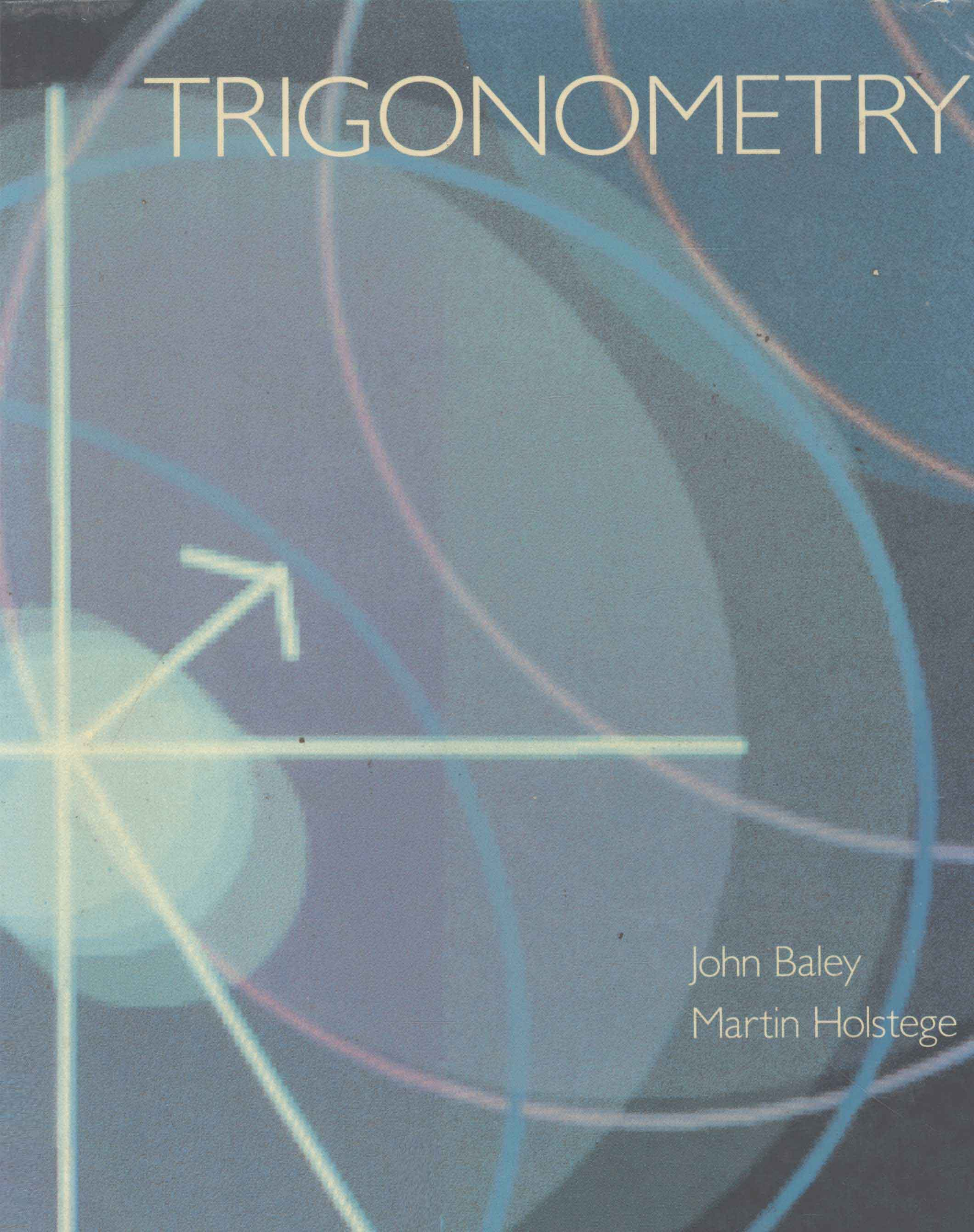
TRIGONOMETRY



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Martin Holstege

TRIGONOMETRY

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RANDOM HOUSE

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First Edition

987654321

Library of Congress Cataloging in Publication Data

Baley, John D.
Trigonometry / John Baley, Martin Holstege.
p. cm.
ISBN 0-394-35461-3
1. Trigonometry, Plane. I. Holstege, Martin. II. Title.
QA533.B19 1987
516.2′4—dc19 87-22391
CIP

Preface

This text is an in-depth course in trigonometry designed to be understood and used by students. Although the development of trigonometry begins on page one, we, the authors, realize that many students may have completed intermediate algebra and geometry some time ago. Therefore, we have included algebra and geometry reminders throughout where we know from teaching experience that many students need help to recall ideas that are needed to develop trigonometry.

While it assumes no previous knowledge of trigonometry, this book shows how trigonometry can be used in many fields. It also develops algebra skills so that students will be thoroughly prepared to continue their study of mathematics and science.

Features: Organization and Pedagogy

This book is divided into fourteen chapters to allow instructors greater flexibility in arranging course outlines to conform to academic calendars and student needs.

Each Chapter Includes:

Preview: This gives the student an overview of how the chapter relates to previous chapters and how the coming chapter will be developed.

Sections: Each chapter is divided into sections of material that approximate one hour of classroom lecture time.

Numerous Examples: Over 250 examples illustrated with graphics demonstrate the concepts presented in the text. Students are given one or more examples of each task they are expected to perform.

Ample Exercises: Over 2,300 problems, usually in matched odd-even sets, give the students the opportunity to apply the concepts and practice the skills taught in the text. Problem sets are a critical part of the learning process because they not only give the student needed practice but they also show the student the results of subtle variations. By doing problems a student gets an opportunity to internalize mathematics and see the effects of changes in a parameter.

Applications: Trigonometry provides more opportunities for students to see actual applications of the mathematics they are studying than any lower-level course they have encountered. This text uses applications in examples and problems to show students that trigonometry is a very useful branch of mathematics that is used in engineering, electronics, geology, optics, aviation, surveying, construction, forestry, navigation, and physics.

Highlighted Definitions, Properties, Theorems, and Rules: These features, which are essential to remember and understand, are highlighted in boxes throughout the text both to draw the students' attention to important concepts and to make it easy for students to reference key ideas.

Key Ideas: Each chapter concludes with a summary of key ideas to aid students in organizing their knowledge and help them prepare for exams.

Review Tests: To further insure students that they have mastered the concepts of each chapter, there is a review test that closely approximates the questions that are likely to be asked on an exam.

Special Features of This Text

Cumulative Reviews: There are cumulative reviews after Chapter 7 and at the end of the book. These reviews are an excellent opportunity for students to get an overview of the course as they study for midterm and final exams.

Conversational Bubbles: These bubbles are spread throughout the book but generally appear in the context of a worked example. Teachers will quickly recognize that these bubbles anticipate and verbalize the questions that students are likely to ask as they are learning the material. Answering bubbles provide the answers that experienced teachers are likely to give.

Geometry and Algebra Review: It would be nice if every student beginning this course had a complete mastery of algebra and geometry, but for many students it has been several years since they have taken these courses. The geometry and algebra reviews are placed in the text to help students refresh key ideas just before these ideas are needed in trigonometry.

Using Your Calculator: This is a series of special features that show students how to use efficiently a scientific calculator to solve problems. Each calculator feature tells students what they need to know, when they need to know it.

Trigonometric Identities: Identities are one of the most critical topics of a complete trigonometry course. Trigonometric identities are essential for many calculus and physical applications. They not only provide an excellent review of algebraic skills but can be used to teach advanced algebraic manipulation techniques that students will need in later courses. Unfortunately, identities can also be one of the most frustrating topics for both teacher and student. Some texts expect students to fail at identities and they avoid challenging students with serious identities. This text solves the problem of mastering the identities by teaching students the entire hierarchy of skills that are needed to provide trigonometric identities. These skills, which include problem analysis and algebraic manipulation, are carefully developed in Chapters 6, 7, and 8. This text does not avoid identities; it teaches students how to prove them at a very high level.

Graphing Trigonometric Functions: Trigonometric functions are graphics using the concept of a *generic box*, which simplifies the task while providing unifying ideas about period and the effect of parameters on a function. These ideas are applicable throughout mathematics.

Class Testing

This book has been class tested through four revisions by over 1,000 students in both lecture and semi-independent mathematics classes. The authors are grateful to Professor Judy Baham of Cerritos College, who helped with the class testing, and to the many students and instructional aides who provided useful feedback, which helped to improve this text.

Ancillaries

Student Solution Manual

A student solution manual with all the even-numbered problems worked step by step is available. This manual also provides additional hints and explanations about how the problems were solved.

Instructor's Materials

An instructor's manual provides four forms of each chapter test, a midterm exam, a final exam, and suggested tests for Chapters 1–3, 4–6, 7–9, 11–12, and 13–14 to assist teachers who wish to cover multiple chapters on each test. In addition, a separate manual that contains answers to all the exercises is available.

Acknowledgments

The authors wish to thank Debra Pentacost for the level of care and attention she gave to preparing the preliminary editions for class testing. Her efforts have made the development of this book a much more pleasant task for all concerned. The accurate graphs in the book are, in large part, due to the programming efforts of W. Scott London.

The authors are also especially grateful to the reviewers from colleges throughout the country who carefully read, critiqued, and provided many excellent suggestions that have been incorporated into the book you are holding. The quality of this text has been improved by the efforts of:

James Arnold—*University of Wisconsin, Milwaukee*
Jerald Ball—*Chabot College*
Elizabeth Cauley—*Pensacola Junior College*
Betty Ann Daley—*University of Delaware*
Virginia Hamilton—*Ball State University*
Louis Hoelzle—*Bucks County Community College*
Michael Karelius—*American River College*
Anna Marie Lallemont—*Pennsylvania State University*
Peter Lindstrom—*North Lake College*
Beverly Rich—*Illinois State University*
Vicki Schell—*Northern Illinois University*
Cynthia Siegel—*University of Missouri, St. Louis*
Ken Seydel—*Skyline College*
Sharon Walker—*Arizona State University*

Many thanks are due to the entire editorial and production team at Random House, particularly our editor, Wayne Yuhasz, whose experienced guidance has helped shape every aspect of this book. His able and trusty assistants Anne Wightman and Louise Bush provided much appreciated coordination and support for the project. The production staff at Random House in Cambridge—Margaret Pinette, Project Manager; her assistant, Pamela Niebauer; Michael Weinstein, Production Manager; and his assistant, Susan Brown—have done an admirable and highly-professional job. We also thank Lucy Jenkins and all the people at Quarasan for their excellent services and support.

Contents

Preview

Many applications of trigonometry deal with rotations which are measured by angles. This chapter will show four ways to measure angles. It will show how to determine the length of an arc like a curve on a freeway bridge. We will use a point on the terminal side of an angle to define the six trigonometric functions that will be used throughout this course. To avoid dependence on calculators and provide reference points to easily visualize examples, we will examine 0°, 30°, 45°, 60°, and 90° angles. We will then use the location of a point on the terminal side of each of these frequently used angles to find values for their trigonometric functions.

■ 1.1 Measurement of Angles

Trigonometry is a branch of mathematics with many applications. Early trigonometric applications included land surveys, building construction, astronomy, and navigation. Later developments in optics, electronics, mechanics, engineering, and communications using radio waves or fiber optics increased trigonometric applications.

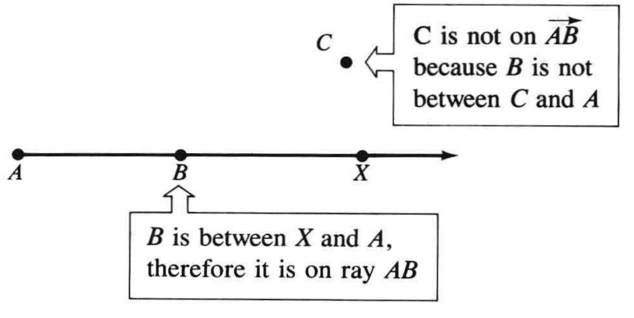
There are several valid ways to begin the study of trigonometry. We will start by looking at the coordinates of a point rotated through an angle because most applications of trigonometry involve analysis of this type of motion.

Some practical examples of this rotation are the moving armature of an electric motor or generator, a piston or a crankshaft, an automobile traveling around a curve, and a hand on the end of an arm. To study rotation we need to define a few terms.

Definition 1.1A Ray $\overrightarrow{AB}$

Ray $\overrightarrow{AB}$ is the set of points consisting of line segment $\overline{AB}$ and all other points, X, such that point B lies between point X and point A.

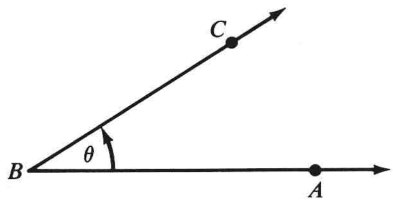
■ Figure 1.1



A ray is a half-line with one end point. It continues in one direction forever.

Definition 1.1B Angle

An angle consists of two rays with a common end point called the *vertex*.



This angle may be referred to as

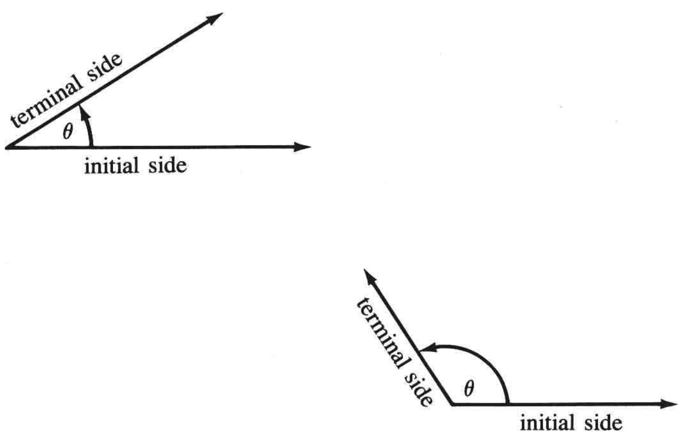
$\angle B$	read "angle B"
$\angle ABC$	read "angle ABC"
$\angle \theta$	read "angle theta"

Most letters in the Roman alphabet have standard meanings, such as **t** for **time.** Therefore, to identify angles we use Greek letters. Common letters used are:

α	Alpha	γ	Gamma
β	Beta	θ	Theta

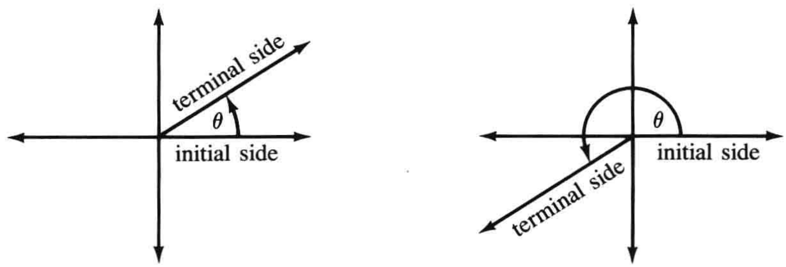
We can think of an angle as a ray rotated about its end point. The angle consists of the starting or initial position and the ending or terminal position of the ray. The size of the angle is determined by the amount of rotation.

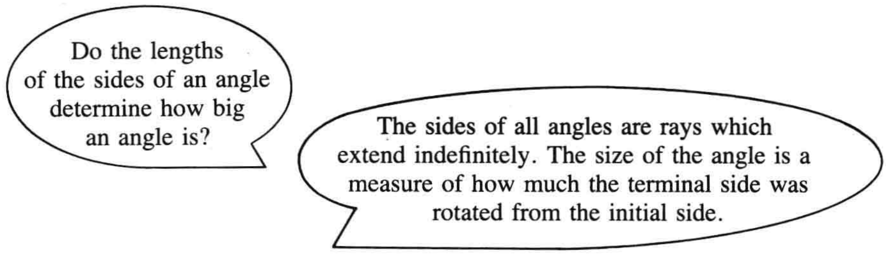
■ Figure 1.2



1.1C Standard Position for an Angle

An angle is in standard position if its vertex is at the origin and its initial side lies on the positive x-axis.





Geometry Reminder

About Angles

Angles are defined as two rays with a common end point. Therefore an angle is a set of points.

Strictly speaking we cannot say that $\angle A$ above is equal to $\angle B$ because each angle is a different set of points.

We *can* say that if you pick up $\angle A$, you could move it so that it fits exactly over $\angle B$. To express this idea we say $\angle A \cong \angle B$. An equal sign with a wavy line above it is read "congruent," so this is read "angle A is congruent to angle B." In trigonometry we are interested in comparing the amount of rotation or the size of two angles. To compare we use a number called the **measure of an angle** for each angle.

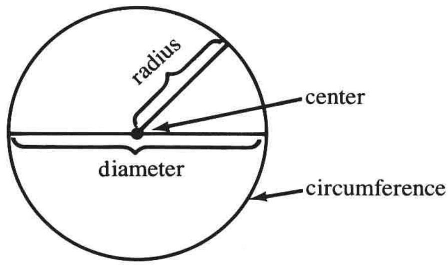
Then we can say: $m\angle A = m\angle B$.

In trigonometry, however, we make this comparison so frequently that we usually refer to "angle θ" rather than "the measure of angle θ". Throughout this book, unless we specifically say otherwise, we will use $\angle\theta$ to mean the measure or "size" of $\angle\theta$ rather than the set of points that makes up $\angle\theta$.

Geometry Reminder

About Circles

A circle is defined as the set of points in a plane all the same distance from a fixed point. The fixed point is called the **center of the circle.** The given distance is called the **radius.**



Neither the center nor the radius is part of the circle; they are used only to define the circle.

The circle is a set of points.

Each of the following are distances associated with a circle.

Radius—distance from the center of the circle to a point on the circle.

Diameter—distance across the circle passing through the center.

$$d = 2\pi$$

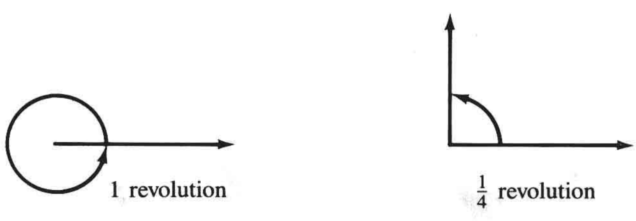
Circumference—distance around the circle.

$$C = 2\pi r \qquad \text{or} \qquad C = \pi d$$

The size or measure of an angle is a real number associated with the amount of rotation. There are four basic ways to measure rotation. All are based on a complete circle.

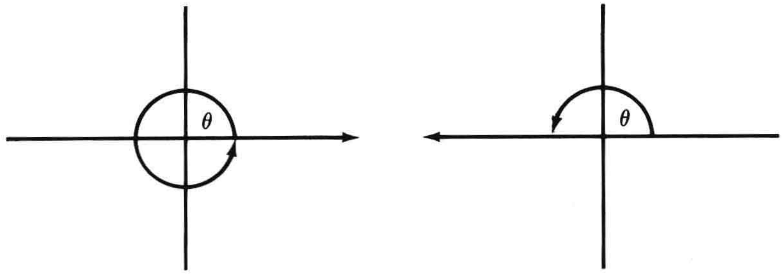
1. REVOLUTIONS: 1 revolution = 1 circle rotation A point rotated through a complete circle is called 1 revolution.

■ Figure 1.3



2. DEGREES: Earliest astronomers thought a year was 360 days long, so 360 degrees seemed a logical number of divisions for a circle.

■ Figure 1.4

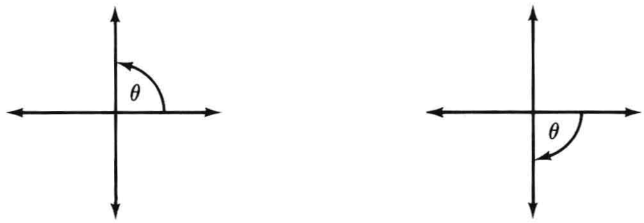


$360° =$ rotation in a complete circle

$180° =$ rotation through half a circle

Counterclockwise rotations are considered positive. Clockwise rotations are considered negative.

■ Figure 1.5



$90° =$ rotation through $\frac{1}{4}$ of a circle

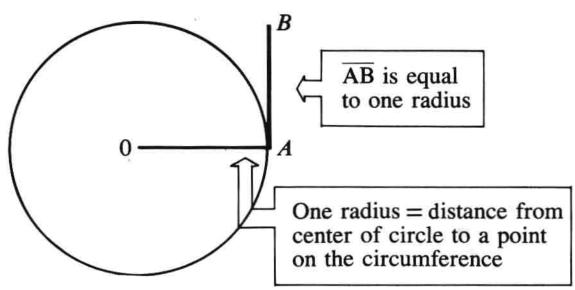
$-90° =$ rotation through $\frac{1}{4}$ of a circle in a clockwise direction

3. GRADS: Your calculator may have a "grads" key. The grad system is used by the military for artillery calculations. It divides a circle into 400 grads. This text will not use grads.

4. RADIANS: In higher mathematics, the most convenient measure of an angle is the ratio of the intercepted arc to the radius of a circle. The measure of a central angle that intercepts an arc on a circle equal to the length of a radius is one radian. To draw an angle of one radian follow these steps:

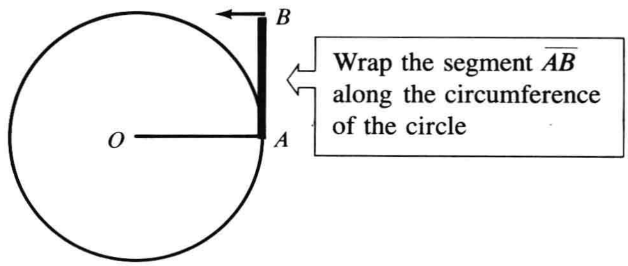
Step 1.

■ Figure 1.6



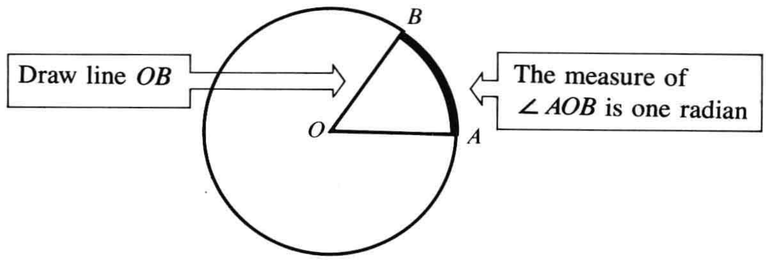
Step 2.

■ Figure 1.7



Step 3.

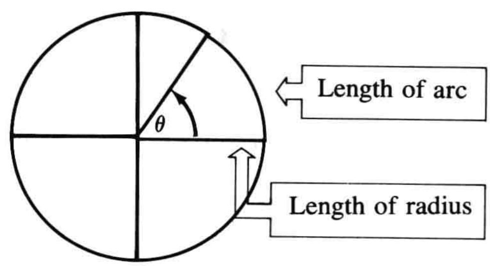
■ Figure 1.8



In general

$$\text{Angle in radians} = \frac{\text{length of intercepted arc}}{\text{length of radius}}$$

■ Figure 1.9



$$\theta = \frac{s}{r}$$

Where θ = measure of angle in radians

s = length of intercepted arc

r = radius of circle

Why doesn't θ have an $\angle$ sign?

$\angle\theta$ is the set of points that make the figure. In this book θ refers to the number used to measure the angle.

Next we will determine the number of radians in one revolution. Since the circumference of a circle is $2\pi r$, an angle in radians equal to one revolution is as follows.

$$\text{One revolution} = \theta = \frac{\text{length of arc intercepted in one revolution}}{\text{length of radius}}$$

$$\theta = \frac{2\pi r}{r}$$

$$\theta = 2\pi$$

$$\theta \approx 6.2832 \qquad \text{since } \pi \approx 3.1416$$

2π what?

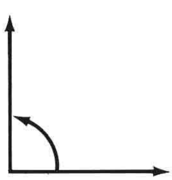
Just 2π. Radians don't have dimensions. Notice any units used to measure the arc length in the numerator cancel with the units used to measure the length of the radius in the denominator.

Here are a few other angles in radian measure:

■ Figure 1.10

$$\frac{1}{2}\text{ revolution} = \frac{1}{2}(2\pi) = \pi$$

■ Figure 1.11



$$\frac{1}{4}\text{ revolution} = \frac{1}{4}(2\pi) = \frac{1}{2}\pi$$

Some common angles and their radian measure are drawn in Figure 1.12 to help you visualize radian measure.