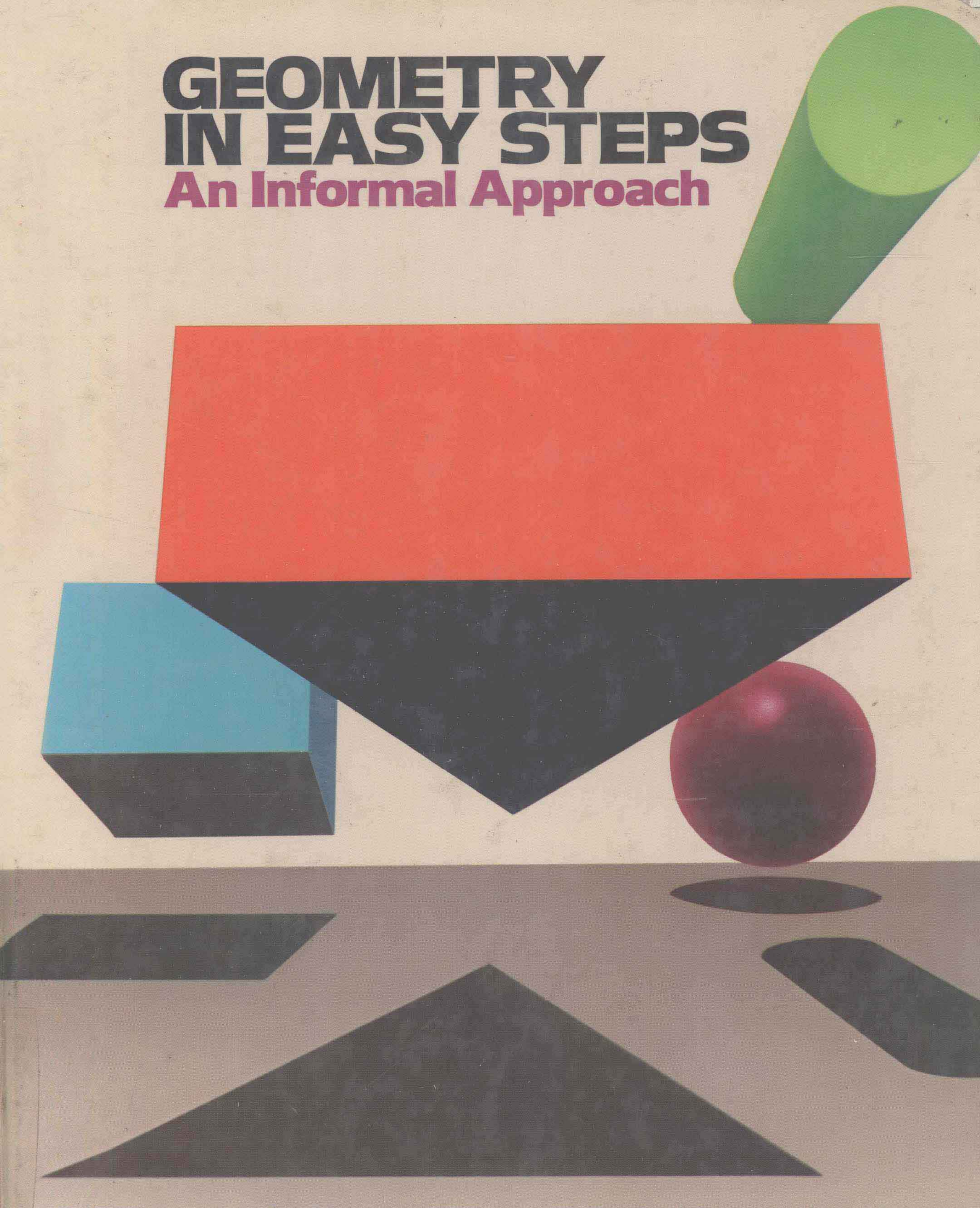
GEOMETRY IN EASY STEPS

An Informal Approach



GEOMETRY IN EASY STEPS

An Informal Approach

Philip L. Cox

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PREFACE

Geometry comes from two Greek words meaning "measurement of the earth." There is nothing physical that does not have shape and size. Geometry then is a mathematical study of the shapes and sizes of figures. It is made the more attractive by attention to its presence in the things around us and its application in practical ways to the solution of human problems.

Geometry in Easy Steps uses an informal approach to the study of shapes and sizes and its applications. The student begins with given facts and/or observations and induces step-wise certain and useful conclusions. In this way, the student will not only learn all the geometry content covered in most high school courses but in the process learn as well to think clearly and logically. However, because the course is based on an informal approach, it will be within the grasp of any interested student.

Some of the features which make the text more meaningful and enjoyable include the following:

Everyday Geometry—consumer applications of geometry to the needs of everyday life.

Applications—historical and/or technical uses of geometry in the service of humankind.

Geometry on the Job—people in careers that make use of geometry.

Did You Know That . . .—interesting, little known facts about geometry and related mathematics.

To assist further the learning process, the text contains the following aids:

Arithmetic (or *Algebra*) *Review*—reteaching and practice of basic skills utilized in later lessons.

Test Yourself—half-page quizzes occurring after every 3 or 4 lessons.

Chapter Review—Fully-referenced three-part review consisting of Vocabulary, Skills Checklist, and Exercises.

Cumulative Review—comprehensive semester reviews.

End-of-text material—useful tables (squares and square roots, trigonometric functions, and symbols), a fully-referenced listing of constructions and major conclusions, an illustrated glossary of terms, and answers to Class Practice and Test Yourself.

Acknowledgments

The author wishes to acknowledge and thank those teachers who participated in the research for *Geometry in Easy Steps* by teaching the prototype version and providing their comments and suggestions based thereon. Special thanks are extended to the principals and supervisors of the participating schools for authorizing the use of their classes in the field-testing program.

Other persons who deserve special thanks include: Don Junak and Gary Lundquist of the L'Anse Public Schools, Mt. Clemens, Michigan, who sparked the author's interest in an informal geometry course; Gerald Wallace, principal of Walled Lake Central High School, for his support; staff members of Oakland Schools, Pontiac, Michigan; Art Coxford, Professor of Mathematics Education, University of Michigan, for his patient guidance; and Sylvia Gelb, the book's editor, for her competent editing and cooperative spirit.

Most of all, I wish to thank my wife Judy for her continual assistance and my daughter Andrea for her understanding. Without their support and encouragement, this text could not have been completed.

Philip L. Cox

CONTENTS

Chapter 1

BUILDING A FIRM FOUNDATION

Points, Lines, and Planes; Polygons; and Prisms

1-1
Points, Lines, and Planes

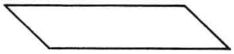
In this book you will learn about geometric figures on a flat surface. These properties are useful even though Earth's surface is curved. Any small part of Earth's surface is almost flat.

Geometric figures are sets or collections of points.

Three basic geometric figures are described and pictured below.

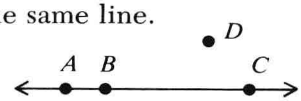
Point •
has no size.
indicates a definite location.

Line 
is straight.
has no thickness.
extends indefinitely in two opposite directions.

Plane 
is a flat surface.
has no thickness.
extends indefinitely in all directions.

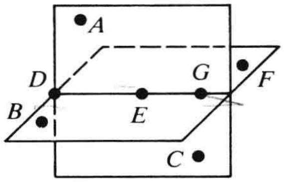
The two terms defined next will be useful in describing other geometric figures and their properties.

Collinear points are points that are on the same line.



Points A, B, and C are collinear.
Points A, B, and D are noncollinear.
Points D and C are collinear. *If a line can be drawn through some points, they are collinear even though the line is not shown.*

Coplanar points are points that are in the same plane.



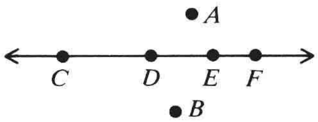
In the drawing you see two planes that *intersect*, or meet.
Points B, D, and E are coplanar.
Points A, B, D, and E are noncoplanar.
Points D, E, and G are coplanar. These three points are contained in both planes.
Points A, C, and F are coplanar. *If a plane can be drawn through some points, they are coplanar even though the plane is not shown.*

DISCUSS

1. Is there a real object that has the properties of a point? of a line? of a plane?
2. Look at the three drawings at the beginning of this lesson. In what way is the drawing of a point different from a point? the drawing of a line different from a line? the drawing of a plane different from a plane?
3. Name some objects that represent a point, a line, and a plane.

CLASS PRACTICE

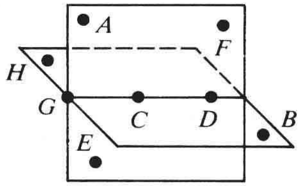
State whether each set of points is collinear.



1. *D, E, F*
2. *C, B, D*
3. *D, C, F*
4. *A, C, B*

For Exercises 5–13, look at the drawing of two intersecting planes.

Identify the labeled points contained in the indicated plane(s).

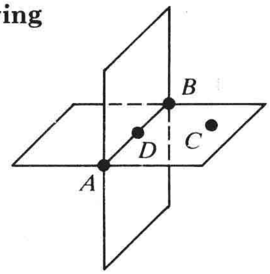


5. Horizontal plane
6. Vertical plane
7. Both planes

State whether each set of points is coplanar.

8. *B, E, F*
9. *A, C, F*
10. *A, H, F, B*
11. *A, E, F, D*
12. *B, C, E, D*
13. *H, C, D*

Look at the drawing of two intersecting planes and answer the following questions for each set of points.



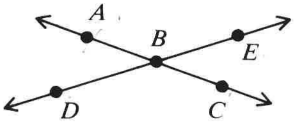
a. Are the points coplanar?
b. How many planes shown contain all the points?
c. How many other planes can be drawn that contain all the points?

14. *A, B*
15. *A, B, C*
16. *A, B, D*

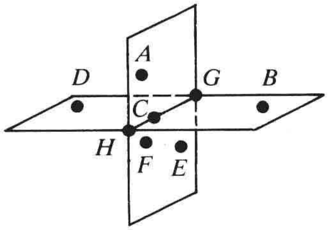
EXERCISES

A

1. Look at the drawing of two intersecting lines. State whether each set of points is collinear.

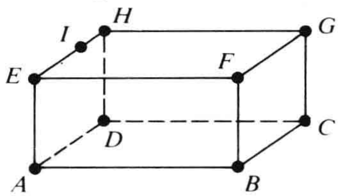


   a. *A, B* b. *A, E*
   c. *D, E* d. *D, B, E*
   e. *A, B, C* f. *A, D, B*
   g. *E, C* h. *D, C*
2. Can two points be noncollinear?
3. Can three points be collinear? noncollinear?
4. Can two different lines be drawn through two different points?
5. Look at the drawing of two intersecting planes. State whether each set of points is coplanar.



   a. *A, F, E* b. *A, C, G*
   c. *H, C, G* d. *B, F, E*
   e. *A, G, H, B* f. *A, C, G, E*

6. Six planes are shown in the drawing of a box. How many of the six planes contain all points of each set listed?



a. A, B	b. A, B, C
c. A, B, D	d. A, B, C, D
e. A, B, C, G	f. E, A
g. E, A, F	h. E, A, B
i. $E, A, B, F,$	j. E, A, B, C
k. E, I, H, G	l. E, I, H, G, F

Classify each statement as sometimes true, always true, or never true.

7. Two distinct points are collinear.
8. Three distinct points are collinear.
9. Two distinct points are coplanar.
10. Three distinct points are coplanar.
11. Four distinct points are coplanar.

— B —

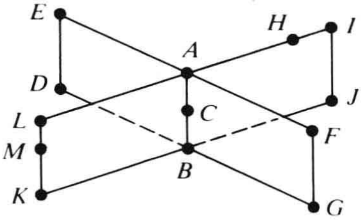
Draw a figure to fit each description, *if possible*.

12. Two points that are collinear
13. Two points that are noncollinear
14. Three points that are collinear
15. Three points that are noncollinear
16. Four points that are collinear
17. Three points that are coplanar
18. Three points that are not coplanar
19. Three points that are coplanar and are contained in exactly one plane

— C —

20. Look at the picture of two intersecting planes. Which of these four phrases best describes each set of points?

collinear *coplanar*
neither collinear nor coplanar
both collinear and coplanar

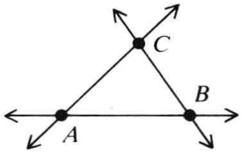


a. A, C, E	b. A, C, B
c. A, H, I	d. K, A, C
e. M, H, E, F	f. M, H, I, J
g. D, C	h. G, B, F
i. G, B, C, A, F	j. E, L, M, K

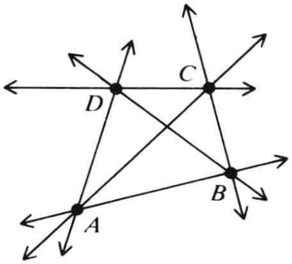
21. a. Can collinear points be noncoplanar? Why or why not?
    b. Can coplanar points be noncollinear? Why or why not?

22. Use one or more of these words—*collinear*, *coplanar*, *noncollinear*, or *noncoplanar*—to complete each statement.
    a. Any two points are ? .
    b. Any three points are ? .
    c. Three ? points lie in many planes.
    d. Three ? points lie in exactly one plane.

23. Two points determine one line. That is, exactly one line can be drawn containing both points. The figures show the number of lines determined by three noncollinear points and by four points, no three of which are collinear.



3 noncollinear points determine 3 lines. ($\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{AC}$)



4 points, no 3 of which are collinear, determine 6 lines. ($\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{DA}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BD}$)

a. Draw 5 points, no 3 of which are collinear. Draw all lines determined by the points. How many are there?

b. Repeat step a for 6 points, no 3 of which are collinear.

c. Predict how many lines are determined by 7 points, no 3 of which are collinear. Check your prediction.

24. Do an activity like that in Exercise 23 but remove the restriction that no three points are collinear. Find all possibilities for the number of lines determined by each given number of points. (Hint: There are two different possibilities for 3 points.)

a. 3 points **b.** 4 points
c. 5 points **d.** 6 points
e. 7 points

Applications

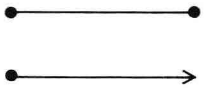
Stability in Design

As you know, when it is placed on an uneven or slanted surface, any four-legged table will wobble. To support something that must be kept absolutely steady, three legs are used for stability. The reason for this is as follows: Think of the end of each leg as a point. Any three points that are noncollinear lie in exactly one plane.

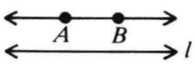
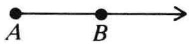
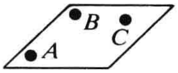
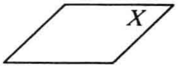
This property is used in designing many commonly used objects. Tripods are used by photographers to steady their camera. Surveyors also use a tripod to steady their transit. Some other objects whose design is based upon this property include a stand for a movie projection screen, a painter's easel, and a tricycle.

1-2
Segments and Rays

The drawings represent segments and rays.
A *segment* is part of a line with two endpoints.
A *ray* is part of a line with only one endpoint.
A ray extends indefinitely in one direction.



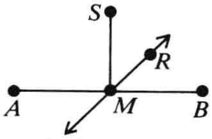
Special symbols are used to represent geometric figures.

Geometric Figure	Identified by:	Drawing	Symbol
Point	a capital letter		A
Line	any two points on the line or a lowercase letter		$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ l
Segment	its endpoints		$\overline{AB}$ or $\overline{BA}$
Ray	its endpoint and any other point on the ray (The symbol for the endpoint is given first.)		$\overrightarrow{AB}$
Plane	three noncollinear points or a capital letter (which does not represent a point)	 	plane ABC plane X

The *midpoint of a segment* is the point that divides the segment into two segments of equal length. A segment has exactly one midpoint.

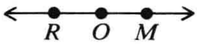
M is the midpoint of $\overline{AB}$.

A *bisector of a segment* is a line, segment, or ray that passes through the midpoint of the segment. A segment has many bisectors.



$\overline{SM}$, $\overrightarrow{MR}$, and $\overleftrightarrow{RM}$ are bisectors of $\overline{AB}$.

Opposite rays are two rays that have the same endpoint and form a line.



$\overrightarrow{OM}$ and $\overrightarrow{OR}$ are opposite rays.

DISCUSS

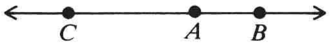
1. Is there a real object that has the properties of a segment? of a ray?
2. Name some objects that represent a segment and a ray.

CLASS PRACTICE

1. $\overrightarrow{XY}$ is a correct symbol for the ray shown. Why is $\overrightarrow{YX}$ an incorrect symbol for this ray?

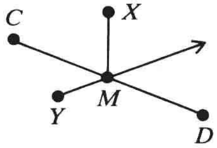
Look at line $\overleftrightarrow{CB}$.

2. $\overrightarrow{CA}$ and $\overrightarrow{BC}$ form a line. Are they opposite rays? Why or why not?



Look at the figure. *M* is the midpoint of $\overline{CD}$.

3. Identify two segments that have the same length.
4. Identify three different bisectors of $\overline{CD}$.
5. How many midpoints does a segment have? How many bisectors?



Symbols often suggest properties of geometric figures.

6. How many points are needed to determine a segment? a ray? a line?
7. In what ways do the symbols for segment, ray, and line suggest properties of each figure?
8. How many points are needed to determine a plane?
9. Will any three points always determine exactly one plane?

EXERCISES

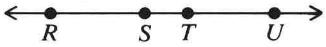
A

1. Look at this line:

   a. Are points *A*, *B*, and *C* collinear?
   b. Write six different correct symbols for the line.

2. Name all segments shown in the drawing. (There are three.)

3. Look at this line:

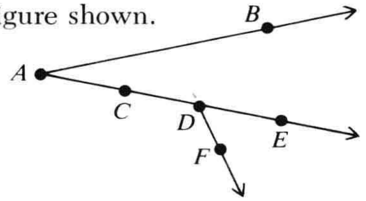


   a. Name the endpoints of $\overline{ST}$.
   b. Name three different segments with an endpoint of *S*.

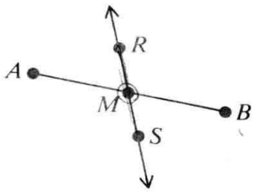
4. Look at this line:

   a. Does $\overrightarrow{AR}$ pass through *B*?
   b. Is *A* on $\overrightarrow{RB}$? on $\overrightarrow{BR}$?
   c. Write a symbol for the ray through *A* with endpoint *R*.

5. Identify five different rays in the figure shown.



Look at the intersecting line and segment. M is the midpoint of $\overline{AB}$. Classify each statement as true or false.



6. R and A are collinear.
7. R, A, and M are collinear.
8. R, A, and M are coplanar.
9. $\overline{RM}$ is a bisector of $\overline{AB}$.
10. $\overleftrightarrow{RS}$ is a bisector of $\overline{AB}$.
11. $\overrightarrow{SR}$ is a bisector of $\overline{AB}$.
12. $\overline{AM}$ and $\overline{MB}$ are the same length.
13. $\overrightarrow{RM}$ and $\overrightarrow{RS}$ are the same ray.
14. $\overrightarrow{MR}$ and $\overrightarrow{MS}$ are opposite rays.
15. $\overrightarrow{RS}$ and $\overrightarrow{SR}$ are opposite rays.

Classify each statement as true or false. (A, B, and C are different points.)

16. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BA}$ are the same line.
17. $\overline{AB}$ and $\overline{BA}$ are the same segment.
18. $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are the same ray.
19. Two rays with the same endpoint must be opposite rays.
20. Points A, B, and C are located in only one plane.
21. Each segment has exactly one midpoint.
22. Each segment has exactly one bisector.

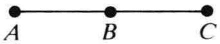
B

Draw and label a figure to fit each description.

23. $\overleftrightarrow{HI}$
24. $\overline{HI}$
25. $\overrightarrow{HI}$
26. $\overrightarrow{IH}$
27. $\overline{IH}$
28. $\overleftrightarrow{RT}$ intersecting $\overleftrightarrow{RS}$ (Identify the point of intersection.)
29. Opposite rays $\overrightarrow{OR}$ and $\overrightarrow{OT}$ (Write a symbol for the figure formed.)
30. Two rays that are not opposite rays but have the same endpoint (What kind of figure is formed?)
31. Point C on both $\overleftrightarrow{AB}$ and $\overrightarrow{AB}$
32. Point C on $\overleftrightarrow{AB}$ but not on $\overrightarrow{AB}$
33. Two lines bisecting $\overline{AB}$
34. $\overline{AB}$ and $\overline{CD}$ bisecting each other

C

35. Two points determine one segment. The figures show the number of segments determined by 3 collinear points and by 4 collinear points. (Some segments overlap.)



($\overline{AB}$, $\overline{BC}$, $\overline{AC}$)

3 collinear points determine 3 segments.

($\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AC}$, $\overline{AD}$, $\overline{BD}$)

4 collinear points determine 6 segments.

a. Draw 5 collinear points. Name all segments determined by the points. How many are there?

b. Repeat step a for 6 collinear points.

c. Predict how many segments are determined by 7 collinear points. Check your prediction.

d. State how the results of this activity compare with the results of Exercise 23 on page 4. Explain why this is true.