

Management Science

An
Introduction

Fadil H. Zuwaylif
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MANAGEMENT SCIENCE: AN INTRODUCTION

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PREFACE

This text is intended for a one or two semester introductory course in Management Science, either at the undergraduate or M. B. A. level. The text is geared primarily for business majors, but it is also suitable for engineering or mathematics majors who seek an applied course in Management Science.

The prerequisites are minimal: high school algebra is sufficient. While most students taking this course will have had a one or two semester course in probability and statistics, the text has been written to accommodate students who have not had such a course. For these students, the necessary probability topics are covered in Chapters 2 and 3. The remainder of the text requires no further prerequisites. If, on the other hand, a statistics prerequisite is imposed, experience has nonetheless shown that probability, like grammar or algebra, is not a topic of everyday conversation among most students and, consequently, does not remain fresh in their minds for too long. Hence, we recommend, for students having had a previous course in probability and statistics, that Chapters 2 and 3 be at least reviewed. With a prerequisite of this nature, however, more time can be devoted to the linear programming material in Chapters 6, 7, 8, and 9 as well as the simulation topics in Chapter 13, or perhaps even the inventory control (Chapters 11 and 12) and/or dynamic programming (Chapter 10) material. In either case, we suggest, for a one semester course, covering Chapters 1 through 9 and, if time permits, one additional chapter, at the discretion of the instructor, selected from among the last four chapters. The entire text should provide enough material for a two semester course.

In view of the modest prerequisite, we have tried to present the material with a minimum of mathematical rigor. At the same time, we have tried not to compromise the text by omitting or diluting difficult topics. Instead, we have attempted to develop each concept from the beginning by providing plenty of (relatively) realistic but pedagogically tractable examples, along with detailed but easy-to-follow explanations. Additionally, we have stressed the economic applications of the concepts as opposed to their mathematical properties. We have also provided an abundant number of exercises, at various levels of difficulty, to give students the opportunity to test their understanding of the subject matter.

We wish to thank Addison Wesley Publishing Company for permission to reprint, in a somewhat revised form, Chapters 2 and 3 which first appeared in F. H. Zuwaylif, *Applied Business Statistics*, Addison Wesley, 1974. We would also like to thank a number of our CSUN colleagues for their support and helpful suggestions: Professors Max Astrachan, Abe

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CHAPTER ONE

INTRODUCTION

Motivated to a large extent by making a profit, the business sector of our economy provides an extensive variety of goods and services to satisfy the whims and desires of consumers. In the course of producing these goods and services, business enterprises face numerous decision problems. For example, a manufacturing concern must decide whether to produce commodity X, commodity Y, or commodity Z. How many units of each commodity should it produce? What is the appropriate inventory level of both raw materials and final products? How should the finished goods be distributed?

Decision problems arise in any aspect of business activity—in finance, in production, and in marketing. But regardless of the nature of the problem, the decision process is generally the same.

In any decision problem, the decision maker is faced with several alternative courses of action. The decision maker must then choose among the alternatives using some choice criterion. Most often, the choice criterion in business is either to maximize profit or to minimize cost.

Decision problems are generally classified into two kinds: decisions under uncertainty and decisions under certainty. We give an example of each type of decision problem in turn.

DECISIONS UNDER UNCERTAINTY

As an illustration of decision under uncertainty, let us consider the following example: A florist orders carnations one day in advance. The cost of carnations is \$1 per dozen, and the selling price is \$3 per dozen. Unsold carnations

at the end of the day are worthless. The florist estimates his daily demand as shown below.

<u>Daily Demand (in dozens)</u>	<u>Probability of Demand</u>
10	.60
11	.40

How many dozen carnations should he stock each day?

This simple problem contains all the elements of decision making under uncertainty. First, there is the decision maker, the florist in this case. Second, the decision maker must choose between two alternative courses of action: whether to stock 10 dozen or 11 dozen carnations each day. Third, there is an element of uncertainty concerning the quantity of carnations demanded by customers: the florist is not sure how many dozen carnations he can sell on any given day. However, based on his past experience and judgment, the florist assigns subjective or personal probabilities to the various levels of customers' demand. Thus, he personally believes that, on any given day, the probability is .6 that customers would demand 10 dozen carnations, and that the probability is .4 that they will demand 11 dozen.

In order to choose between the two alternative courses of action (whether to stock 10 dozen or 11 dozen carnations each day), the florist must adopt some choice criterion. Using profit maximization as a criterion, we may now compare the two acts.

Act 1 (stock 10)

If he stocks 10 dozen carnations each day, the florist is able to sell all these carnations, and he would realize a daily profit of \$20.

Act 2 (stock 11)

If the florist stocks 11 dozen carnations each and every day, he would realize a profit of \$19 when the demand is only 10 dozen (with probability of .6); and he would realize a profit of \$22 when demand is 11 dozen (with probability of .4). Thus, when stocking 11 dozen carnations each day, the florist's weighted average profit, called expected profit, is:

$$\text{Expected profit} = 19 \times .6 + 22 \times .4 = \$20.20 \text{ per day}$$

Hence, using the maximization of expected profit as a criterion for optimality, the optimal act is act 2, and the florist's expected profit is \$20.20 per day.

It is very important to point out here, that when the florist stocks 11 dozen carnations each day, his *actual* profit on any given day is either \$19.00

or \$22.00; but in the long run, he *expects* to realize an average profit of \$20.20 per day. This is his expected profit under uncertainty.

DECISIONS UNDER CERTAINTY

As an example of decisions under certainty, let us consider the following production problem. A manufacturer uses two machines, A and B, in his production process. Machine A has an operating capacity of 24 hours each week, and machine B has an operating capacity of 16 hours a week. In order to utilize these two machines, the manufacturer is considering the production of two new products, called product I and product II.

The production of *each* unit of product I requires 2 hours from machine A and 2 hours from machine B. The production of each unit of product II, on the other hand, requires 3 hours of machine A and 1 hour of machine B. Thus, each of two products must be processed on *both* machines. The production requirements of each product are summarized in the table below.

	Number of Machine Hours Required to Produce 1 Unit of Product I	Number of Machine Hours Required to Produce 1 Unit of Product II	Hours of Operating Capacity Available
Machine A	2	3	24
Machine B	2	1	16

Now, assuming that product I contributes \$5 per unit to the manufacturer's profit, and product II contributes \$7 per unit, how many units of each product should the manufacturer produce?

There are several combinations of product I and product II that the manufacturer can produce; each represents an alternate course of action. For example, the manufacturer can produce only product I; and if this is the case, he can produce no more than 8 units of product I (8 units require 16 hours of machine B, i.e., all available hours of that machine). Similarly, he can produce only product II, but with a maximum amount of 8 units (due to limitations imposed by availability of time on machine A). Finally, the manufacturer can produce any combination of the two products as long as the time required to produce this combination is within the existing capacities of both machines. For instance, the manufacturer can produce a combination of 6 units of product I and 4 units of product II (this combination utilizes all time on both machines).

In fact, there is an infinite number of combinations of product I and product II that can be produced utilizing some or all of the time on the two machines. Each of these combinations represents an alternative course of

action that can be adopted by the manufacturer. Once again, the manufacturer must choose among these alternatives using an appropriate choice criterion. Now, by using profit maximization as a criterion, we will discover that the manufacturer should produce a combination of 6 units of product I and 4 of product II; the profit contribution of this combination is \$58. (The reader should not be concerned at this point with the problem of finding such a solution. This matter is thoroughly discussed in Chapter 6.)

It may be useful now to compare our two illustrative problems. The optimal act in the carnation problem is to stock 11 dozen carnations each day, and the florist's expected profit under uncertainty is \$20.20 per day. (His actual profit on any given day is either \$19.00 or \$22.00.) By comparison, the manufacturer's optimal act is to produce 6 units of product I and 4 units of product II, and he is *certain* to realize a profit of \$58.00 each week.

ORGANIZATION OF TEXT

The general theme of this text is business decision making under uncertainty, and under certainty. Since probability plays a fundamental role in decision making under uncertainty, Chapters 2 and 3 are devoted entirely to the subject of probability and probability distributions.

Bayesian analysis is the subject matter of Chapters 4 and 5. In these chapters, we provide many examples of decision problems under uncertainty. Binomial distribution models are used to illustrate the Bayesian decision process, with and without sample information, in Chapters 4 and 5.

Decision making under certainty is illustrated by a variety of problems in the areas of linear and dynamic programming. Chapters 6 and 7 are devoted to the formulation and solution of a linear programming problem. Chapters 8 and 9 deal with the economic significance of linear programming, and Chapters 10 and 11 are devoted to inventory theory.

Chapter 12 deals with dynamic programming. The text concludes with a chapter on simulation.

CHAPTER TWO

PROBABILITY

PROBABILITY AND DECISION MAKING

Probability plays a fundamental role in decision making under uncertainty. In fact, the mathematical theory of probability is the foundation upon which a rational theory of decision making under uncertainty is developed.

Once again, in a decision problem, the decision maker must choose among several alternative courses of action, called acts. The consequence of adopting any act is dependent on some future events whose occurrences are subject to considerable uncertainty. Probability theory provides the decision maker with the means to measure the uncertainty concerning these future events and, hence, enables him to determine the *expected consequence* of each possible course of action.

Most often the consequence of any act, also called its payoff, is expressed either in terms of profit or in terms of cost. The decision maker uses probability to determine the expected payoff of each possible course of action, and he then chooses the act with the optimal payoff (highest expected profit or lowest expected cost).

OBJECTIVE AND SUBJECTIVE PROBABILITIES

The concept of probability can be approached in two ways: an objective approach and a subjective approach. In the objective approach, the probability of an event is defined as the relative frequency of its occurrence in the long run. To clarify this notion, let us assume that a balanced coin is tossed 50

times, and the event “head” appears 23 times. The relative frequency of the event “head,” denoted by $RF(H)$ is

$$RF(H) = \frac{23}{50} = .46$$

Now, the probability of the event H , denoted by $P(H)$, is the limit of its relative frequency as the coin is tossed an infinite number of times. Since the relative frequency of the event “head” approaches .5 as the number of tosses approaches infinity, we may then conclude that the probability of getting a head on a single toss of a balanced coin is .5. Similarly, we can also conclude that the probability of getting a 4 in a single throw of a fair die is $\frac{1}{6}$.

Initially, probability theory was developed in connection with games of chance and here the objective approach to probability is most appropriate. In contrast, let us now consider some other situations, namely business, where the objective interpretation of probability may no longer be possible. We may state, for example, that the probability of having a recession next year is .3, and that the probability that the price of a given stock will rise tomorrow is .9. Clearly, these probabilities can hardly be viewed as relative frequencies. More appropriately, these probabilities measure the degree of belief we attach to the occurrence of such events. Hence, in contrast to the objective approach, which defines probability as a relative frequency of an event, a subjective or a personalistic approach to probability defines the probability of an event as a measure of the degree of belief that an individual attaches to the occurrence of that event.

Subjective probabilities are individualistic. An individual assesses the probability of an event based on whatever information (past and current) is available to him together with his personal biases, experience, judgment, and expectations. Subjective probabilities are also personal probabilities in the sense that two individuals may assign different probabilities to the same event due to differences in their attitudes, experiences, and outlooks.

Before concluding this discussion, however, the reader should clearly note that all probabilities, whether objective or subjective in nature, must satisfy the same theorems and obey identical rules.

SETS

Our understanding of probability will be greatly enhanced if we first understand some elementary notions of sets.

A *set* is a collection of objects. The positive integers 1, 2, 3, 4, . . . form a set, and the students in this college are a set. These objects are called elements or members of the set. Thus, the integer 4 is an element of the set of positive integers of the number system.