POLARONS AND EXCITONS

EDITED BY

C. G. KUPER &
G. D. WHITFIELD

POLARONS AND EXCITONS

Scottish Universities' Summer School
1962

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EARLIER VOLUMES IN THIS SERIES:

Dispersion Relations, edited by G. R. Screaton, M.A., Ph.D. (Scottish Universities' Summer School, 1960)

Fluctuation, Relaxation and Resonance in Magnetic Systems, edited by D. ter Haar, M.A., Dr.Sc. (Scottish Universities' Summer School, 1961)

PREFACE

The third Scottish Universities' Summer School in Physics was held in St Andrews from 30th July to 18th August 1962, the subject chosen being that of " Excitations in Semiconductors: Polarons and Excitons ". Generous financial support from the NATO Science Committee helped us to assemble a distinguished panel of lecturers, and to offer a number of bursaries enabling students to come from far afield. In addition to the seventy-three lecturers and students attending the School, several notable physicists were able to join us for all too brief periods—J. Bardeen, E. L. Hahn, M. Lax, N. H. March and J. J. Markham.

The number of applications for student places greatly exceeded the number who could be accommodated at the School, and it was with much regret that the Selection Committee was forced to exclude so many deserving applicants. There was, nevertheless, at least the partial compensation that those finally selected formed a more homogeneous group as regards standard, and we were able to assure the lecturers that they could assume that all participants possessed, in addition to a deep interest in solid state physics, a thorough knowledge of quantum mechanics up to and including the theory of second quantization.

As will be seen from this record of the scientific proceedings of the School, the basic idea in the planning of the lectures was to encourage the broadest possible approach to both polaron and exciton problems. Thus the theory of the polaron was developed successively from the standpoints of perturbation expansions, successive canonical transformations (intermediate coupling), path-integral formulations, Green function techniques, and strong-coupling theory; and the theory of the exciton was approached from the standpoints of both band theory and atomic excitation. In the lectures dealing with experimental aspects, evidence on the behaviour of polarons and excitons was drawn from measurements of drift and Hall mobilities, cyclotron resonance and optical absorption.

The reader of this volume will appreciate in some measure the debt the School owed to the lecturers, note-takers, editors and secretarial staff, who cooperated in issuing successively pre-prints, post-prints and now this collected record of the scientific proceedings As Director, I must also express my gratitude to all those who helped in the accommodation and entertainment of the lecturers and students, and accompanying wives and children—totalling well over a hundred individuals—

over a period of three weeks: particularly to the Housekeeper and staff of St Regulus Hall, where the great majority of participants and wives were lodged, and which formed a centre for the School; to the Master and Council of St Salvator's College for agreeing to hold an Official Dinner in association with the School; to the Ladies' Committee (Helen Dingle, Jennifer Green and Marie Kuper), indefatigable in arranging baby-sitting, lessons in Scottish Country Dancing, and additional excursions, coffee mornings and tea afternoons for the wives; to the Treasurer and members of the Executive Committee for their vigilance and exertions extending over several months; and above all to the Secretary, Dr C. G. Kuper, whose combination of enterprise and hard work over a very long period contributed so largely to the success of the School.

R. B. DINGLE
Professor of Theoretical Physics
University of St Andrews

EDITORS' NOTE

THE manuscripts of the courses by Fröhlich and Haken, and of McLean's seminar, were prepared largely from notes taken during the lectures. Although these lecturers have had a limited opportunity to amend and correct the manuscripts, the Editors must assume ultimate responsibility for any errors and obscurities which these lecture notes contain. The manuscripts of Brown's, Platzman's, Pines's and Toyozawa's lectures were prepared by the authors in collaboration with note-takers.

We will take this opportunity to thank the note-takers, the editorial committee and the editorial staff for the efficient execution of their often arduous duties; in particular we express our gratitude to Miss Lorna Guild, who continued to assist us for a considerable period after the end of the Summer School.

C. G. KUPER
G. D. WHITFIELD

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INTRODUCTION TO THE THEORY OF THE POLARON

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HISTORICAL INTRODUCTION

THE problem of the motion of an electron in an ionic crystal has been of interest ever since solid state physics began to develop in the early 1930's. The interest in this problem has continued to the present day for two main reasons. Firstly, and more obviously, it is relevant to the applied physics of semiconductors. Secondly, it provides a simple model of a particle interacting with a quantum field, and serves as a testing ground for new methods.

In this historical sketch we hope to illustrate how the attitudes and procedures involved in the development of a physical theory may fail to be reflected in the final solution; conversely, much time and effort may have been taken on what, in retrospect, seems an obvious step. As an example let us look briefly at the concept of free electrons in the theory of metals. By neglecting Coulomb interactions, the free-electron model appeared for many years to be based on inconsistent approximations; Wigner's calculation of the correlation energy (in 1933) showed that Coulomb effects were far from negligible. It was only in the early 1950's that the application of the plasma concept and the understanding of the screening effect in the electron gas effectively validated the independent-electron concept. Both the physical concepts and the mathematical techniques had been familar for many years, yet the essential step was slow in coming.

The earliest direct approach to the polaron problem was by Landau [1] in 1933. In an attempt to explain *F*-centres, he introduced the idea of the self-trapped electron. The idea was that an electron, by its Coulomb interaction with the ions of an ionic crystal, produces a polarization. This polarization constituted a potential " hole " surrounding the electron, which was then trapped (or bound) in this potential. Because the ions are much heavier than the electron, they will not return to their equilibrium positions during a half-period of the electron's motion in the trapping potential, it was thought. The effect may be qualitatively described by

† The manuscript for these lectures was prepared from notes taken by Mr D. M. Eagles and Dr J. M. Vail; in the final lecture they were assisted by Dr R. A. Moore.