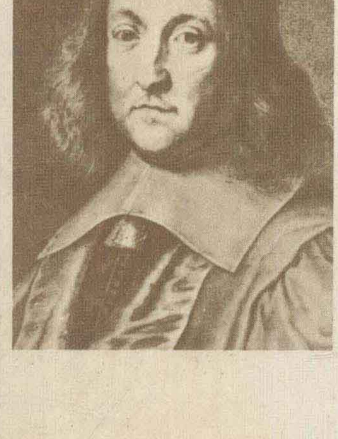
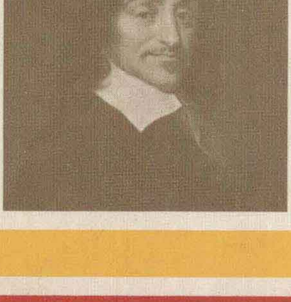
MATHEMATICS
The Alphabet of Science

Willerding & Hayward






SECOND EDITION

MATHEMATICS

The Alphabet of Science

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MATHEMATICS
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To our loyal friends
Gai
Suzie
Meg
Maggie

PREFACE

This revision of *Mathematics: The Alphabet of Science* is based on many letters, comments, reviews, and evaluations received by the authors, and upon the classroom experiences of many instructors and students since the first edition in 1967. The original text has been rewritten and supplemented with these comments, criticisms, and suggestions as a guide.

The book is intended for liberal arts students and other students who wish to know what mathematics is about, but who have no desire to be mathematicians. The presentation is designed so that even persons who have little or no high school mathematics, or those whose mathematics study is so far in the past that it is completely forgotten, will have no trouble following the explanations.

The topics selected for discussion are simple yet profound. Many have applications in fields other than mathematics. Explanations have been made as detailed as is reasonably possible. The manner in which the subjects are treated will cause no problems even to the most mathematically unsophistocated student. For a complete understanding of the topics presented, the reader should verify the calculations and ponder the arguments as he encounters them. But even if the reader reads this book as he reads a novel, he will find enough to acquaint him with some exciting topics of mathematics both ancient and modern.

Numerous illustrative examples are presented throughout the book to clarify the use of mathematical ideas. The number of exercises in all chapters has been substantially increased and the answers to all odd-numbered problems are included in the text.

Chapter 1 presents the basic notions of mathematical logic and gives the reader a respect for correct reasoning and a clear understanding of the type of deductive reasoning used in mathematics. In this edition a new section on negation of compound statements has been added and deMorgan's Laws are presented.

Chapters 2, 3, and 4 present topics from Number Theory. To justify the inclusion of these chapters, if such a justification is necessary, we quote an address of G. H. Hardy given before the American Mathematical Society.

> The elementary theory of numbers should be one of the very best subjects for early mathematical instruction. It demands very little previous knowledge; its subject matter is tangible and familiar; the processes of reasoning which it employs are simple, general and few; and it is unique among the mathematical sciences in its appeal to natural, human curiousity. A month's intelligent instruction in the Theory of Numbers ought to be twice as useful and at least ten times more entertaining than the same amount of calculus for engineers.

Chapter 5, Sets, Relations, and Functions, is completely new in this edition. Here we review sets, set operations, Cartesian products, and the Cartesian plane. Relations and functions are defined; function notation is presented; and linear and quadratic functions are discussed in detail and graphed.

Chapters 6 and 10 study abstract mathematical systems and stress the postulational method. Chapter 6, Groups and Fields, has been completely rewritten and supplemented. The modulo-seven system discussed in Chapter 6 is presented here as an example of a finite field (the F_7 field). Solution sets of linear equations over F_7 are found and graphed. Polynomials over F_7 are also discussed and graphed.

Chapter 10, Finite Geometry, gives the reader an opportunity to study a geometry with only a finite number of points as contrasted with the infinite geometry studied in high school. The numerous letters received by the authors praising this chapter have resulted in the presentation being virtually unchanged from the first edition.

Chapter 7, The Pythagorean Theorem, is substantually the same. The only change being the addition of a section on the distance formula from analytic geometry. This chapter, like Chapter 10, seems to have many fans among the users of the first edition.

Chapter 8 has, to a large extent, been rewritten. The fundamental counting principle and a discussion of permutations and combinations have been added. The discussion of these topics facilitates a deeper understanding of the probability theory presented and makes possible a slightly more sophistocated presentation of probability measure.

Chapter 9, Matrices, has been lengthened to include the solution of matrix equations, solutions of systems of linear equations using matrices, and determinants of square matrices.

Chapter 11, Analytic Geometry: The Straight Line and the Circle, is entirely new. This chapter includes the midpoint formula, the analytic geometry of the line and the circle, and analytic proofs of theorems from elementary geometry.

Chapters 12, 13, and 14, present some of the fundamental principles and the mathematical concepts related to the digital computer. The materials presented are those basics necessary in the effective use of a computer. These so-called "thinking" machines have become an integral part of life in this scientific age, and the authors feel that every educated person should understand their workings. The old adage "first things first" was primary in the selection of the materials included in these chapters.

One of the first prerequisites of a working knowledge of the sewing machine is how to thread it; similarly, to understand the workings of a computer one must have some idea of how the machines themselves operate and how they are programmed to solve problems. Many of the students using this text will someday have positions in which such knowledge will be invaluable as well as necessary. With these objectives in mind Chapters 12, 13, and 14 were written.

Chapter 12 covers systems of numeration used by computers and persons working with computers. All present-day, large-scale digital computers use the binary system of numeration to represent data. The binary system and other related systems (those based on powers of two) used in the computer field are covered. The decimal system is presented before the other positional numeration systems as an aid in the understanding of numeration systems with bases other than ten.

Chapter 13 attempts to explain some of the basics of the digital computer without becoming technical. Some topics covered are basic machine language (bit patterns) related to assembly language, complement arithmetic, and representation of real numbers versus integers. These topics show the need for studying the systems of numeration presented in Chapter 12.

Chapter 14 is a brief treatment of a higher level compiler language, FORTRAN. Compiler languages are used in most programs written today for computers. They provide a relevant set of mnemonics to simulate the problem to be solved. Although these symbols must be translated into machine language by the compiling program, the user need not know details about either of these. FORTRAN was chosen to represent the world of compiler languages since its structure is very much like the structure of mathematical equations.

The authors would like to thank all those students and instructors who used the first edition of Mathematics: The Alphabet of Science and particularly those who wrote and made suggestions as to its improvement. We would like to thank Professor Walter J. Gleason, Bridgewater State College, Bridgewater, Massachusetts; Dr. Kelvin Casebeer, Southwestern State College, Weatherford, Oklahoma; Professor C. Ralph Verno, West Chester State College, West Chester, Pennsylvania; and Professor Henry Harmeling, Jr., North Shore Community College, Beverly, Massachusetts for their critical reviews of the manuscript and for their helpful suggestions for improving the book.

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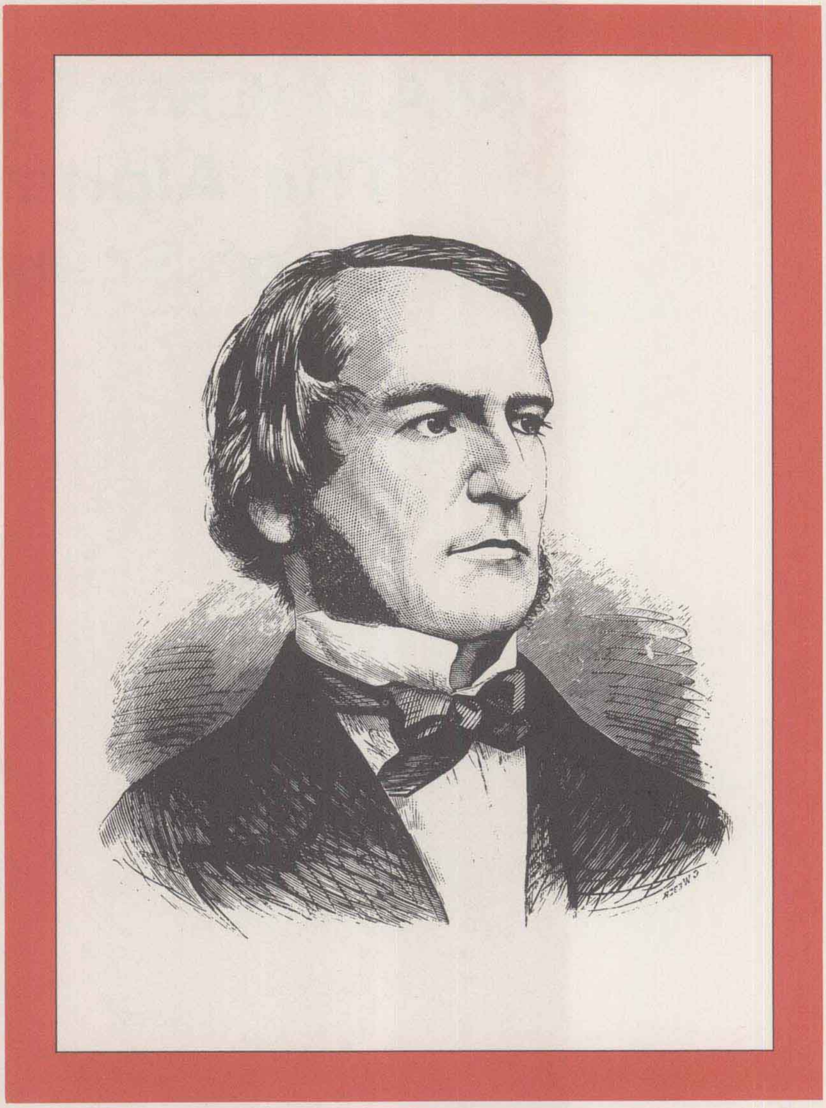

George Boole (1815–1864) discovered that the symbolism of algebra can be used not only for making statements about numbers, but also about mathematical logic. In his book called *The Mathematical Analysis of Logic*, Boole developed the idea of formal logic and introduced an algebra into the field of logic. Today, Boolean algebra is important not only in the field of logic, but also in the geometry of sets, the theory of probability, and other fields of mathematics.

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