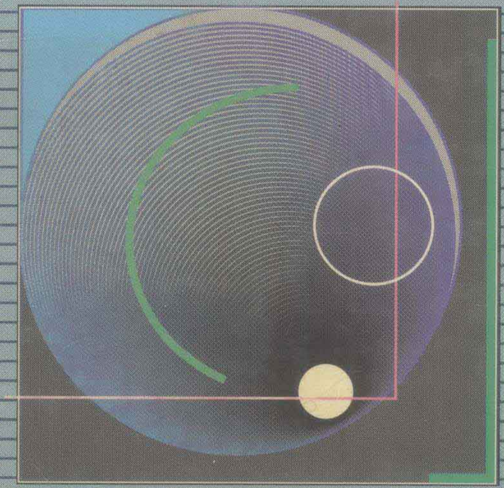
CALCULUS

One Variable

6th Edition



SALAS / HILLE

S. L. SALAS   EINAR HILLE

CALCULUS

One Variable

6th Edition

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In fond remembrance of
Einar Hille

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Sir Isaac Newton

PREFACE

Over the years we have heard a continuing murmur of criticism: SALAS/HILLE does not have enough contact with science and engineering, not enough physical applications. We have finally addressed the problem.

In this edition you will find simple physical applications scattered throughout the text and here and there, listed as optional, some applications that are not so simple. Perhaps some of these may pique the interest of the more serious student.

Notwithstanding the increased presence of applications, the book remains a text on mathematics, not science or engineering. The subject is calculus and the emphasis is on three basic ideas: limit, derivative, integral. All else is secondary; all else can be omitted.

S. L. SALAS

ACKNOWLEDGMENTS

Many have contributed to this edition. First of all I want to thank my collaborator (and now good friend) John D. Dollard of the University of Texas. Many of the improvements in the last six chapters (13–18) originated with him. My guru for physical applications was Richard W. Lindquist of Wesleyan University. Our head-to-head meetings were always productive and always fun. The computer programs that appear in the text are the generous contribution of Colin C. Graham of Northwestern University.

Good ideas came from old friends. Edwin Hewitt suggested an elegant little argument relating to simple harmonic motion. [Proof of (7.8.2)] W. W. Comfort called to my attention a remarkable property of the sphere of which I had been totally unaware. (Exercise 25, p. 559) James D. Reid simplified a proof that had stood for several editions. (Theorem 7.2.2)

We learn from our critics. The following reviewers (their criticisms were not always gentle) guided me through this edition.

David Ellis	*San Francisco State University*
William P. Francis	*Michigan Technological University*
Lew Friedland	*SUNY at Geneseo*
Colin C. Graham	*Northwestern University*
Ed Huffman	*Southwest Missouri State University*
John Klippert	*James Madison University*
Michael McAsey	*Bradley University*
Hiram Paley	*University of Illinois, Urbana*
Dennis Roseman	*University of Iowa*
Stephen J. Willson	*Iowa State University*

Each edition has developed from the previous one. The present book owes much to those who reviewed previous editions and encouraged me to further effort. I find a certain nostalgic pleasure in recalling their names and affiliation.

John T. Anderson	*Hamilton College*
Elizabeth Appelbaum	*University of Missouri, Kansas City*
Victor A. Belfi	*Texas Christian University*
W. W. Comfort	*Wesleyan University*
Louis J. Deluca	*University of Connecticut*
Garret J. Etgen	*University of Houston*
Eugene B. Fabes	*University of Minnesota*
Fred Gass	*Miami University (Ohio)*
Adam J. Hulin	*Louisiana State University, New Orleans*
Max A. Jodeit, Jr.	*University of Minnesota*
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Harvey B. Keynes	*University of Minnesota*
Jerry P. King	*Lehigh University*
Clifford Kottman	*Simpson College*
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John W. Lee	*Oregon State University*
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Giles Maloof	*Boise State University*
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Carl David Minda	*University of Cincinnati*
C. Stanley Ogilvy	*Hamilton College*
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Mark A. Pinsky	*Northwestern University*
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Ted Scheick	*Ohio State University*
Thomas Schwartzbauer	*Ohio State University*
Donald R. Sherbert	*University of Illinois, Urbana*
George Springer	*University of Indiana*
Robert D. Stalley	*Oregon State University*
Norton Starr	*Amherst College*
James White	*University of California, Los Angeles*
Donald R. Wilken	*SUNY, Albany*

In particular I want to acknowledge my debt to John T. Anderson (who collaborated with me on the fifth edition), to Harvey B. Keynes (who made major contributions to the fourth edition), to Donald R. Sherbert (whose reviews sustained me over many years), and to George Springer (who gave me the encouragement I needed at the very beginning some twenty years ago).

My thanks to Edward A. Burke. He designed the book, he designed the cover, and he supervised production. Valerie Hunter, Mathematics editor at Wiley, kept the process going. It was a pleasure to work with her and I look forward to working with her again.

SPECIAL ACKNOWLEDGMENT

For years now my son Charles G. Salas has collaborated with me on all my projects. Everything I write passes through his hands. He criticizes, objects, and suggests improvements. Usually I agree immediately. Sometimes I don't and we fight it out. More often than not I find that he is right and the book is improved. Once again, thank you, Charlie.

A Note from the Publisher: Answers/solutions to all the odd-numbered exercises are available in a *Student's Solution Manual.*

THE CHANGES

CHAPTERS 1–12: ONE VARIABLE

• Greater variety of exercises on limits, including calculator exercises. • More attention to numerical approximations. Calculator exercises and, thanks to Professor Colin Graham of Northwestern University, some computer programs written in BASIC. (These programs serve to illustrate certain procedures explained in the text and may prove helpful to some students. All the computer programs are *optional.* Access to a computer is not necessary for an understanding of the text.) • Early consideration of infinite limits and limits as $x \to \pm\infty$. • Fine-tuning of sections on maxima-minima and simpler prescriptions for graphing. • More emphasis on motion with constant acceleration. Conservation of energy during free-fall. • Introduction to angular velocity and uniform circular motion. • Early explanation of the role of symmetry in integration (odd functions, even functions). • Weighted averages; the mass of a rod, center of mass. • A new section on the centroid of a region culminates in Pappus's theorem on volumes. • New material on gravitational attraction. *(optional)* • Strong emphasis on exponential growth and decline. • The section on simple harmonic motion has been revised to give the student more insight into oscillatory phenomena. • The chapter on differential equations has been discontinued. Those parts most relevant to elementary calculus have been rewritten and distributed in the appropriate chapters. • Detailed examination of the conic sections in polar coordinates. (*optional*; necessary only for a later section on planetary motion, which too is *optional*) • The centroid of a curve and Pappus's theorem on surface area. (The centroid of a solid of revolution is introduced in the exercises.) • A brief explanation of the gravitational force exerted by a spherical shell. *(optional)* • The inverted cycloid as the tautochrone and the brachystochrone. *(optional)*

CHAPTERS 13–18: SEVERAL VARIABLES

• Although vectors are still introduced as ordered triples of real numbers, we have placed increased emphasis on working with vectors and vector functions in a component-free manner. • Earlier introduction to the cross product. The cross product is now defined geometrically, and its components are derived from that definition. • The chapter on vectors contains a brief introduction to matrices and determinants (2 by 2 and 3 by 3 only). • An elementary discussion of curvilinear motion from a vector viewpoint (followed by some rudimentary vector mechanics) culminates in an *optional* section on Kepler's three laws of planetary motion. • Two intermediate-value theorems that prove useful in later chapters. • Students are invited to derive Snell's law of refraction from Fermat's principle of least time. • Early exploitation of symmetry in multiple integration. • Moments of inertia are introduced by examining the rotation of a rigid body. Frequent use of the parallel axis theorem. • All the material on changing variables in multiple integration has been totally rewritten. A more unified treatment ends with Jacobians and the general theorem. • A revised introduction to line integrals precedes an *optional* section on the work–energy formula and the conservation of mechanical energy. • More attention is given to line integrals with respect to arc length (mass of a wire, center of mass, moments of inertia). • Green's theorem for Jordan regions is followed by Green's theorem for regions bounded by two or more Jordan curves. • Surfaces are introduced in parametrized form. Care is taken to help the student understand various ways of parametrizing the more common surfaces. • Paramount in our discussion of surfaces is the fundamental vector product. • Surface area and surface integrals are defined initially for parametrized surfaces. [Surfaces of the form $z = f(x, y)$ then appear as a special case.] • Surface integrals are used first to calculate the mass, center of mass, and moments of inertia of a material surface. Later we focus on two-sided surfaces and the notion of flux. • The basic differential operators are presented in terms of the operator ∇. The divergence and curl of a vector field $\mathbf{v}$ are introduced as $\nabla \cdot \mathbf{v}$ and $\nabla \times \mathbf{v}$. The Laplacean is written ∇^2. • The divergence theorem first stated for a solid bounded by a single surface is extended to solids bounded by two or more surfaces. (The idea is then applied to find the flux of the electric field that surrounds a point charge.) • Stokes's theorem is made more intelligible by working first with polyhedral surfaces.

THE GREEK ALPHABET

Α	α	alpha
Β	β	beta
Γ	γ	gamma
Δ	δ	delta
Ε	ϵ	epsilon
Ζ	ζ	zeta
Η	η	eta
Θ	θ	theta
Ι	ι	iota
Κ	κ	kappa
Λ	λ	lambda
Μ	μ	mu
Ν	ν	nu
Ξ	ξ	xi
Ο	o	omicron
Π	π	pi
Ρ	ρ	rho
Σ	σ	sigma
Τ	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
Χ	χ	chi
Ψ	ψ	psi
Ω	ω	omega

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Gottfried Leibniz

CONTENTS

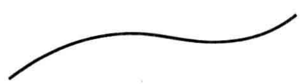
CHAPTER 1

INTRODUCTION

1.1 WHAT IS CALCULUS?

To a Roman in the days of the empire a "calculus" was a pebble used in counting and in gambling. Centuries later "calculare" came to mean "to compute," "to reckon," "to figure out." To the mathematician, physical scientist, and social scientist of today calculus is elementary mathematics (algebra, geometry, trigonometry) enhanced by *the limit process.*

Calculus takes ideas from elementary mathematics and extends them to a more general situation. Here are some examples. On the left-hand side you will find an idea from elementary mathematics; on the right, this same idea as extended by calculus.

Elementary Mathematics	*Calculus*
	
slope of a line $y = mx + b$	slope of a curve $y = f(x)$