

Frontiers in
*Applied General
Equilibrium Modeling*

Edited by

Timothy J. Kehoe • T. N. Srinivasan • John Whalley

CAMBRIDGE

Frontiers in Applied General Equilibrium Modeling

In Honor of Herbert Scarf

Edited by

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FRONTIERS IN APPLIED GENERAL EQUILIBRIUM MODELING

This volume brings together fifteen papers by many of the most prominent applied general equilibrium modelers to honor Herbert Scarf, the father of equilibrium computation in economics. It deals with new developments in applied general equilibrium, a field that has broadened greatly since the 1980s. The contributors discuss some traditional as well as some newer topics in the field, including nonconvexities in economy-wide models, tax policy, developmental modeling, and energy modeling. The book also covers a range of new approaches, conceptual issues, and computational algorithms, such as calibration, and new areas of application, such as the macroeconomics of real business cycles and finance. An introductory chapter written by the editors maps out issues and scenarios for the future evolution of applied general equilibrium.

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Herbert Scarf has been the guiding spirit in several ways for each of us. Kehoe and Whalley did their doctoral dissertations under Scarf and have continued to be involved with him. Then a graduate student at Yale, Srinivasan had the privilege of hearing Scarf present his paper (Scarf 1960) on examples of global instability of competitive equilibrium at the summer meetings of the Economic Society at Stanford in 1959. Since joining the faculty of the Department of Economics at Yale in 1980, Srinivasan has constantly learned from and interacted with Scarf, including jointly teaching a course on Applied General Equilibrium twice in the past several years. All of us owe a deep intellectual debt to him.

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Introduction

Timothy J. Kehoe, T. N. Srinivasan, and John Whalley

This volume honors Herbert Scarf and his contributions to economics. It deals with new developments in applied general equilibrium (AGE) modeling, a field in which Scarf's contributions have played a decisive role. All but two of the chapters in the volume were presented at a conference held at Yale University in April 2002. The chapter by Herbert Scarf and Charles Wilson was written afterward; it demonstrates the uniqueness of equilibrium in an important class of international trade models. The chapter by Lars Ljungqvist and Thomas Sargent is an outgrowth of Sargent's discussion at the conference of the paper presented there by Edward Prescott. The chapters presented here build on a well-known earlier volume in applied general equilibrium, edited by Herbert Scarf and John Shoven in 1984 (Scarf and Shoven 1984), which in turn grew out of Scarf's pioneering contributions in general equilibrium computation in the 1960s and early 1970s (Scarf 1967a, Scarf and Hansen 1973). Kenneth Arrow's chapter in this volume points out that the ability to deploy AGE models is the product of research advances, going back at least 130 years, in which progress in economic theory and vastly improved availability of economic data have played crucial roles. According to Arrow, equally crucial inputs were improvements in computing power and the development of algorithms for computing equilibrium, in which Scarf's (1967b) algorithm based on simplicial subdivisions was the crucial step.¹

Since the 1980s, applications of AGE have broadened. They now include international trade, public finance, development, energy, and climate change and broader environmental concerns, as well as other fields. A range of new approaches and conceptual issues, not to mention computational algorithms, has evolved. These include calibration and expanded areas of application, such as macroeconomics of real business cycles and finance. In addition, the techniques of AGE modeling – namely calibrating and benchmarking observed data on economies into an initial

¹ In an as yet unpublished paper, Scarf (2002) provides a fascinating account of his involvement in the computation of economic equilibria and the contribution of his interaction with faculty and students at Yale in the late sixties and seventies.

equilibrium data set and then doing counterfactual policy analysis – have spread into other areas, such as game theory and even partial equilibrium models of industrial organization.

After the initial phase of demonstrating the potential of using AGE models for policy analysis, searching questions were raised as to the performance and robustness of AGE as a basic tool in policy and other work. Most of the policy applications had been performed *ex ante*, in anticipation of a policy change being enacted, such as the implementation of NAFTA or of the Uruguay Round agreement on international trade. These applications provided valuable estimates of the likely consequences of a policy change. For the methodology of AGE to become a widely accepted and useful policy tool, however, cross checking of the projections of models with actual outcomes, after the policy change has been put in place, is essential. Such cross checking (which is an analogue of cross checking of out-of-sample predictions of an econometric model with its actual realizations) has to allow for the fact that projections of an AGE model are conditional in that they are based on particular assumptions about values of variables exogenous to the model, and, as such, the projections could deviate from the actual outcomes if the realized values of exogenous variables differed from their assumed values. Also, in actual implementation, aspects of a policy could differ from those assumed in the model, and other policies not included in the model could be implemented at the same time. Nonetheless, with appropriate allowance for these factors, it should be possible to look backward, after the model's policy change has been implemented, and evaluate how accurate and useful the model projections were. Timothy Kehoe's evaluation of models of NAFTA in this volume is one such attempt.

This volume builds on existing AGE literature and consciously aims to go well beyond it and to look to the future. Scarf's research agenda of making the elegant theoretical general equilibrium models fully operational, implementable with actual data, and useful to practitioners such as policy makers is relevant to all theoretical models of economics. All analytical structures should, in principle, have their numerical analogues implementable with data. The practical issues are how to do this and what conclusions can be drawn from simulations or projections from the numerical model. Similar issues of how models are parameterized or calibrated arise even when models other than the general equilibrium model are used.

The chapters in this volume illustrate both the progress in AGE modeling since the 1980s and applications to new areas, as well as challenges that remain to be addressed. We start with a discussion of the origins of applied general equilibrium modeling and Herbert Scarf's contributions to this field. We then provide brief descriptions of the individual chapters.

ORIGINS OF APPLIED GENERAL EQUILIBRIUM MODELING

Numerical applications of general equilibrium began with the work of Arnold Harberger (1962) and Leif Johansen (1960). Harberger used a model with two

production sectors, one corporate and one noncorporate, calibrated to U.S. data from the 1950s, to calculate the incidence of the U.S. corporate income tax. Johansen used a model with nineteen production sectors, calibrated to Norwegian data from 1950, to identify the sources of economic growth in Norway over the period 1948–53. Both linearized the model and solved it analytically without worrying whether an equilibrium of the original nonlinear model actually existed near the benchmark equilibrium. Neither Harberger nor Johansen raised the possibility of multiple equilibria in the model or attempted to check for multiplicity in any way. Interestingly, in many of the more recent contributions to AGE, proofs of existence are also forgone, and instead a computational algorithm is presented, which, in practice, converges to an approximate equilibrium, given some specified measures for closeness of approximation. The check for multiplicity, if at all attempted, is often rudimentary – it is simply whether the algorithm, starting from different initial positions, converges to the same or different final positions.

The first rigorous approach to developing a computational algorithm that was guaranteed to find equilibria to any desired degree of approximation dates to the pioneering work of Herbert Scarf, first published in 1967. Although Scarf himself did not ever put together an AGE model and solve it for its equilibrium using his algorithm, he clearly had applications in mind. In fact, describing his involvement in the computation of equilibrium, Scarf (2002) has this to say about the numerical example with six commodities and eight activities in his 1967 paper (Scarf 1967a):

The example was meant to suggest to my colleagues, at Yale and elsewhere, that these novel numerical techniques might be useful in assessing consequences for the economy of a change in the economic environment, or in a major policy variable – to engage in comparative statics where the equilibrium model was too large to solve graphically or by hand.

He adds that:

... it was some time before this suggestion was taken seriously. We were in the 1960s, in the era of large Keynesian macro models in which specific scarce resources and relative prices were not included; there was, in the air, a suggestion that the economy could actually be fine-tuned by prescient economic advisors.

It turned out that building large macro econometric models would no longer engage academic macroeconomists, although private economic forecasters and some public agencies continue to use such models. Whether this development is to be applauded or regretted, it is a fact. In contrast, the use of AGE models has grown far beyond what Scarf might have foreseen in 1967.

We would like to supplement the intellectual history of AGE modeling, narrated by Arrow in his paper, with an account of the contributions of Scarf and his students at Yale to this history, drawing on the account in Scarf (2002). Kenneth Arrow and Gerard Debreu (1954) and Lionel McKenzie (1959) provided a careful definition of a competitive equilibrium, a rigorous proof of its existence under certain sufficient conditions, and a characterization of the equilibrium (often called the two

fundamental theorems of neoclassical welfare economics) and its extension to cover transactions over time and involving uncertainty. The first fundamental theorem established that the set of competitive allocations without lump sum redistribution is a subset of Pareto efficient allocations. The second theorem shows that with lump sum redistributions, any Pareto efficient allocation can be sustained as a competitive equilibrium, thus the set of competitive allocations and the set of Pareto efficient allocations are the same, once lump sum redistributions are allowed. In 1881, Francis Edgeworth had developed the idea of the core as the set of allocations upon which no coalition of agents in the economy can improve, in the sense of doing better for all of its members by an alternate allocation in an economy of its own with its own endowments and technology. The core is obviously a subset of the set of Pareto efficient allocations, but in general, the core is much smaller than the Pareto efficient set. Debreu and Scarf (1963) proved a deeper result than the first welfare theorem by showing that the core converges to the set of competitive allocations – without lump sum income redistributions – as the economy is replicated.

The issue of whether there is a mechanism that will lead an economy to a competitive equilibrium is related to the development of an algorithm to compute equilibrium. Léon Walras, the founding father of general equilibrium theory, had proposed in 1874 a process that he called *tâtonnement*, or groping, to find an equilibrium. Paul Samuelson later formalized this *tâtonnement* process as a system of differential equations. This process raises the price of a good in positive excess demand and lowers the price of a good in negative excess demand. Research in the late 1950s by Kenneth Arrow, H. D. Block, and Leonid Hurwicz (1959) and by Hirofumi Uzawa (1960) established that the Walrasian *tâtonnement* process for an exchange economy was globally stable provided either that the market excess demands exhibited gross substitutability or that they satisfied the weak axiom of revealed preference. Unfortunately, although gross substitutability in the excess demand of each individual consumer ensures that it holds also for the aggregate market excess demand, it is not satisfied for individual demand functions that exhibit some complementarity, however modest. On the other hand, although the weak axiom is satisfied by individual excess demands, it need not be satisfied by aggregate excess demands. In 1960, Scarf produced the first examples of global instability of the competitive equilibrium woven around preferences that exhibit complementarity. Scarf's examples come as no surprise now, because a later series of papers by Hugo Sonnenschein (1973), Rolf Mantel (1974) (a student of Scarf), and Gerard Debreu (1974) showed that, with a sufficient number of consumers, aggregate excess demand is essentially arbitrary and hence the behavior of the *tâtonnement* can be made to follow arbitrary curves. At the time, however, Scarf's (1960) paper had considerable influence in discouraging enthusiasm for the *tâtonnement* process. It also had the effect of focusing Scarf's own attention on the need for developing an algorithm for calculating competitive equilibria.

Scarf had first thought that computation of competitive equilibria could be found by finding allocations in the core of an economy and then replicating. He had developed an algorithm for finding allocations in the core (Scarf 1967c) using techniques similar to those developed by Carlton Lemke and Joseph Howson (1964) for finding Nash equilibria for two-person non-zero-sum games. Scarf then developed an alternative algorithm avoiding the core and closely related to Sperner's argument demonstrating Brouwer's fixed point theorem. This later work was the beginning of homotopy, or path-following, computational algorithms for calculating equilibria. Scarf himself later made important contributions to the theory and implantation of these algorithms, most notably in his joint work with Curtis Eaves (Eaves and Scarf 1976).

Some students of Scarf at Yale – Terje Hansen, Timothy Kehoe, Rolf Mantel, Michael Todd, and Ludo van der Heyden – wrote Ph.D. theses on computation or on theoretical topics related to computation. But Scarf encouraged even more of his students to search for ways in which to apply general equilibrium theory and these novel computational techniques. Indeed, there is what many would characterize as the Yale school of economists, who use AGE models to do economic policy analysis. Students of Scarf in this group include Andrew Feltenstein, Timothy Kehoe, Ana Matirena-Mantel, Marcus Miller, Donald Richter, Jaime Serra-Puche, John Shoven, John Spencer, and John Whalley. A feature that characterizes the research of the Yale school of AGE modeling – and distinguishes it from some other AGE modelers – is its heavy interaction with the general equilibrium theory of Arrow, Debreu, McKenzie, and Scarf. Members of the Yale school rely on rigorous theory to guide the development of their models, and they carefully modify and develop new theory when the existing theory is not adequate for their particular applications.

As applications of AGE modeling progressed, they were taken up by governments and international organizations around the world. The World Bank, the World Trade Organization, the International Monetary Fund, and government agencies in the United States, Australia, Canada, Mexico, the United Kingdom, the Netherlands, and many other countries all had general equilibrium models. The field of AGE modeling as an operational tool in government and policy circles was launched.

CONTRIBUTIONS IN THIS VOLUME

We have grouped the contributions into parts in order to bring coherence to our discussion and to draw on any overlaps among them. Part 1 has two chapters. The first, by Kenneth Arrow, is an expanded version of his talk at the conference dinner. It is a fascinating recapitulation of the intellectual history of general equilibrium theory and its use in AGE. The chapter of Herbert Scarf and Charles Wilson follows Arrow's chapter. It is on pure general equilibrium theory as applied to the classic Ricardian model of international trade. It provides elegant proofs for the uniqueness (assumed by most trade economists) of equilibrium for the model under the well-known

sufficient condition of gross substitution in aggregate demand. One of the proofs relies on the fixed point index theorem developed by Timothy Kehoe (1980) in his Ph.D. thesis, written under Scarf's supervision. Kehoe's work had been inspired by the results obtained by Eaves and Scarf (1976).

Part 2 consists of two chapters on developments in computation methods. The first, by Kenneth Judd, presents an alternative algorithm for solving dynamic stochastic models, combining convergent methods for solving finite systems of equations with convergent dynamic programming. The second, by Michel Ferris, Steven Dirkse, and Alexander Meeraus, describes a new suite of methods for solving problems that combine facets of optimization and complementarity using a unifying framework of mathematical programs with equilibrium constraints.

Part 3 is devoted to applications in macroeconomics and finance. It consists of four chapters. Edward Prescott reviews the role of nonconvexities at the micro level in macro business cycles. In contrast to a long tradition of viewing business fluctuations as disequilibrium phenomena, in contemporary stochastic dynamic general equilibrium macroeconomic models, of which Prescott's is one, the cycles emerge from the stochastic processes that are essential elements of the models. Thus random, but persistent, changes in the factors that determine the level of output give rise to fluctuations that approximate those observed in real economies. The chapter by Lars Ljungqvist and Thomas Sargent both complements and challenges the results presented by Prescott. It shows that models in which unemployment is frictional have very different implications for the data than do models, such as that of Prescott, in which lotteries transform the economic environment into a standard Arrow-Debreu-McKenzie general equilibrium setting. The chapter by Makoto Nakajima and José-Víctor Ríos-Rull focuses on borrowing and lending by individual agents with endogenous default and credit limits and explores the extent to which aggregate events are amplified or smoothed by bankruptcy filings. The parameters of this model are estimated using U.S. data and the model replicates aggregate fluctuation of the U.S. economy. The chapter by Alosio Araujo and Mário Páscoa also models default penalties and collateral and credit restrictions. It extends the received theory of general equilibrium with incomplete markets that has been used to analyze the stochastic volatility of asset prices and the risk premium puzzle to incorporate default, credit risk, and institutions to deal with them. This chapter, which is theoretical, shows that Ponzi schemes and asset price bubbles may occur. The authors provide sufficient conditions for the nonexistence of bubbles in equilibrium.

Part 4 consists of three chapters on applications of AGE to public finance, development, and climate change. Dale Jorgensen and Kun-Young Yun employ an aggregate dynamic general equilibrium model of the U.S. economy to analyze the economic impact of alternative tax reform proposals. Equilibrium is characterized by an intertemporal price system that clears markets for labor, capital services, consumption goods, and investment goods. Starting from the base case solution of a unique steady state for the tax policy existing in 1996 and the associated