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MODELING FLUCTUATIONS IN SCATTERED WAVES

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Preface

Fluctuations in scattered light constitute some of the most familiar natural optical phenomena, including the twinkling of starlight, the glittering of sunlight on a rippled water surface, and the shimmering of distant objects on a hot day. These effects are geometrical in origin, being a consequence of refraction by large-scale random variations in refractive index. Fluctuations of this kind may be caused by density variations within a propagation medium, such as the atmosphere, or by roughness of the interface between media having differing dielectric constants. Although significant dispersion may also be observed (for example, in the case of the colorful twinkling of a star low in the sky), for the most part they are white light optical effects that are independent of wavelength within the visible spectrum. With the advent of the laser, a new range of coherent-light scattering phenomena became visible. In particular, the random interference effect that has come to be known as *laser speckle* commonly occurs in laboratory laser light scattering experiments (Figure 0.1). Indeed, laser light scattering generates many beautiful diffraction, interference, and geometrical optics effects that can be observed with the naked eye. Figure 0.2 shows the effect of scattering multifrequency laser light from a piece of ground glass in the laboratory. Figure 0.3 shows the optical intensity pattern obtained when laser light is passed through turbulent air rising above a heating element, and is rather reminiscent of the pattern sometimes observed on the floor of a swimming pool.

It is important to recognize that a similar range of effects is generated at other frequencies of the electromagnetic spectrum and in the scattering of sound waves. These can and have been measured, but of course they cannot be seen with the naked eye. However, they obviously contain information about the scattering object and they also limit the performance of sensing and communication systems. One familiar example of this is the fading of shortwave radio reception due to ionospheric fluctuations. Many early theoretical results were derived as a consequence of the observation of fluctuations at radio and radar wavelengths. For example, an early theoretical description of random interference effects was developed to explain the fluctuating radar return from raindrops twenty years before the visual appearance of the equivalent laser-generated phenomenon gave rise to the term "speckle." It was soon recognized that intensity patterns of the form shown in Figure 0.1 were manifestations of Gaussian noise, a well-known statistical model that is readily characterized and amenable to calculation. Unfortunately, in practice the more complicated radiation patterns illustrated in Figure 0.2 and Figure 0.3 are common and the Gaussian noise model

FIGURE 0.1
Random interference pattern generated when laser light is scattered by ground glass. (See color insert following page 14.)

cannot adequately describe these. The aim of this book is to provide a practical guide to the phenomenology, mathematics, and simulation of non-Gaussian noise models and how they may be used to characterize the statistics of scattered waves.

The plan of the book is as follows: After Chapter 1, in which the statistical tools and formalism are established, Chapter 2 reviews the properties of Gaussian noise including some lesser known results on the phase statistics. Chapter 3 describes processes derived from Gaussian Noise that are commonly encountered, while Chapter 4 discusses deviation from Gaussian statistics in the context of the random walk model for discrete scattering centers. The random phase-changing screen is a scattering system that introduces random distortions into an incident wave front. It provides an

FIGURE 0.2
The effect of focusing multifrequency laser light onto a small area of the same kind of scatterer as in Figure 0.1. (See color insert following page 14.)

excellent model for a wide variety of continuum scattering systems, ranging from ionospheric scattering of radio waves to light scattering from rough surfaces, and is a valuable aid to understanding the phenomenology in non-Gaussian scattering regimes. Chapter 5 through Chapter 8 provides an overview of the predictions of this ubiquitous model, making various assumptions for the properties of the initial phase distortion. Chapter 9 addresses aspects of propagation through an extended medium while Chapter 10 discusses some multiple scattering effects. The scattering of vector waves and polarization fluctuations are discussed in Chapter 11. Chapter 12 is devoted to a discussion of perhaps the most widely used non-Gaussian model: *K*-distributed noise. Chapter 13 outlines some of the practical limitations encountered in experimental measurement and how they affect the interpretation of results, detection, and measurement accuracy. Finally, Chapter 14 will describe techniques for numerical simulation.

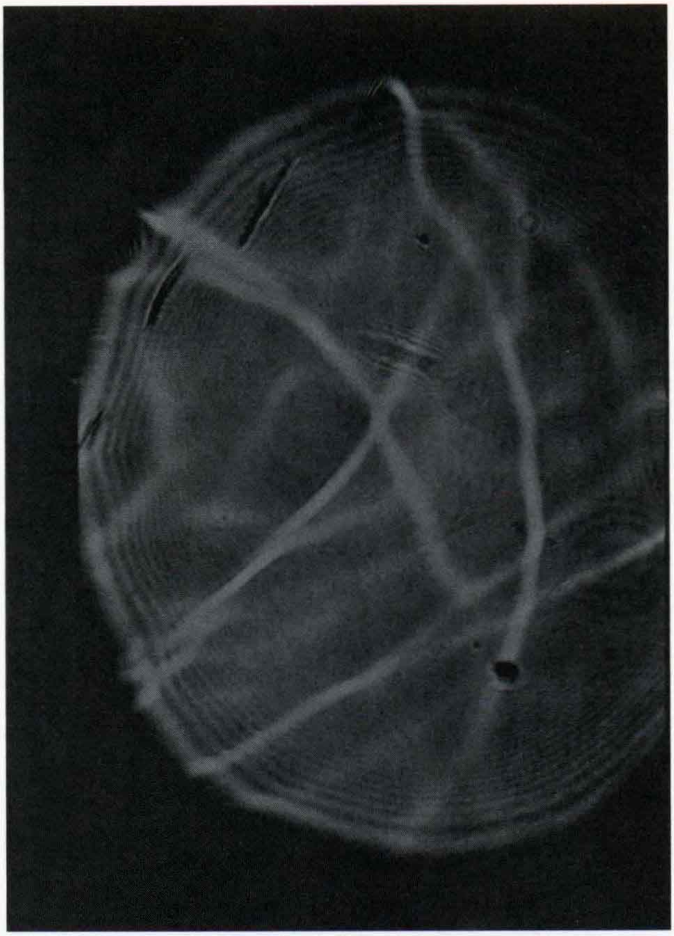

FIGURE 0.3
Intensity pattern obtained when laser light is passed through air convecting above a heating element. (See color insert following page 14.)

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Authors

Eric Jakeman is an emeritus professor of the University of Nottingham, associated with the School of Electrical and Electronic Engineering and the School of Mathematical Sciences. He graduated with a degree in mathematical physics at Birmingham University in 1960 and obtained his PhD on superconductivity theory in 1963. After a year researching low-energy nuclear physics at the University of California, Los Angeles, he joined the Royal Radar Establishment (now QinetiQ) Malvern in 1964 where he eventually rose to the position of chief scientist in the electronics business sector. His interests have ranged from heat and mass transfer problems in crystal growth to photon-counting statistics, but since the mid-1970s his principal area of work has been the analysis of fluctuations in scattered waves. Professor Jakeman has published more than 180 papers in refereed journals. He was elected Fellow of the Institute of Physics in 1979, Fellow of the Optical Society of America in 1988, and Fellow of the Royal Society of London in 1990.

Professor Jakeman was a member of the Council of the Institute of Physics from 1985 until 2003 and was vice president for publications and chairman of the publishing board 1989 to 1993 and again 2002 to 2003. He was honorary secretary from 1994 to 2003. He has also served on the executive committee of the European Physical Society. University positions held include visiting professor in the physics department at Imperial College of Science Technology and Medicine, London, and special professor at the University of Nottingham where he took up a permanent position in 1996.

Kevin Ridley is currently employed as a laser physicist at QinetiQ Malvern. He graduated from the University of Bath with a degree in applied physics in 1987. He then joined the Royal Signals and Radar Establishment (now QinetiQ) Malvern and obtained his PhD as an external student at Imperial College London in 1993 on the subject of optical phase conjugation and stimulated Brillouin scattering. He has worked on a wide range of laser-related topics including nonlinear optics, statistical optics, laser light propagation through the atmosphere, laser vibrometry, and photon-counting detectors. Dr. Ridley has published over 40 papers in refereed journals. He is a QinetiQ Fellow and was elected a Fellow of the Institute of Physics in 2002.

Contents

1

Statistical Preliminaries

1.1 Introduction

The purpose of this chapter is to provide a brief introduction to the statistical quantities and notation that will be used in the remainder of the book. There are many excellent texts on probability theory, noise, and stochastic processes, covering the subjects at various depths and levels of sophistication. The treatment here is aimed at a nonspecialist user community and we shall present a straightforward engineering exposition that covers only the essentials required to understand the principles, significance, and application of the statistical models that are described in the following chapters. A more comprehensive treatment, taking the same general approach, is to be found in the book *An Introduction to the Theory of Random Signals and Noise* by Davenport and Root [1], while for the ultimate encyclopedic treatment the reader is referred to *An Introduction to Statistical Communication Theory* by Middleton [2]. Optical engineers may find the treatment given in the excellent treatise *Statistical Optics* by Goodman [3] more to their taste.

1.2 Random Variables

We start with the description of a simple one-dimensional signal $V(t)$ (Figure 1.1) that is a random function of the time t, that is, a random *process*. Fluctuations in the value of V may be the result of many rapidly changing underlying variables or may be the chaotic outcome of a complicated system of nonlinear differential equations. Although the functional dependence of V on time may be random, the result of a measurement of V can be expressed statistically in terms of the single-time probability density function (pdf) $P(V(t))$ which defines the likelihood of obtaining a value V at time t. Similarly, the double-time density $P(V_1(t_1),V_2(t_2))$ defines the joint probability of obtaining the values V_1 and V_2 by making measurements at times t_1 and t_2,