INTRODUCTION TO THE Optical Transfer Function

Charles S. Williams
Orville A. Becklund

Reprinted from the 1989 original publication

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Cover images: A representation of the two-dimensional, perfect OTF (Chapter 5, Ref. 6), and a plot of intensity distribution in the image of a sinusoidal test object showing (1) an ideal image, (2) a "perfect" image having reduced amplitude, and (3) an aberrated image having a further reduced amplitude and a phase shift (page 157, Fig. 5.7).

Dedication

We dedicate a second publication of this book to the memory of spouses—to Orville, deceased co-author and husband of Mrs. Alma Becklund; and to Dorothy, wife of co-author Charles Williams.

Orville was proud of the finished book; he was always pleased to say, "It took us 13 years to write that book."

Dorothy was too deeply ill with Parkinson's disease to appreciate whatever she did see of the printed book. Even so, let no one underrate the need for a spouse to the success of a book.—CSW, ABB

Preface to the Reprinted Edition

When Orville Becklund and I began writing our book, a powerful and rapid computer was not available to us. The best we had was a hand-held programmable calculator. We used it to calculate solutions in which each solution consisted of a series of series. Many times I had to program the calculator to run all night. I would turn it on to calculate until morning, and go to bed. Finally, I had a bunch of partial answers to put together. I think it should have been expected that we would always be uneasy about the accuracy of data that finally found its way into the text.

Computers, programs, and programmers have come a long way since then. One of the best for calculating problems relating to optics, is from the work of Dr. David F. Edwards of Tracy, California. Much of his work in optics programming was after his retirement from Lawrence Livermore National Laboratory as head of the Optical Sciences and Engineering Group. Our calculations are updated by Dr. Edwards's, and are found in Appendix D (p. 401).

Charles Williams
July 2002

Preface

An abundance of knowledge about the optical transfer function (OTF) has been published in many excellent articles during the past 35 years or so, but somehow a niche for this knowledge has never been found in the engineering and scientific structure. As a result, OTF publications are scattered throughout the archival literature of scientific and technical journals. Our book aims to bring together into one source much of this wealth of information.

Those concerned with grounding engineers and scientists in the procedures of optical evaluation have found that *spatial frequency*, *wave-front distortion*, and *optical transfer function*, though not particularly difficult concepts to understand, do not as easily become part of one's thinking, and therefore practice, as the concepts of *rays*, *ray tracing*, and *ray aberrations*. The word *ray* (geometrical optics), for example, in contrast with *spatial frequency* (physical optics) is used so commonly in our language that it is no longer an esoteric term reserved for optics. Actually, there are advantages peculiar to each of the two viewpoints, and an optical analyst is handicapped by a lack of facility with either. We hope that our book is articulate enough in the art to bring practitioners up to speed in the realm of spatial frequency and the OTF.

Specifically, our text dwells on such fundamental concepts as spatial frequency, spread function, wave aberration, and transfer function—how these are related in an optical system, how they are measured and calculated, and how they may be useful. In the early chapters we review the historical background for the OTF, the related concepts, and the necessary nomenclature and coordinate systems. We discuss in some detail the wave aberration function, which is a measure of an optical system's ability to produce an image that is a "reasonable facsimile" of the object and which, therefore, is a fundamental characterization of the system's excellence of performance. We derive the optical transfer function and related concepts mathematically, and we discuss some ways that the OTF can be used for assessing the quality of an optical system both during its design and during testing of the manufactured system.

We show how the OTF can be used: when specifications for the optical system are being drawn up, when the OTF is part of a merit function while the system is being designed by computer, and when the optical system is being tested to verify adherence to specifications. Finally, we show how the OTF can be calculated mathematically, both by analytical procedures and by numerical methods of integration.

In the appendixes some pertinent mathematical basics are reviewed, and we document a number of OTF calculations that other workers have made.

Our book makes liberal use of illustrations. For the reader who wishes to pursue studies beyond the scope of our text, we provide a full complement of references at the end of each chapter.

The reader of our mathematical chapters should have had courses in calculus; a course in transform theory would be helpful but not necessary because the mathematics in the appendixes provide a review of all the Fourier transform theory that the reader will need. Besides the professional nonexpert in physical optics, the level of our text is intended to suit undergraduates with limited exposure to optics, such as juniors and seniors in science, mathematics, and engineering.

We have purposely avoided certain OTF topics: We do not treat the geometrical approximation of the OTF, the OTF of sampled images, or the polychromatic OTF, because we feel that the state of the art concerning each of these topics is not quite ready to be included in a tutorial book on the optical transfer function.

We make no pretense that the ideas in this book are original with us. Our information has come through various paths and from many sources, and we have tried to give credit at the appropriate places in the text to the many whose work we have used.

CHARLES S. WILLIAMS
ORVILLE A. BECKLUND

Dallas, Texas
May 1988

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1

OTF Historical Background

INTRODUCTION

The Optical Transfer Function (OTF) is the frequency response, in terms of spatial frequency, of an optical system to sinusoidal distributions of light intensity in the object plane; the OTF is the amplitude and phase in the image relative to the amplitude and phase in the object as a function of frequency, when the system is assumed to respond linearly and to be space invariant. The OTF depends on and potentially describes the effect of diffraction by the aperture stop and the effects of the various aberrations.

The concepts related to the OTF, which are considered in some detail in the next chapter, evolved very slowly. In fact, our whole civilization developed so gradually that only rarely can we clearly mark the beginning or the end of a stage in the evolutionary process. Similarly, any historical stage through which our modern institutions evolved was so like the preceding and the succeeding stages that a date of birth can hardly be established. We think it of some importance, therefore, that in the world of optical design and evaluation a new era has emerged in which once familiar terms like *circle of least confusion*, *blur circle*, *resolution*, and *bar chart* have become obsolete and, instead, the optical transfer function is being accepted as a criterion for the performance of optical systems.

An analysis of any given optical system using the OTF must necessarily consider the shape of wave fronts at the exit pupil; that is, it must use wave optics rather than, or at least as well as, ray optics. The new emphasis on wave optics has undoubtedly been a handicap in the advancement of the OTF; there is evidence that a traditional dependence on geometrical optics and ray tracing for design and analysis has delayed the acceptance of the OTF. The people who made their living and built their reputations by tracing rays may have felt themselves too busy to explore the possibilities of the OTF. ''Prosperity, like the other creations of technology, is a tiger whose riders do not dare to dismount'' [1]. Among the senior practitioners of optics there has been a tendency to regard the OTF as interesting but too theoretical to be of much practical use for deciding either how to design an optical instrument or how to evaluate one. Perhaps the complementary new approach has somehow been seen as a threat to the old ways, and, as Robert Frost suggests in "Reluctance,"

> Ah, when to the heart of man
> Was it ever less than a treason
> To go with the drift of things,
> To yield with a grace to reason,
> And bow and accept the end
> Of a love or a season.

Nevertheless, substantial progress in the knowledge of the OTF and its potential usefulness seems to have begun during the mid 1950s; and the art of OTF has steadily continued to advance since that time so that now the OTF can be applied to the procedures of optical system design, specification, and evaluation. Therefore, this book must give special consideration to these topics along with the main topic, *the OTF*. The OTF is now recognized as a means of refining an optical system during the final stages of design optimization; its application has the potential of going beyond the optimum design that can be obtained with ray optics alone. In the sections that follow, a brief history is given of early optical design and evaluation; and the history of OTF concepts and of the OTF potential is outlined.

For a list of the principal contributors to OTF concepts, we offer the authors in the chapter reference lists of this book. Prominent among these contributors are H. H. Hopkins and his associates, including his pupils, at the Imperial College of Science and Technology of London. For guidance in assembling the brief historical background of OTF for this chapter, we are particularly indebted to Hopkins, Baker, and Smith [2–4].

THE EARLY HISTORY OF OPTICAL DESIGN AND IMAGE EVALUATION

The theory of optical instruments and the evaluation of their performance have been studied ever since the first useful systems were assembled early in the seventeenth century. Long before that time, Pliny and other ancient writers indicated that people knew about burning glasses, which were glass spheres filled with water. However, it was not until the thirteenth century that any mention was made of lenses deliberately made for a purpose, for example, spectacles. In about 1608, a Holland spectacle maker, Hans Lippershey, is said to have been holding a spectacle lens in each hand, and when he happened to align them before his eye with the steeple of a nearby church, he was astonished to find that the weathercock appeared nearer. Then when he fitted the two lenses in a tube to maintain their spacing, he had constructed the first telescope.

Galileo Galilei in Venice heard about the new telescopes in June of 1609, and immediately began to make telescopes of his own. His first had a magnification of 3, but his instruments were rapidly improved until he achieved magnifications of about 32. He manufactured hundreds with his own hands for observers and experimenters throughout Europe. Galileo's own application of the telescope turned to the heavens. He startled the world with observations of Jupiter's satellites, the spots on the sun, the phases of Venus, the hills and valleys on the moon, and the nature of the Milky Way.

Spherical aberration was soon recognized by the developers of the early telescopes as a reason for defective images, and considerable effort was spent in experimenting with various aspherical refracting elements to overcome this fault. In 1666 Isaac Newton discovered that refraction by a given lens depended upon color, and he correctly concluded that the most significant defect of the then current telescopes was what we now know as chromatic aberration. He hastily concluded that all glasses had the same relation between refraction and color, so he turned to reflectors to solve the color problem. This decision prevailed in the telescope art for almost seventy years. Then Chester Moore Hall, a gentleman from Essex, realized that the refracting humors of the human eye are balanced optically to avoid color separation in visual images. Why shouldn't an appropriate combination of optical glasses solve this problem in telescopes? In 1733 he demonstrated a refracting telescope essentially free of chromatic aberration.

Long focal lengths and parabolic mirrors were also early means for alleviating aberrations in the telescope art.

Inasmuch as the first optical systems were mostly designed for astronomical work, it is not surprising that the star (an almost perfect point source) became the standard test object. Although many other objects have since been used for testing optical systems, the study of star and other point-source images has persisted to the present time in evaluating the response of image-forming systems. As later chapters in this book indicate, the OTF represents the latest organized approach to judging systems by the nature of their point-source images.

Even systems that are almost free of aberrations produce highly complex star images. A skilled observer, particularly one who specializes in one kind of optical system, still finds the star test one of the most sensitive methods of evaluating aberration residuals; however, the image is extremely difficult to interpret quantitatively. In general, the star test provides so much data in its own peculiar code that considerable data reduction is required before a particular star image can be said to qualify the system for, say, a television or an image intensifier application. The precision required in data reduction can be appreciated when we realize that a star image rarely extends over more than one or two hundred micrometers in diameter and that the range of flux densities to be measured and recorded can extend over a few orders of magnitude. If we choose to apply the star test by calculating reference images for a sequence of design parameters for an assumed system, we find that calculation of the distribution of flux density in the star image is still a formidable problem.

As applications broadened from the study of sky objects to predominantly *extended* terrestrial objects, the difficulties of evaluating optical systems by interpreting star images led to the use of various extended test objects. In recent decades, a variety of test charts consisting of black and white bars of varying dimensions has been a favorite type. The bar chart has one notable advantage: Performance can be specified by giving a single number, which is the acceptable bar spacing (either the bar interval or the number of bars, or lines, per unit distance perpendicular to the bars) in the image of the chart. However, the method has a number of shortcomings. Results are highly dependent on the nature of the image detector (human eye, photographic emulsion, etc.). Near the resolution limit, the boundary between bars of the bar chart image becomes a gradual transition from maximum to minimum intensity (or reflectance) rather than a sharply defined boundary between black and white; so repeatable observations of the resolution limit for a given system are difficult. A phenomenon called *spurious resolution*, in which the color of bars is reversed, that is, a black bar becomes a white bar and a white becomes black, is often seen in bar charts. It is also hard to predict the results expected from optical design data or to describe the quality of reproduction of another kind of test object once the bar chart resolution is known. In contrast to star images, which seem to give too much information, bar chart images tend to tell too little.

As he reflected on the limitations of the bar chart in 1938, Frieser [5] suggested that the resolution test could be improved by substituting a one-dimensional sinusoidal variation of grays in the test object for the abrupt variation from black to white characteristic of the bar pattern. He saw an advantage resulting from the nature of optical systems to produce a sinusoidally distributed image from a sinusoidally distributed object over a wide range of spatial frequencies. This concept brought with it the first expression of a related idea, that the transfer function connecting the sinusoidal image with the sinusoidal object would be a good way to assess the performance of optical systems.

Since the beginning of optical system fabrication, designers have relied on the accepted concept of straight-line propagation of light with a change of direction (refraction) at boundaries between unlike media. Directed lines called *rays* depict light paths from object to image. Sometimes *pencils* of rays are traced to evaluate discrepancies between the configurations of object and image. Perhaps less frequently, ray density has represented light flux density, which allowed a comparison of image distribution of light flux with the corresponding object distribution. For the most part, the art of optical design has depended on simple algebra and plane trigonometry, generally referred to as *geometrical optics*.

Even modern computer three-dimensional ray-tracing programs, for all their speed, precision, and self-modification toward optimum design, are fundamentally the same process as the paper-and-pencil, cut-and-try procedures used for a couple of centuries.

About a hundred years ago, a study of intersection patterns on the image plane by pencils of rays from the object led to an analytical theory of aberrations. The image plane was usually defined as the plane of best focus, that is, the plane on which a point source would produce the minimum *circle of confusion*.

Geometrical optics, though recognized as only approximate almost from its inception, has remained the mainstay of optical design simply because it has produced excellent results. In fact, application of geometrical optics to the elimination of aberrations finally produced such fine systems that actual star images, in their departure from a simple point configuration, could no longer be explained by geometrical optics. In 1835, Airy [6], who was familiar with the wave theory of light, developed the formula for the diffraction pattern, thereafter known as the *Airy disk*, that constituted the image of a point source in an aberration-free optical system.

It would have seemed reasonable if the optical designers, impressed by Airy's accounting for the performance of their ultimate systems, had turned to wave theory for further advancement of their art. In the main, however, they did not. Their reluctance to incorporate the wave theory of light, included in what is known as *physical optics*, into their calculations could not be attributed to a lack of scholars and well-documented work in the field. Among the leaders were Grimaldi (1665), Huygens (1678), Young (1802), and Fresnel (1815). In fact, in 1818 Fresnel, by using Huygens' concept of secondary wavelets and Young's explanation of interference, developed the diffraction theory of scalar waves essentially in the form that we know it today. Nor could the resistance to wave theory by designers result from the adequacy of geometrical optics to handle *all* design problems. Besides the particular need satisfied by the Airy disk, general geometrical optics treatment of optical processes in the vicinity of the focus and near the edge of a beam in limited-aperture systems is particularly lacking.

The dichotomy of geometrical optics and physical optics has persisted to the present day. A frequent question for graduate students preparing to extend their optical background is, ''physical or geometrical?'' It is interesting to speculate on this division. The solution of diffraction problems associated with nonspherical waves, which are characteristic of nonideal optical systems, is a difficult mathematical problem. This fact, in combination with the scarcity of corre-