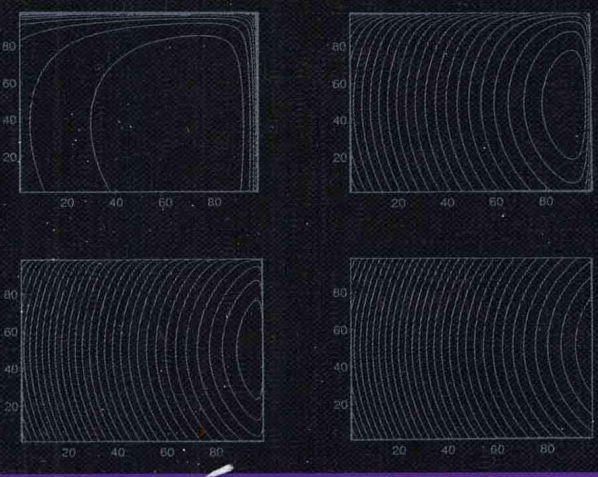
Numerical Methods in Finance

A MATLAB®-Based Introduction

Paolo Brandimarte



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This book is dedicated to Commander Straker, Lieutenant Ellis, and all SHADO operatives. Thirty years ago they introduced me to the art of using both computers and gut feelings in making decisions.

Preface

Crossroads are hardly, if ever, points of arrival; but neither are they points of departure. In some sense, crossroads may be disappointing, indeed. You are tired of driving, you are not at home yet, and by Murphy's law there is a far-from-negligible probability of taking the wrong turn. In this book, different paths cross, involving finance, numerical analysis, optimization theory, probability theory, Monte Carlo simulation, and partial differential equations. It is not a point of departure, because although the prerequisites are fairly low, some level of mathematical maturity on the part of the reader is assumed. It is not a point of arrival, as many relevant issues have been omitted, such as hedging exotic options and interest rate derivatives.

The book stems from lectures I give in a Master's course on numerical methods for finance, aimed at graduate students in economics, and in an optimization course aimed at students in industrial engineering. Hence, this is not a research monograph; it is a textbook for students. On the one hand, students in economics usually have little background in numerical methods and lack the ability to translate algorithmic concepts into a working program; on the other hand, students in engineering do not see the potential application of quantitative methods in finance clearly.

Although there is an increasing literature on high-level mathematics applied to financial engineering, and a few books illustrating how cookbook recipes may be applied to a wide variety of problems through use of a spreadsheet, I believe there is some need for an intermediate-level book, both interesting to practitioners and suitable for self-study. I believe that students should:

- Acquire *reasonably* strong foundations in order to appreciate the issues behind the application of numerical methods
- Be able to translate and check ideas quickly in a computational environment
- Gain confidence in their ability to apply methods, even by carrying out the apparently pointless task of using relatively sophisticated tools to pricing a vanilla European option
- Be encouraged to pursue further study by tackling more advanced subjects, from both practical and theoretical perspectives

The material covered in the book has been selected with these aims in mind. Of course, personal tastes are admittedly reflected, and this has something to do with my operations research background. I am afraid the book will not please statisticians, as no econometric model is developed; however, there is a wide and excellent literature on those topics, and I tried to come up with a complementary textbook.

The book is structured basically on three groups of chapters. Some of them are relatively independent, so that the instructor may choose the material to cover based on the time available and her preferences.

- Chapters 1 and 2 deal with finance and numerical analysis, respectively. The first chapter is aimed primarily at students of engineering; the second, at students of economics. The purpose of this first group of chapters is to lay down a homogeneous background for the heterogeneous groups of students at which the book is targeted.
- Chapters 3, 4, and 5 deal with the three main topics of the book: optimization theory, Monte Carlo simulation (including low-discrepancy sequences), and finite difference methods for partial differential equations.
- Finally, chapters 6, 7, and 8 give a few illustrative examples of the application of each of the three methodologies covered in the book.

The text is interspersed with MATLAB snapshots and pieces of code, to make the material as lively as possible and of immediate use. MATLAB is a flexible high-level computing environment which allows us to implement nontrivial algorithms with a few lines of code. It has also been chosen because of its increasing potential for specific financial applications.

It may be argued that the book is more successful at raising questions than at giving answers. This is a necessary evil given the space available to cover such a wide array of topics. Yet if after reading this book, students will want to read others, my job will have been accomplished. This was meant to be a crossroads, after all.

PS1. Despite all of my effort, the book is likely to contain some errors and typos. I will maintain a list of errata, which will be updated based on reader feedback. Any comment or suggestion on the book will also be appreciated. Please contact me at the following e-mail address: `brandimarte@polito.it`.

PS2. The list of errata will be posted on a Web page which will also include additional material and MATLAB programs. The current URL is

- `http://www.polito.it/~brandimarte`

An up-to-date link will be maintained on Wiley Web page:

- `http://www.wiley.com/mathematics`

PS3. And if (what a shame ...) you are wondering who Commander Straker is, take a look at the following Web sites:

- `http://www.ufoseries.com`
- `http://www.isoshado.org`

P. Brandimarte

Contents

Part I

Background

1

Financial problems and numerical methods

The purpose of this chapter is to outline some basic financial problems that can be tackled by numerical methods, to provide the reader with some background and motivation for the rest of the book. Some of these problems are rather easy to solve, at least in their basic form, by standard numerical methods; indeed, there are different software packages, including (but not limited to) MATLAB,[1] that offer built-in functionalities to cope with the basic versions of portfolio optimization problems and option pricing. However, when the limitations of such basic problem formulations are recognized, the need for the more sophisticated methods described in the rest of the book is apparent. Besides motivating the use of numerical methods such as mathematical programming (including mixed-integer and stochastic programming), Monte Carlo and quasi-Monte Carlo simulation methods, and finite differences to solve partial differential equations, we have to get familiar with MATLAB. The basic use of the MATLAB interactive environment and its syntax as a high-level programming language are described in appendix A, to which the reader is referred. Some background on probability and statistics is given in appendix B. This chapter is also meant as a reference to those unfamiliar with financial problems. In fact, there are (at least) two different potential users of this book:

- Students in finance wishing an introduction to numerical methods

[1]MATLAB is a registered trademark of The MathWorks, Inc.

- Students in a quantitative discipline (such as mathematics, engineering, or computer science) who would like to see how their skills may come handy when dealing with finance

It is a bit difficult to write a book suitable to both these groups without boring one and/or losing the other one in the first chapter. Here we have tried to come up with a compromise by structuring the chapter around the core functionalities of the Financial toolbox of MATLAB, i.e., the analysis of simple fixed-income securities (section 1.2), mean-variance portfolio theory (section 1.3), and the pricing of vanilla options (section 1.4). We also introduce the value-at-risk (VaR) concept (section 1.5).

The treatment will be rather brief, and it is certainly not meant as a substitute for a good book on finance (see the references at the end of the chapter), but it will be complemented by short MATLAB snapshots to make it immediately useful. A last caveat is that we deal here with concepts such as portfolio immunization, mean-variance efficiency, and VaR, which do have many well-known limitations and have been the subject of quite a bit of controversy. The point here is not to suggest that they should be used as they are stated, but just to pave the way for further developments.

1.1 MATLAB ENVIRONMENT

MATLAB is an interactive computing environment, providing both basic and sophisticated functions to cope with numerical problems. You may use built-in functions to solve possibly complex but standard problems, or devise your own programs by writing them as M-files, i.e., as text files including sequences of instructions written in a high-level matrix-oriented language. MATLAB also has a rich set of graphical capabilities, which we will use in a very limited fashion. Refer to appendix A for a quick tour of MATLAB programming.

Some classical numerical problems are readily solved by MATLAB built-in functions. They include:

- Solving systems of linear equations
- Solving nonlinear equations in a single unknown variable (including polynomial equations as a special case)
- Finding minima and maxima of functions of a single variable
- Approximating and interpolating functions
- Computing integrals
- Solving ordinary differential equations

Some background of the underlying methods is given in chapter 2. Although there is no need for a deep knowledge of numerical analysis in order to use