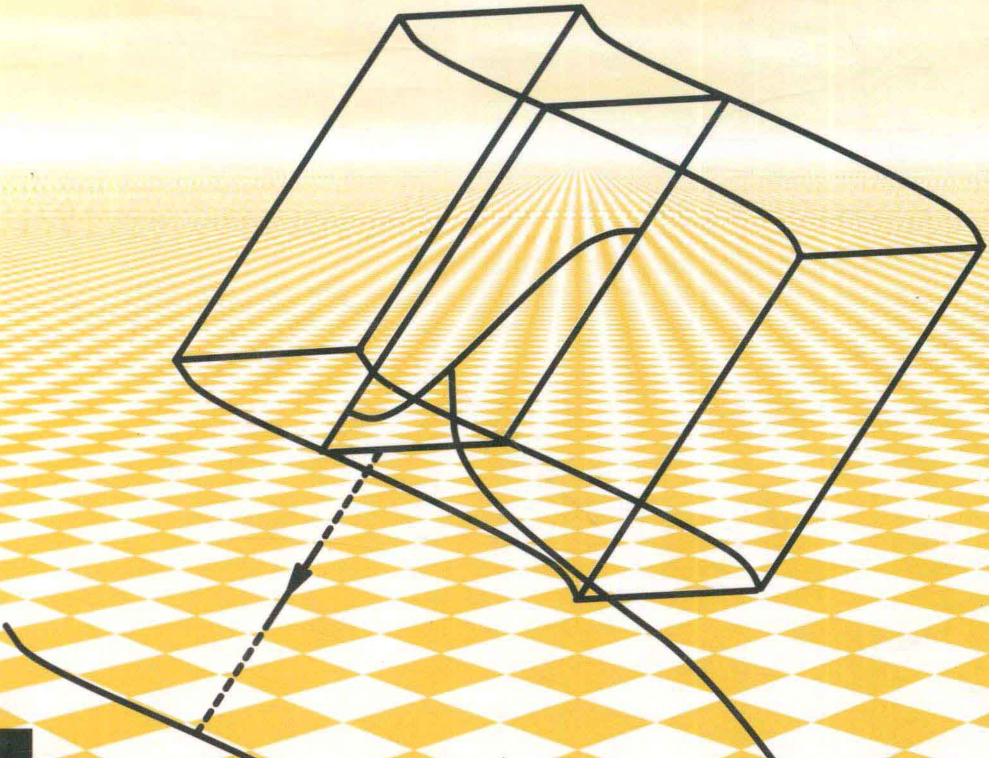
GEOMETRY, SPINORS AND APPLICATIONS

Donal J. Hurley
Michael A. Vandyck



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Preface

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze; but the book cannot be understood unless one first learns to comprehend the language, and read the letters, in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.
>
> Galileo [23]

This book arose from the investigation of a particular question in spinor theory, namely the problem of defining Lie and covariant differentiation of spinor fields. The problem has already been investigated in the literature, mostly in special cases, but some of the approaches adopted differ widely from one another; moreover, a number of results obtained by different authors are in contradiction with one another. It was therefore essential to resolve these contradictions and to present the question, in full generality, in a unified fashion. (Some recent developments by the authors may be found in [13], [41], [42], and the references cited therein.)

In the course of this study, it was found necessary to draw information from several areas of geometry and algebra, for instance from fibre bundles, Lie groups, Clifford algebras, representation theory, etc. Therefore it became clear that, if one attempted to present the theory in a rather self-contained fashion, one would be led to bring together and summarise a certain amount of material of interest to a wide community; furthermore, even the well-known topics, such as covariant differentiation of a tensor, would receive an unusual treatment, as a result of the specific question towards which they would be focused.

When this was realised, it required only a small step to decide to include selected applications to physics of the material presented in the theoretical part of the book. These applications illustrate not only spinorial calculus but also aspects of differential geometry. In this fashion, the authors hope that their work will be helpful both to students and to researchers: students will have at their disposal an account of some aspects of spinors, algebra, and geometry, with applications to physics, whereas researchers, who will be familiar with the standard topics, might concentrate on the specific problem of spinorial differentiation.

It is important to emphasise that our aim was not to present a *complete* account of *all* the aspects touched upon in this book. It would obviously be impossible to present in one volume the whole of differential geometry, fibre bundles, Lie groups, Clifford algebras, etc. Some important topics, such as the bundle theory of curvature and torsion, have been omitted completely. On the other hand, topics that are only briefly mentioned in the more usual treatment of the material, such as metric-incompatible connections, receive here a great deal of attention. This is a result of the emphasis that our specific question puts on the subject. For this reason, the reader is assumed to have some familiarity with differential geometry, so as to be able to see our considerations in a broader context.

It is a pleasure to acknowledge here the people from whom the authors have benefitted, directly or indirectly, in the writing of this book. We wish to express our deep gratitude to Professor D. Speiser for his incisive criticisms, comments, and remarks at various stages of the work, and for suggesting some of the notations adopted in the text. Throughout the years leading up to the final version of the book, we have been enlightened by discussions with Professors L. Ó Raifeartaigh, J. Lewis, P. Hogan, F. Hehl, and T. Laffey, and with Drs. J. D. McCrea, C.T. O'Sullivan, and F.A. Deeney.

We would also like to thank the students who have participated in the investigation, the preparation, and the collation of the material presented hereafter. They were, in chronological order, Messrs. P. Delaney, A. O'Connor, P. Campbell, and T. Philbin. Moreover, Mr. D. Flannery, through the interest that he took in the work, and the searching questions that he asked us, provided much of the encouragement and the stimulation that proved essential for the successful completion of our task.

We greatly appreciate the advice that we received from Dr. P. Cronin and Professor G. Huxley about Greek etymology. The new terms *euthikhode* and *brachisth-*

ode are the tangible fruits of our pleasant and interesting discussions.

For various aspects of computer-assisted typesetting, we are very indebted to Mr. P. Flynn and Dr. J. Morrisson, and Messrs. D. Ó Crualaoich, N. Madden, P. Twomey, J. O'Riordan, and J. Sheehan, as well as Miss K. Lally, with a special mention for Mr. P. Angove, who produced the figures. Dr. C.T. O'Sullivan, Professor M. Mansfield, and Mr. C. Horwood, of Praxis, are gratefully acknowledged for their guidance in the art of book publishing.

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