE

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by
Lawrence G. Gilligan
and
Robert B. Nenno

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This book is dedicated with love to
Corine and Sue

Preface

to the Instructor

As with the first edition of *Finite Mathematics*, our primary goal with this edition was to create a text that is truly readable. We believe we have achieved that goal. Its intended audience is the student with little (one year) of high school mathematics majoring in the social sciences, business, economics, data processing, or the liberal arts.

The major changes in this edition are:

1. The topics of sets and logic have been combined into one, more concise, beginning chapter; additional material on logic appears in Chapter II.
2. The number of completely solved examples and the number of exercises have been greatly increased.
3. The material on matrices has been entirely rewritten. Much of the motivation for using matrices has been provided by systems of linear equations (Chapter 4).
4. Chapter 5, Applications of Vectors and Matrices, is all new.
5. The Transportation Problem has been added to Linear Programming (Chapter 6).
6. The Simplex Method has been used to expand the Theory of Games (Chapter 7) material to include solving $m \times n$ games.
7. Baye's formula has been added to Chapter 4.
8. Chapter 10, Mathematics of Finance is all new.
9. Finally, to help make the text more appealing and more interesting, each chapter offers a motivating lead-in photograph as well as various photographs and drawings with historical notes within each chapter.

to the student

This book was written for you; USE IT! There is more involved when using a book than just reading it; you have to dissect it. We have included, to assist your understanding of the textual matter, more worked-out examples than any other text of this type. Read them carefully—they are the heart of the book. When studying for exams or when notation becomes a problem, the chapter summaries and the glossary of symbols may be useful.

We have noticed in the past that the best critic is the student. So, at any point in your experience with this text, should you care to offer any criticism or should you have any questions or want to make any kind of comment, feel free to contact us. We promise a reply.

Lawrence G. Gilligan
Robert B. Nenno
c/o Goodyear Publishing Co., Inc.
1640 Fifth Street
Santa Monica, CA 90401

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Several people at Goodyear Publishing Company worked long hours on this project. We thank them all and especially our editor, Jack Pritchard, who has been a constant source of encouragement for the last six years.

Susan Gilligan displayed an unusual amount of courage as she risked her marriage to type the manuscript for this edition. She's a good typist and a dynamite wife. Thanks Sue.

Finally and foremost, to the thousands of students who managed through the first edition, we owe the most appreciation to you. Many thanks.

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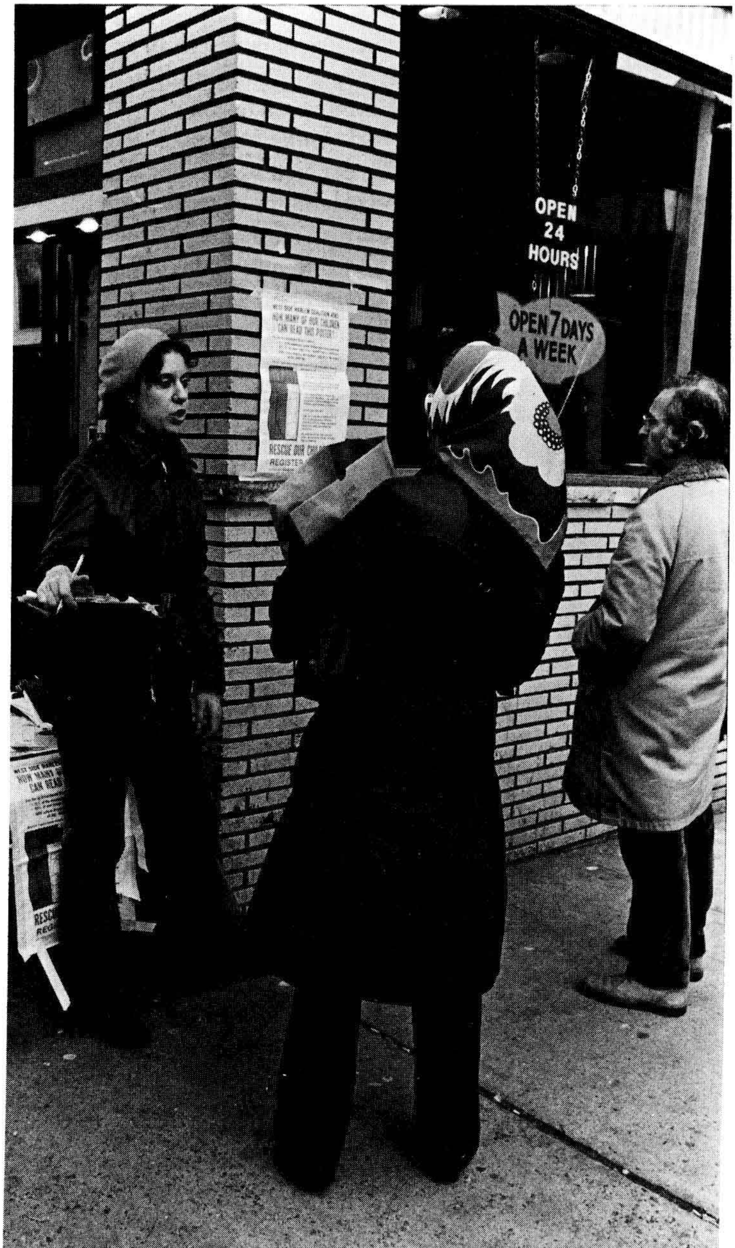
Preliminaries: Logic and Sets



Polls and surveys, like the one being taken in this photograph, often offer useful and interesting information. In Section 1.8 the reader will find how to use sets to represent and interpret survey information. (Photo: Elizabeth Hamlin, Stock Boston.)

Section 1.1

INTRODUCTION TO LOGIC

Logic is the systematic study of the basic formal principles of reasoning. More meaningfully, logic deals with statements, with ways of combining statements to form "compound" statements, and with "proving" things.

First, let us examine exactly what is meant by a statement. Recall from elementary grammar that all sentences in the English language can be classified as one of the following types: declarative, exclamatory, imperative, or interrogative. In logic, a **statement** is a declarative sentence that is either true or false (but not both simultaneously). Exclamatory, imperative, and interrogative sentences will be of no use to us.

Today is Thursday.
Your nose is running.
Bob and Larry wrote this book.
The Dow Jones Industrial Average dropped three points today.

are all examples of statements, while

Are you for real?
Look out!
Make love, not war.

are not.

As in grammar, we have **simple statements** and **compound statements.** A *simple statement* is a statement that conveys exactly one thought, while a *compound statement* is a connection of simple statements. The word or words that perform this "connection" are called **connectives.** Consider the following statements:

1. Jesus wept.
2. Harold is an A student.
3. Ernie is an attorney.
4. Gloria and Vinnie are both attending school.
5. The Soytex Corporation will relocate or I will lose $10,000.
6. If we plan to strike, then we must do it tomorrow.

We see that statements 1, 2, and 3 are simple statements; 4, 5, and 6 are compound. In 4 the connective is "and." In 5 the connective is "or." In 6 it is "if . . . , then." Let us examine further some of the connectives.

Section 1.2

BASIC CONNECTIVES

The logic connective negation is translated as the word "not"

When we wish to change a statement to its **negation,** one with the opposite meaning, we introduce the word "not." So,

Today is Thursday

can be negated to

Today is not Thursday

The word "not" is commonly called a connective even though it isn't really connecting simple statements here.

It is important to determine the **truth value** (the "trueness" or "falseness") of a statement. We can see that the negation of a statement has the opposite truth value of the given statement. To simplify matters we will use the lower case letter p to represent "Today is Thursday." The connective "not" will be symbolized by "$\sim$." So "$\sim p$" (not p) represents "Today is not Thursday."

In order to display truth values, it is common to list all possibilities in table form. Such a configuration is called a **truth table.** The figure below is the truth table for $\sim p$, where T stands for "true" and F for "false."

p	$\sim p$
T	F
F	T

A common connective in the English language is "and." Consider the following statements:

p: The card I am holding is a queen
q: The card I am holding is a heart

The logic connective conjunction is translated as the word "and"

The **conjunction** of the two statements p and q, denoted $p \wedge q$, is read, "The card I am holding is a queen *and* the card I am holding is a heart." This can be shortened to, "The card I am holding is a queen and a heart."

It is important to realize just when $p \wedge q$ is true and when it is false. Common sense prevails. In order for $p \wedge q$ to be true, the card I am holding must actually be the queen of hearts. That is, statement p must be true *and* statement q must be true in order for $p \wedge q$ to be true. Otherwise, $p \wedge q$ will be false. The figure below is the truth table for $p \wedge q$.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Note that there are now *four* rows in the truth table (as opposed to two previously). This is because all possibilities of truth values for p and q must be listed. Notice that in the column headed $p \wedge q$

there is just one "T" and that occurs in the first row of the table, where both p and q are true.

Another common connective in our language is "or." For example, consider these statements:

p: I will buy Eastman Kodak at $62\frac{1}{2}$
q: I will buy General Motors at 52

The logic connective disjunction is translated as the word "or"

The **disjunction** of these two statements is read, "I will buy Eastman Kodak at $62\frac{1}{2}$ or I will buy General Motors at 52." This can be shortened to "I will buy Eastman Kodak at $62\frac{1}{2}$ or General Motors at 52." We will now examine the truth value of this compound statement.

We don't know whether the person who makes the above statement plans to buy stock in *only* one company or *at least* one company (possibly both). Thus, the word "or" can be used in two different senses. To avoid confusion, we distinguish between these two disjunction processes, namely,

There are two types of "or" statements

Exclusive disjunction: the disjunction connective that *excludes* the possibility of buying both stocks. In symbols we write $p \veebar q$ (p or q, but not both).

Inclusive disjunction: the disjunction connective that *includes* the possibility of buying both stocks. In symbols we write $p \vee q$ (p or q or both).

We can now summarize the truth values of each of the two disjunction processes in truth tables.

p	q	$p \veebar q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Note that the only difference occurs in the first case, where $p \veebar q$ is false when both p is true and q is true. Unless otherwise specified, we will translate "p or q" into the inclusive disjunction form, $p \vee q$.

Many statements that we use in our English language are **conditional** statements. Some examples are:

Conditional statements are "if. . . , then" statements

If today is December 24, then tomorrow is Christmas Day.
If I eat too much, then I'll need a Bromo-Seltzer.
If you're not here after what I'm here after, then you'll be here after I'm gone.
If I get a good summer job, then I'll return to college.

In order to further realize the prevalence of conditional statements in our language we should observe that the "If . . . , then . . ." type of statement is the basic form of mathematical theorems. Again, some examples:

If two angles are right angles, then they are equal.
If $a = b$, then $a^2 = b^2$.
If the figure is a rectangle, then the area is the product of its length and width.

Let's take a closer look at a conditional statement. Let p be "I get a good summer job" and let q be "I'll return to college." Suppose Bill makes the following compound statement:

If I get a good summer job, then I'll return to college.

We will symbolize this as $p \rightarrow q$, which is often read in either of the two ways:

1. If p, then q
2. p conditional q

Consider that Bill gets a good summer job, and that he also returns to college. Then $p \rightarrow q$ will be true (see the first row of the truth table below). Suppose Bill gets a good summer job, but he fails to return to college. We would have to say that when Bill makes the statement $p \rightarrow q$, he lies. That is, $p \rightarrow q$ is false in the case where p is true but q is false (see the second row of the truth table below).

It is possible that Bill does not get a good summer job, in which case he might still return to college (he inherits $10,000) or he might not (he has no money). In either of these two cases, Bill's original statement $p \rightarrow q$ would not be considered untrue. Hence, the last two rows of the truth table below have T's under $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

As a brief summary, the only time $p \rightarrow q$ is false is when p is true and q is false.

The conditional, although used commonly in our language, is often misused and misunderstood. For example, out of the advertising industry often come statements like these:

If you use Crust, then you'll have 47% fewer cavities.
If you use Proteen 27, then your hair will be manageable all day.
If you use Scape in the morning, then your breath will be fresh all day.

Consider the last of these statements and further presume the claim to be true. When we hear such a statement we often equate "using Scape" with "fresh breath" and "not using Scape" with "bad breath." The shock value of this could motivate us to scurry to the store immediately. However, consider the truth table below:

use Scape	*fresh breath*	*use Scape* $\rightarrow$ *fresh breath*
T	T	T
T	F	F
F	T	T
F	F	T

It becomes evident that such a reasoning process considers only two (1st and 4th rows) of the three cases (1st, 3rd, and 4th rows) in which "use Scape $\rightarrow$ fresh breath" is really true. We should also consider the third row when we hear the original claim: "If you use Scape, then your breath will be fresh all day." Logically, this third row tells us that we can have fresh breath without using Scape at all!

A connective closely related to the conditional just studied is the **biconditional.** A few examples are:

An "if and only if" statement is a biconditional

I will marry you if and only if you love me.
Fill out the information if and only if you have a change of address.
A quadrilateral is a parallelogram if and only if its opposite sides are parallel.

Let's work out the truth table of the biconditional by considering a specific example and using a little common sense. Use p to represent "I will marry you" and q to stand for "you love me." Imagine that Bill tells Addie, "I will marry you if and only if you love me." We will symbolize this statement as $p \leftrightarrow q$ and read it as:

1. p if and only if q
2. p if q
3. p biconditional q

In any case, what Bill is saying is that Addie's love will lead him to marry her and also that if Addie does not love him, he will not marry her. That is, any biconditional $p \leftrightarrow q$ states that when p is true, q is true, and that when p is false, q is false. For these reasons, we see T's in the first and fourth rows of the truth table along with F's in the second and third rows of the truth table.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

In brief, $p \leftrightarrow q$ is true when p and q have the same truth values (both true or both false).

Earlier we mentioned that the biconditional is closely related to the conditional. It also is related to the conjunction and disjunction. In fact, there are many interrelationships among all six of the connectives ($\sim$, $\wedge$, $\vee$, $\underline{\vee}$, $\rightarrow$, $\leftrightarrow$) we have studied, but we are going to defer this until Chapter 11.

Right now, let's summarize all six of the truth tables studied so far; we will save a little writing by presenting them in slightly compact form.

$\sim$	p
F	T
T	F

p	$\wedge$	q
T	T	T
T	F	F
F	F	T
F	F	F

p	$\underline{\vee}$	q
T	F	T
T	T	F
F	T	T
F	F	F

p	$\vee$	q
T	T	T
T	T	F
F	T	T
F	F	F

p	$\rightarrow$	q
T	T	T
T	F	F
F	T	T
F	T	F

p	$\leftrightarrow$	q
T	T	T
T	F	F
F	F	T
F	T	F

We conclude this section with an example.

Example 1.2.1. Assume p is true, q is false, and r is false. Find the truth value of each of the following compound statements:

a. $p \wedge \sim q$ **b.** $p \rightarrow (q \vee r)$ **c.** $\sim p \rightarrow (q \vee r)$
d. $\sim[\sim p \rightarrow (q \vee r)]$

SOLUTION. Below each compound statement we substitute the appropriate truth values and use the tables listed previously to arrive at the answer:

a. $p \wedge \sim q$
$T \wedge T$
T (Answer)

b. $p \rightarrow (q \vee r)$
$T \rightarrow (F \vee F)$
$T \rightarrow F$
F (Answer)

c. $\sim p \rightarrow (q \vee r)$
$F \rightarrow (F \vee F)$
$F \rightarrow F$
T (Answer)

d. $\sim[\sim p \rightarrow (q \vee r)]$
$\sim[F \rightarrow (F \vee F)]$
$\sim[F \rightarrow F]$
$\sim[T]$
F (Answer)

EXERCISES: SECTIONS 1.1 AND 1.2.

1. Which of the sentences below are statements?

a. Larry, take out the garbage.
b. $2 \neq 3$.
c. If I am elected president, then I will lower taxes.
d. Where have all the flowers gone?
e. I will buy lunch if and only if I lose the coin toss.

2. The sentence, "This sentence is false" is not a statement. Why?

In exercises 3 through 12, assume we know statement p to be true and we know statement q to be false. Determine the truth value of each compound statement.

3. $\sim p$ **4.** $\sim p \vee q$ **5.** $p \wedge q$ **6.** $\sim q$

7. $p \wedge \sim q$ **8.** $\sim(p \wedge q)$ **9.** $\sim p \wedge \sim q$ **10.** $p \veebar q$

11. $p \rightarrow q$ **12.** $q \rightarrow p$

For exercises 13 through 17, classify each statement as either simple, negation, conjunction, disjunction, conditional, or biconditional.

13. You are not the man you pretend to be.

14. The sum of nine and four is thirteen.

15. If you disobey, then you shall be punished.

16. The Health Food Company tripled its quarterly dividend.

17. I will attend Brockport or Harvard next fall, but not both.