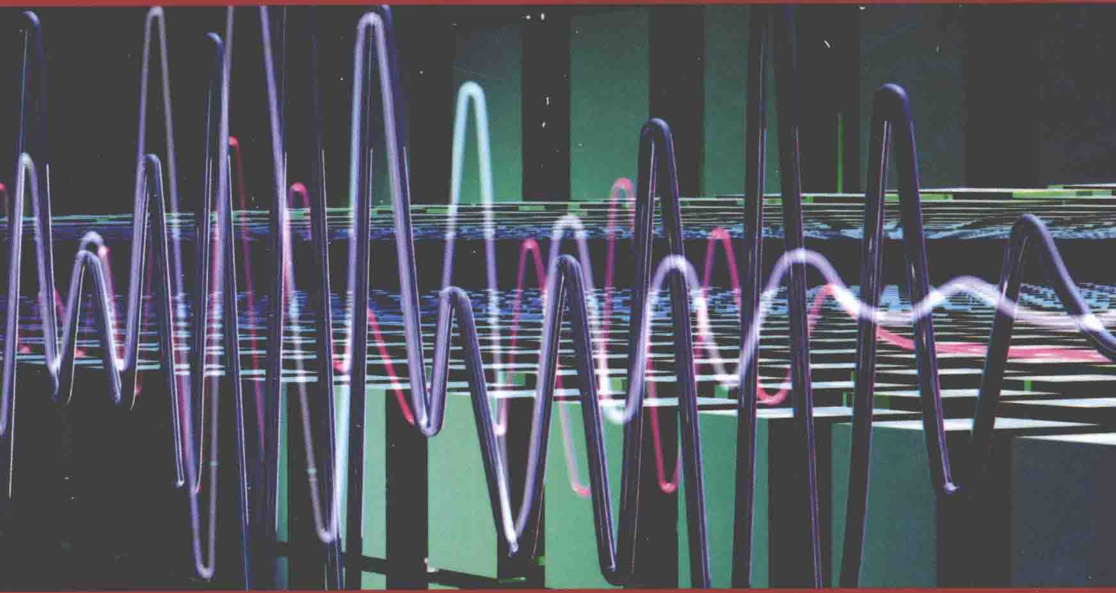
DIGITAL SIGNAL AND IMAGE PROCESSING SERIES



Signal and Image Multiresolution Analysis

Edited by Abdeldjalil Ouahabi

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Series Editor
Francis Castanié

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First published 2012 in Great Britain and the United States by ISTE Ltd and John Wiley & Sons, Inc.

ISTE Ltd
27-37 St George's Road
London SW19 4EU
UK

www.iste.co.uk

John Wiley & Sons, Inc.
111 River Street
Hoboken, NJ 07030
USA

www.wiley.com

Library of Congress Control Number: 2012947123

British Library Cataloguing-in-Publication Data
A CIP record for this book is available from the British Library
ISBN 978-1-84821-257-2

Printed and bound in Great Britain by CPI Group (UK) Ltd., Croydon, Surrey CR0 4YY

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Introduction

Wavelet-based multiresolution analysis can be applied for analysis, processing and synthesis of mono- or multidimensional signals at different levels of resolution, by decomposing them into (orthonormal) "scaling" and (orthonormal) "wavelet" functions. Therefore, multiresolution analysis provides a family of orthonormal wavelets, and reduces any redundancy to nil.

In practice, on the one hand, the field of activity awaited the work of Mallat and Daubechies, which focused on implementation adapted to the pyramidal algorithms of Burt and Adelson, for a concrete use of the wavelets to be generated, using the fast wavelet transform. On the other hand, multiresolution analysis was also contingent on the benefits of sub-band coding, introduced in 1977 by Esteban and Galand.

The concept of multiresolution analysis provides a framework for the decomposition (and reconstruction) of a signal in the form of a series of approximations of decreasing scale, completed by a series of details.

To illustrate this idea, let us take the case of an image constructed of a succession of approximations; details improve this image. Therefore, coarse vision becomes finer and more precise.

In fact, in multiresolution analysis, fast wavelet transform is achieved without wavelets or scaling functions! All that is required are the coefficients of a low-pass filter h (and of a high-pass filter derived from h). That is the wonder of Mallat's fast wavelet transform. In practice, such filters are directly linked to the chosen wavelet.

Introduction written by Abdeldjalil OUAHABI.

Researchers, engineers and practitioners of a diverse range of disciplines (multimedia, telecommunications, medicine and biology, signal and image processing, fracture and fluid mechanics, thermodynamics, astrophysics, finance, etc.) are confronted daily with increasingly challenging technological problems at multiple levels of analysis, such as in classification, segmentation, detection (of contours or parameters of interest), reduction or even elimination of noise, compression in preparation for transmission or storage, synthesis and reconstruction.

The concept of multiresolution analysis constitutes an efficient tool, often without *a priori* knowledge, that is universally applicable to the domains noted above. This tool, sometimes described as miraculous, produces an immediate and readily interpretable and exploitable result. However, for specific applications that require the extraction of targeted information, it is amply clear that it will be necessary to develop and "merge" advanced methods that use existing techniques or optimize analyses (for example, in compression) by taking into account edges or borders, using third-generation waveforms such as ridgelets, curvelets, and strips. In effect, these anisotropic wavelets are automatically oriented and expanded by unifying the geometry of a potential edge.

Since the concept of multiresolution analysis has been well-known for two decades, it is now appropriate to question the utility of further work.

This book aims to provide a simple formalization and new clarity for multiresolution analysis, rendering accessible obscure techniques, and merging, unifying or completing the technique with encoding, feature extraction, compressive sensing, multifractal analysis and texture analysis.

This book is aimed at industrial engineers, medical researchers, university lab attendants, lecturer-researchers and researchers from various specialisms. It is also intended to contribute to the studies of graduate students in engineering, particularly in medical imaging, intelligent instrumentation, telecommunications, and signal and image processing. Given the diversity of the problems posed and addressed, this book paves the way for the development of new research themes, such as brain–computer interface (BCI), compressive sensing, functional magnetic resonance imaging (fMRI), tissue characterization (bones, skin, etc.) and the analysis of complex phenomena in general.

Throughout the chapters, informative illustrations assist the uninitiated reader in better conceptualizing certain concepts, taking the form of numerous figures and recent applications in biomedical engineering, communication, multimedia, finance, etc.

The first chapter of this book briefly recounts the story of the discovery of wavelets, and simply and informatively summarizes the principal themes of multiresolution analysis in one or two dimensions. Abdeldjalil Ouahabi illustrates the interest of the concept of "multiresolution analysis" through several recent applications in feature extraction and classification, adaptive compression, masking encoding and image transmission errors, and suppression of correlated noise, the most notable of which come from the medical (ECG, EEG, BCI and fMRI), telecommunications and multimedia domains.

In Chapter 2, Abdeldjalil Ouahabi introduces the notion of complexity in the context of "multifractal" modeling and analysis of complex, nonlinear and irregular phenomena involving a large hierarchy of scales.

Historically, multifractal analysis was developed in the 1980s to understand and analyze complex phenomena in which regularity varies significantly from one point to another. It also aims to extract new quantitative parameters or multifractal attributes to be used in the more general tasks of analysis of complex phenomena, for example in classification or characterization.

It is interesting to note that multifractal analysis and wavelet transforms are two initially distinct concepts, but were born in the same period at the beginning of the 1980s, and address common concepts: oscillation of localized functions, the concept of scale and localization of singularities. The use of wavelets, and more particularly multiresolution analysis, since the first work in multifractal analysis of turbulent signals, has since facilitated the refining of the practice of multifractal analysis.

After suggesting some practical tools and their related algorithms, Abdeldjalil Ouahabi revisits the "multifractal formalism" based on multiresolution analysis, with an emphasis on algorithm and statistical validation. He illustrates the topic with several applications of signal processing to the analysis of turbulent high-speed signals, financial data, Internet traffic, food quality, medical images, in particular in BCI, fMRI, bone density (osteoporosis) and in ultrasonic tissue characterization (melanoma).

Chapter 3 is devoted to multimodal compression. It consists of joint compression of several signals of different modalities using a unique encoder (here, the JPEG 2000 standard). The supervised insertion approach presented by Régis Fournier and Amine Naït-Ali in this chapter is based on quasi-optimal insertion of one signal into another (that is the insertion of a signal into an image) by exploiting the oversampled characteristics of certain zones, known as insertion zones. The future of this novel approach seems promising, notably for signals particularly rich in potential insertion zones such as high-definition (HD) images.

Chapter 4 details the use of multiresolution analysis via wavelet packets in the medical domain, and specifically in the detection of microembolisms.

Migration of these microembolisms can result in a cerebral artery aneurysm, disrupting circulation; this can cause an ischemic stroke, cerebrovascular accident (CVA). The number of CVAs is increasing and presents a serious problem in public health. Therefore, their timely detection is particularly important due to their correlation with an increased risk of accidents.

Rather than immediately presenting the best multiresolution analysis-based microembolism detector, Jean-Marc Girault first explains and justifies the entire process involved in the implementation of a detector: from analysis to the modeling of ultrasonic Doppler blood processes, via existing research and data extraction. Such synchronous detectors based on multiresolution analysis by wavelet packets are compared with other detectors, from the most recent to more classic systems, in the case of synthetic signals and audio recording from clinical investigations.

The reliable performance of such detectors offers today's medical practitioner a new "offline" tool for revealing hitherto undetectable microembolisms. However, detection of these "inaudible" microembolisms raises the question of the validity of the current gold standard.

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Chapter 1

Introduction to Multiresolution Analysis

1.1. Introduction

The concept of wavelet analysis can be introduced in two ways: one is the continuous wavelet transform and the other is multiresolution analysis (MRA).

Continuous wavelet transform is calculated on the basis of the scale factor s and the time position u in the set of real numbers (the time-scale plane is therefore continuously traversed), which renders it extremely redundant. In the reconstruction of a signal by continuous inverse transform, this redundancy is extreme in the sense that all the expanded and translated wavelets are used such that they are linearly dependent, therefore reflecting existing signal information without adding new information.

To profit from a non-redundant signal representation while ensuring a perfect reconstruction from its decomposition, an extremely effective tool, i.e. MRA, was defined by Mallat [MAL 99] and Meyer [MEY 90].

Wavelet multiresolution analysis includes the analysis, processing and synthesis of mono- or multidimensional signals at different levels of resolution, by decomposing them into (orthonormal) "scaling" basis functions and (orthonormal) "wavelet" basis functions. MRA, therefore, provides a family of orthonormal wavelets, which reduces any redundancy to zero.

Chapter written by Abdeldjalil OUAHABI.

This powerful concept allows numerical implementation of wavelet decomposition; the definition of the discrete wavelet transform thus necessarily underpins that of the MRA. This chapter concerns wavelet-based MRA.

The introduction of wavelets is closely related to multiresolution. Grasping the concept of wavelets, is therefore, the key to understanding this type of analysis.

Arising from the insight of the geophysicist Jean Morlet of Elf Aquitaine (which became Total following a merger with TotalFina), and formalized jointly by the physicist Alex Grossmann [GRO 84] in 1983, wavelets constitute a powerful tool for signal and image processing for the simultaneous description at multiple scales and for the local properties of complex signals such as non-stationarities, irregularities and rapid transitions, as in image contours. From this perspective, they constitute a "mathematical zoom", providing accurate analysis and a remarkable synthesis of signals and images.

Wavelets[1] have also been extremely useful in the analysis of transients of different durations, in noise reduction (denoising), image compression (JPEG 2000 standard), pattern recognition, data mining and progressive data transmission, and in numerical analysis (solving partial differential equations), computer vision, etc.

The orthonormal wavelet basis was first discovered by mathematician Yves Meyer in 1986 [MEY 90]. At the end of the 1980s, physicist and mathematician Ingrid Daubechies [DAU 92] constructed a family of compactly supported orthonormal wavelet bases (Daubechies wavelets have a support of minimum size). In addition, due to the concept of MRA, algorithms for rapid analysis and reconstruction [MAL 99, MEY 90, BEY 91] based on this have been developed, their implementation requiring only a small number of operations: in the order of N operations for a signal of size N.

The concept of MRA, therefore, provides a framework for signal decomposition in the form of a series of approximations of decreasing resolution together with a set of details.

To illustrate this idea, take the case of an image constructed out of a succession of approximations; the details (see section 1.3) come to sharpen this image. Therefore, coarse vision becomes finer and more precise.

The effervescent enthusiasm for the world of wavelets and their application has existed for almost three decades; it prompted the publication of a number of

1 Strictly speaking, this concerns wavelet transforms rather than wavelets.

influential books and scientific journals on its theoretical foundations and development. In this chapter, the first objective is to introduce the "wavelet" tool via MRA to make it accessible to the widest possible range of users from the academic world (students, early career researchers or non-specialists in the field) as well as from the socioeconomic world. The second objective of the chapter is to provide the reader with the basics through which he/she can approach the concepts of the other chapters in the book. Therefore, it provides keys with which we can unlock the concepts that, due to their roots in applied mathematics, may at first seem obtuse to those who are involved in information processing.

This introductory chapter refers to a rich bibliography. We recommend that readers wishing to deepen their understanding of wavelets and their transforms refer to the primary references cited in the bibliography. We also thank our colleagues Stéphane Mallat and Gabriel Peyré for their kind permission to reproduce several figures found on the website htttp://www.ceremade.dauphine.fr/~peyre/wavelet-tours/. The simulations presented in this chapter can be reproduced using either Matlab® or Scilab the free multiplatform software for numerical calculation.

After briefly recounting the discovery of wavelets, this chapter focuses on the concept of MRA and the equivalence of orthonormal wavelet bases and filter banks, leading to the rapid, linearly complex algorithms used for the calculation of a wavelet transform.

Throughout the chapter, several comments are included to clarify, add necessary depth, draw the reader's attention to undiscussed topics or highlight areas of possible confusion. The multiresolution analysis and the criteria for choosing a wavelet provided underpin the chapter, highlighting the take-home message of the problems discussed (MRA and the criteria for choosing a wavelet). Finally, informative illustrations help the uninitiated reader to better conceptualize certain ideas, manifested in particular applications (biomedical engineering and communication) and frequently accompanied by the corresponding Matlab® code.

1.2. Wavelet transforms: an introductory review

1.2.1. *Brief history*

At the beginning of the 19th Century (between 1807 and 1822), Joseph Fourier discovered that all signals could be decomposed into a set of sine waves (often called waves, see Figure 1.1) of different frequencies. The idea of this decomposition is to weigh (by judicious choice of amplitudes) these sine waves and modify their phases by shifting them such that they become additive or subtractive.

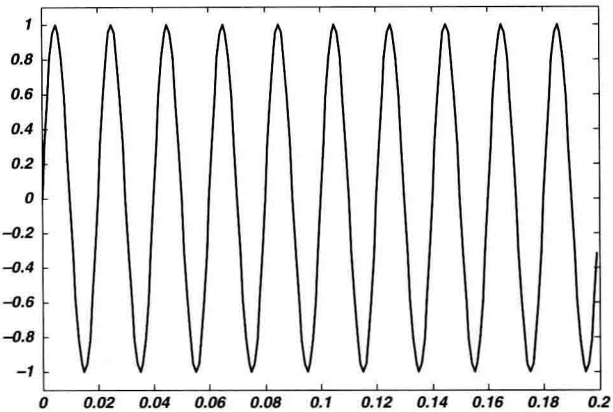

Figure 1.1. *Wave: sine wave Fourier basis*

Mathematical rigor came from Lejeune Dirichlet in 1829 (proof of theorem), and later it was consolidated by Camille Jordan in 1881 (extension to certain functions and convergence criteria).

In practice, Fourier analysis is equally as applicable to periodic signals, better known by the term "Fourier series", as to transients (signals that rapidly decrease to zero) known in this case as "Fourier transforms". Today, in the digital age and due to the fast Fourier transform (FFT), there is no operational difference between Fourier series and Fourier transforms.

The Fourier approach can also be seen as the decomposition of a signal into its frequency components or visualization (or representation) in the frequency or spectral domain of information initially described in the temporal domain. Such transformations are reversible and no information is lost in the conversion from one domain to the other.

These simple ideas provoked astonishment and even hostility from Fourier's contemporaries, such as Lagrange, Laplace, Monge and Lacroix.

To illustrate the "insufficiencies or limitations" of the Fourier concept, take the case of music: each note is literally "noted" on the score so that it can be read and interpreted by the musician. The shape of the note represents its duration, and its position on the staff determines its pitch or its (fundamental) frequency. Clearly, without information about duration (time) and frequency (as well as intensity of the note), it is impossible to perform a piece of music. The central critique of the Fourier approach, therefore, is that information is "hidden" in time. In effect, Fourier is informative about frequency contents (number, value and intensity of the frequencies) but remains silent with respect to the moment of transmission and the duration of each frequency. However, temporal information is not, in fact, lost: the original signal can be "reconstructed" using an inverse Fourier transform (addition of weighted sine waves).

From 1975 onward, Jean Morlet strived to find an alternative to Fourier analysis under the umbrella of petroleum prospecting via seismic reflection. Toward the end of the 1970s and the beginning of the 1980s, Jean Morlet discovered highly temporal localized waves, which he named wavelets (Figure 1.2). These wavelets serve as the mathematical microscope in adapting automatically to the various components of a given signal: they are compressed for the analysis of high-frequency transients, increasing the microscope's magnification to examine the finer details, and expanded for the analysis of long-term, low-frequency components. These wavelets are translated along the time axis in order to "scan" the entire signal.

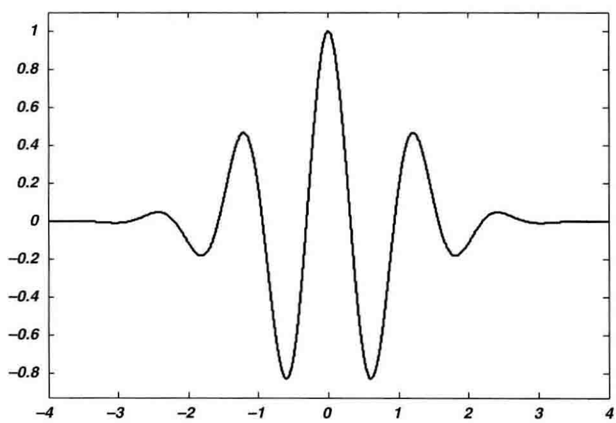

Figure 1.2. *Morlet wavelet: a perfectly localized finite energy function*

In 1984, the physicist Alex Grossman collaborated with Jean Morlet at the advent of the wavelet era, demonstrating the robustness of the wavelet transform and its property of conservation of energy. Shortly afterward, in 1986, the mathematician Yves Meyer added the finishing touches to this formidable saga, created through the insight of Jean Morlet an engineer at Elf Aquitaine, which gave him the recognition he deserves.

Some years later, Stéphane Mallat provided the information processing community with a "filter" approach in which the principal role is played by a scaling function, sometimes called the "father wavelet". The concept of MRA is