PHYSICS TEXTBOOK

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WILEY-VCH

Econophysics

An Introduction

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Cover Design
Cover-Image © Kit Wai Chan (Fotolia.com)
Adam Design, Weinheim, Germany

Library of Congress Card No.: applied for

British Library Cataloguing-in-Publication Data:
A catalogue record for this book is available from the British Library.

Bibliographic information published by the Deutsche Nationalbibliothek
The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available on the Internet at http://dnb.d-nb.de.

Typesetting le-tex publishing services GmbH, Leipzig
Printing and Binding Strauss GmbH, Mörlenbach

Printed in the Federal Republic of Germany
Printed on acid-free paper

ISBN 978-3-527-40815-3

Preface

Physicists have long had a reputation of meddling in areas outside the restricted domain of physics. In some cases, the non-traditional approaches that physicists bring can result in new insights, and in due course, this new inter-disciplinary area of research can become a recognized part of science. Biophysics has been the most spectacular success in this mold; astrophysics and geophysics have also emerged in the last century from the forays of physicists into disciplines outside their own.

However, the use of physics techniques to understand economic phenomena, leading to the development of a new field dubbed "econophysics" (for better or worse), marks a new departure for physicists beyond their traditional domain. Economics, and for that matter, the social sciences in general, are often thought to be unamenable to the analytical tools used in the natural sciences, in particular, physics. As critics of the quantitative approach to socioeconomic issues are always ready to remind us: social sciences deal with issues related to human beings, whose behavior do not follow any reproducible patterns nor can they be reduced to equations. Thus, the early attempts of physicists to address economic questions have been invariably met with, at best, bemused curiosity, or worse, total indifference from the social science community.

However, the ongoing worldwide economic and financial crisis has rudely awakened everyone to the weak theoretical foundations on which the science of economics rests. It has also highlighted alternative approaches to understand social issues, especially econophysics. The past decade has seen an ever-growing number of physicists tackling problems arising in an economic or financial context, under the premise that while individual human behavior is not predictable, those of a large collection of economic agents are likely to follow discernible patterns. The increasing volume and quality of the contributions by econophysicists have now reached a critical level where they cannot be ignored either by mainstream economics or physics communities. We have reached the point where a beginner will find it difficult to figure out where to start learning about the field and even a specialist working in this area can no longer keep track of all the exciting developments that are happening in it.

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Thus, we felt that this is an appropriate time to write a book introducing the main currents of research in econophysics to a broad and non-specialist scientific audience. While the text will possibly be most useful for students and researchers in physics and mathematics interested in socioeconomic phenomena, we hope that economists and social scientists would also find it helpful as an introductory glimpse of econophysics. With this aim in view, we have kept assumptions of requisite background knowledge for reading the book at a minimum. The unusually extensive appendices introduce almost all the basic physics concepts that are necessary for a person not trained in physics to read this book. While it only serves as the first step towards the long journey of acquiring expertise in this rapidly developing inter-disciplinary area, we would be happy if our book serves to give the reader an overall idea of the present state of research in econophysics and hopefully inspire them to join in this effort to use the techniques of physics to understand how society works.

The choice of topics discussed in the book has necessarily been colored by our own areas of interest in econophysics. We do realize that, as econophysics is a rapidly growing field, we have not included many references and have not discussed the contributions of several scientists. We apologize that we could not give equal representation to all the areas of research in econophysics. We have tried to include important references to the literature whenever we have thought that the original material would be indispensable, and have sometimes omitted continued attribution and references which would have been very distracting otherwise.

The idea of writing a book introducing econophysics was suggested to us by our dynamic and enthusiastic editor, Anja Tschoertner of Wiley-VCH. Despite numerous delays and false starts on the way, her patience and support have helped to bring the project to a successful completion. This book is a collective effort among authors across two different continents and the principal responsibility for writing the various chapters were distributed as follows: S. Sinha for Chapters 1, 3, 4, 5, 6, 10 and 11, A. Chatterjee for Chapters 7 and 8, A. Chakraborti for Chapters 2 and 9 and B.K. Chakrabarti for Chapter 1 and the Appendices.

Several friends and collaborators have provided invaluable assistance. In particular, we would like to thank Soumyajyoti Biswas and Anulekha Dutta for transcribing and checking the lecture notes corresponding to the material included in the appendices, and Rajeev Singh for proofreading the entire manuscript and making numerous suggestions on how to improve it. We would also like to thank all our collaborators, including Frédéric Abergel, Pratip Bhattacharyya, Anindya Sundar Chakrabarti, Guido Germano, Asim Ghosh, Els Heinsalu, Antti Kanto, János Kertész, Kimmo Kaski, Subhrangshu Sekhar Manna, Sugata Marjit, Manipushpak Mitra, Jukka-Pekka Onnela, Raj Kumar Pan, Marco Patriarca, Srutarshi Pradhan, Srinivas Raghavendra, M.S. Santhanam, Nisheeth Srivastava, Robin Stinchcombe, Marko Sysi-Aho, Ioane Muni Toke and S.V. Vikram. We would also like to acknowledge helpful discussions with Ginestra Bianconi, Indrani Bose, Arnab Das, Deepak

Dhar, J. Doyne Farmer, Mauro Gallegati, Brian Hayes, Abhijit Kar-Gupta, Thomas Lux, Krishna Maddaly, Matteo Marsili, Pinaki Mitra, Pradeep K. Mohanty, Peter Richmond, Abhirup Sarkar, Parongama Sen, H. Eugene Stanley, Dietrich Stauffer, S. Subramanian, Yarlagadda Sudhakar and Victor M. Yakovenko among others.

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1 Introduction

> "Economic theorists, like French chefs in regard to food, have developed stylized models whose ingredients are limited by some unwritten rules. Just as traditional French cooking does not use seaweed or raw fish, so neoclassical models do not make assumptions derived from psychology, anthropology, or sociology. I disagree with any rules that limit the nature of the ingredients in economic models".
>
> – George A. Akerlof, *An Economic Theorist's Book of Tales* (1984)

Over the past couple of decades, a large number of physicists have started exploring problems which fall in the domain of economic science. The common themes that are addressed by the research of most of these groups have resulted in coining a new term "econophysics" as a collective name for this venture. Bringing together the techniques of statistical physics and nonlinear dynamics to study complex systems along with the ability to analyze large volumes of data with sophisticated statistical techniques, the discoveries made in this field have already attracted the attention of mainstream physicists and economists. While still somewhat controversial, it provides a promising alternative to, and a more empirically based foundation for the study of economic phenomena than, the mainstream axiom-based mathematical economic theory.

Physicists have long had a tradition of moving to other fields of scientific inquiry and have helped bring about paradigm shifts in the way research is carried out in those areas. Possibly the most well-known example in recent times is that of the birth of molecular biology in the 1950s and 1960s, when pioneers such as Schrödinger (through his book *What is Life?*) inspired physicists such as Max Delbruck and Francis Crick to move into biology with spectacularly successful results. However, one can argue that physicists are often successful in areas outside physics because of the broad-based general nature of a physicist's training, rather than the applicability of physical principles as such in those areas. The large influx of physicists since the late 1990s into topics which had traditionally been the domain of economists and sociologists have raised the question: does physics really contribute towards gaining significant insights into these areas? Or, is it a mere fad, driven by the availability of large quantities of economic data which are amenable to the kind of analytical techniques that physicists are familiar with?

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The coining of new terms such as econophysics and sociophysics (along the lines of biophysics and geophysics) have hinted that many physicists do believe that physics has a novel perspective to contribute to the traditional way of doing economics. Others, including the majority of mainstream economists, have been dismissive until very recently of the claim that physics can have something significant to contribute to the field. Physics is seen by them to be primarily a study of interactions between simple elements, while economics deals exclusively with rational agents, able to formulate complex strategies to maximize their individual utilities (or welfare).

However, even before the current worldwide crisis revealed the inadequacies of mainstream economic theory, economists had realized that this new approach of looking at economic problems cannot be simply ignored, as indicated, for example by the entry of the terms "econophysics" and "economy as a complex system" in the *New Palgrave Dictionary of Economics* (Macmillan, 2008). The failure of economists by and large to anticipate the collapse of markets worldwide in 2008 over a short space of time has now led to some voices from within the field of economics itself declaring that new foundations for the discipline are required. The economists Lux and Westerhoff in an article published in *Nature Physics* in 2009 [1] have suggested that econophysics may provide such an alternative theoretical framework for rebuilding economics. As Lux and other economists have pointed out elsewhere [2], the systemic failure of the standard model of economics arises from its implicit view that markets and economies are inherently stable. Similar sentiments have been expressed by the econophysicist Bouchaud in an essay in *Nature* published in the same year [3].

However, worldwide financial crises (and the accompanying economic turmoil) are neither new nor as infrequent as economists would like to believe. It is therefore surprising that mainstream economics have ignored, and sometimes actively suppressed, the study of crisis situations. The famous economist Kenneth Arrow even tried to establish the stability of economic equilibria as a mathematical theorem; however, what is often forgotten is that such conclusions are crucially dependent on the underlying simplifying assumptions, such as, perfectly competitive markets and the absence of any delays in response. It is obvious that the real world hardly conforms to such ideal conditions. Moreover, the study of a wide variety of complex systems, e.g., from cellular networks to the internet and ecosystems, over the past few decades using the tools of statistical physics and nonlinear dynamics has led to the understanding that inherent instabilities in dynamics often accompanies increasing complexity.

The obsession of mainstream economics with the ideal world of hyper-rational agents and almost perfect competitive markets has gone hand in hand with a formal divorce between theory and empirical observations. Indeed, the analysis of empirical data has ceased to be a part of economics, and has become a separate subject called econometrics. Since the 1950s, economics has modeled itself more on mathematics than any of the natural sciences. It has been reduced to the study of self-consistent theorems arising out of a set of axioms to such an extent that it is probably more appropriate to term mainstream economics as *economathematics*, that is mathematics inspired by economics and that too, having little connection to reality. This is strange for a subject that claims to have insights and remedies for one of the most important spheres of human activity. It is a sobering thought that decisions made by the IMF and World Bank which affect millions of lives are made on the basis of theoretical models that have never been subjected to empirical verification. In view of this, some scientists (including a few economists) have begun to think that maybe economics is too important to be left to economists alone. While a few have suggested that econophysics may provide an alternative theoretical framework for a new economic science, we think that the field as it stands is certainly an exciting development in this direction, and intend to give an introduction to it here.

Before describing in this book how physicists have brought fresh perspectives on understanding economic phenomena in recent times, let us point out here that despite the present divorce of economics from empirical observation, there has been a long and fruitful association between physics and economics. Philip Mirowski, in his book, *More Heat Than Light* [4] has pointed out that the pioneers of neoclassical economics had indeed borrowed almost term by term the physics of 1870s to set up their theoretical framework. This legacy can still be seen in the attention paid by economists to maximization principles (e.g., of utility) that mirrors the framing of classical physics in terms of minimization principles (e.g., the principle of least action). Later, Paul Samuelson, the second Nobel laureate in economics and the author of possibly the most influential textbook of economics, tried to reformulate economics as an empirically grounded science modeled on physics in his book *Foundations of Economic Analysis* (1947). While the use of classical dynamical concepts such as stability and equilibrium has also been used in the context of economics earlier (e.g., by Vilfredo Pareto), Samuelson's approach was marked by the assertion that economics should be concerned with "the derivation of operationally meaningful theorems", that is those which can be empirically tested. Such a theorem is "simply a hypothesis about empirical data which could conceivably be refuted, if only under ideal conditions". Given the spirit of those times, it is probably unsurprising that this is also when the engineer-turned-economist Bill Philips (who later became famous for the Philips curve, a relation between inflation and employment) constructed the Moniac, a hydraulic simulator for the national economy (Figure 1.1) that modeled the flow of money in society through the flow of colored water. The mapping of macroeconomic concepts to the movement of fluids was a direct demonstration that the economy was as much a subject of physical inquiry as other more traditional subjects in physics.

This was however the last time that physics would significantly affect economics until very recently, as the 1950s saw a complete shift in the focus of economists towards proving existence and uniqueness of equilibrium solutions in the spirit of mathematics. A parallel development was the rise of mathematical game theory, pioneered by John von Neumann. To mathematically inclined economists, the language of game theory seemed ideal for studying how selfish individuals constantly devise strategies to get the better of other individuals in their continuing endeavor to maximize individual utilities. The fact that this ideal world of paranoid, calculating hyper-rational agents could never be reproduced in actual experiments carried

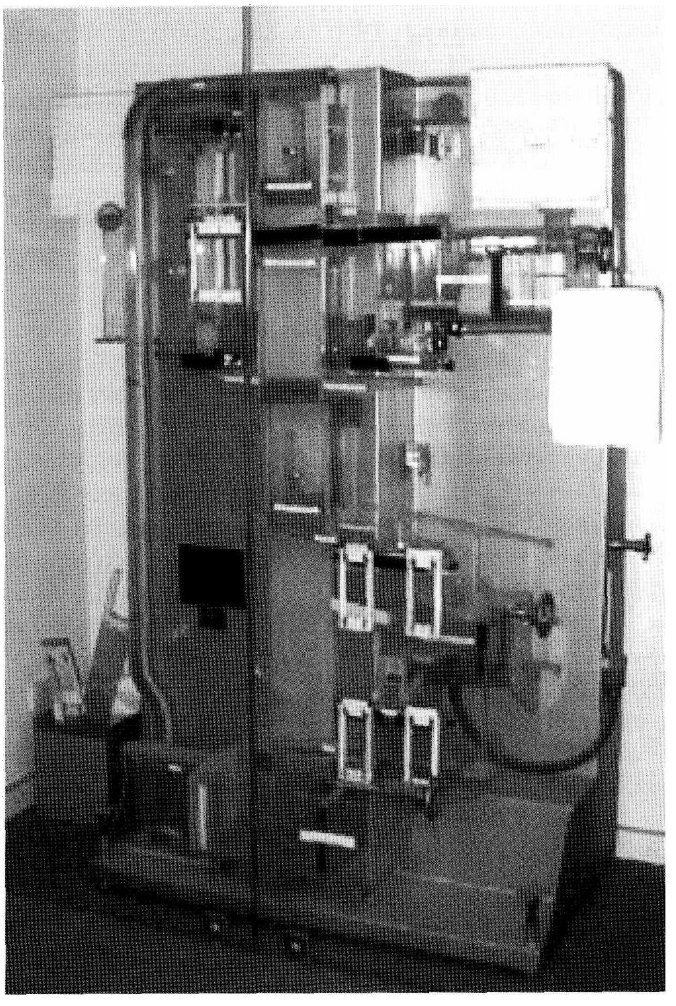

Figure 1.1 The economy machine. A reconstruction of the Moniac (at the University of Melbourne), a hydraulic simulator of a national economy built in 1949 by A.W.H. Phillips of the London School of Economics, that used the flow of colored water to represent the flow of money. It is currently again being used at Cambridge University for demonstrating the dynamic behavior of an economic system in economics first-year lectures. [Source: [5], Photo: Brett Holman]

out with human subjects where "irrational" cooperative action was seen to be the norm, could not counter the enthusiasm with which economists embraced the idea that society converges to an equilibrium where it is impossible to make someone better off without making someone else worse off. Further developments of rational models for interactions between economic agents became so mathematically abstract, that an economist recently commented that it seems (from an economic theorist's point of view) even the most trivial economic transaction is like a complicated chess game between Kenneth Arrow and Paul Samuelson (the two most famous American economists of the post-war period). The absurdity of such a situation is clear when we realize that people rarely solve complicated maximization equations in their head in order to buy groceries from the corner store. The concept of bounded rationality has recently been developed to take into account practical constraints (such as the computational effort required) that may prevent the system from reaching the optimal equilibrium even when it exists.

It is in the background of such increasing divergence between economic theory and reality that the present resumption of the interrupted dialogue between physics and economics took place in the late 1980s. The condensed matter physicist Philip Anderson jointly organized with Kenneth Arrow a meeting between physicists and economists at the Santa Fe Institute that resulted in several early attempts by physicists to apply the recently developed tools in nonequilibrium statistical mechanics and nonlinear dynamics to the economic arena (some examples can be seen in the proceedings of this meeting, *The Economy as an Evolving Complex System*, 1988) [6]. It also stimulated the entry of other physicists into this inter-disciplinary research area, which, along with slightly later developments in the statistical physics group of H. Eugene Stanley at Boston University, finally gave rise to econophysics as a distinct field, the term coined by Stanley in 1995, in Kolkata. Currently there are groups in physics departments around the world who are working on problems related to economics, ranging from Japan to Brazil, and from Ireland to Israel. While the problems they work on are diverse, ranging from questions about the nature of the distribution of price fluctuations in the stock market to models for explaining the observed economic inequality in society to issues connected with dynamical fluctuations of prices as a consequence of delays in the propagation of information, a common theme has been the observation and explanation for scaling relations (or power laws). Historically, scaling relations have fascinated physicists because of their connection to critical phenomena; but more generally, they indicate the presence of universal behavior. Indeed, the quest for invariant patterns that occur in many different contexts may be said to be the novel perspective that this recent incursion of physicists has brought to the field of economics, and that may well prove to be the most enduring legacy of econophysics.

1.1 A Brief History of Economics from the Physicist's Perspective

When physics started to develop, around the time of Galileo Galilei (1564–1642), there were hardly any fully matured fields in science from which to get help or inspiration. The only science that was somewhat advanced was mathematics, which is an analytical science (based on logic) and not empirical (based on observations/experiments carried out in controlled environments or laboratories). Yet, developments in mathematics, astronomical studies in particular, had a deep impact on the development of physics, of which the (classical) foundation was almost single-handedly laid down by Isaac Newton (1643–1727) in the seventeenth and early eighteenth century. Mathematics has remained at the core of physics since then. The rest of "main stream" sciences, like chemistry, biology, etc., have all tried to obtain inspiration from, utilize, and compare with physics since that time.

In contrast, development in the social sciences started much later. Even the earliest attempt to model an agricultural economy in a kingdom, the "physiocrats' model", named after the profession of its pioneer, the French royal physician Francois Quesnay (1694–1774), came only in the third quarter of the eighteenth century when physics was already put on firm ground by Newton. The physiocrats made the observation that an economy consists of the components like land and farmers, which are obvious. Additionally, they identified the other components as investment (in the form of seeds from previous savings) and protection (during harvest and collection, by the landlord or the king). The impact of the physical sciences in emphasizing these observations regarding components of an economy is clear. The analogy with human physiology then suggested that, like the healthy function of a body requiring proper performance of each of its components or organs, and the (blood) flow among them remaining uninterrupted, each component of the economy should be given proper care (suggesting rent for land and tax for protection!). Although the physiocrats' observations were appreciated later, the attempt to make conclusions based on the analogy with human physiology was not.

Soon, during their last phase, Mercantilists like Wilhelm von Hornick (1638–1712), James Stewart (1712–1780), and others, made some of the most profound and emphatic observations in economics, leading to the foundation of political economy. In particular, British merchants who traded in the colonies, including India, in their own set terms observed that instabilities arise as a result of growing unemployment in their home country. They also observed that whenever there is a net trade deficit and outflow of gold (export being less than import), this led to the formulation of the problem of effective demand: even though the merchants, or traders were independently trading (exporting or importing goods) with success, the country's economy as a whole did not do well due to lack of overall demand when there was a net flow of gold (the international exchange medium) to balance the trade deficit! This still remains as a major problem in macroeconomics. The only solution in those days was to introduce tax on imports: third party (in this case the government) intervention on the individual's choice of economic activity (trade). This immediately justified the involvement of the government in the economic activities of individuals.

In a somewhat isolated but powerful observation, Thomas Malthus (1766–1834) made a very precise model of the conflict between agricultural production and population growth. He assumed that the agricultural production can only grow (linearly) with the area of the cultivated land. With time t, in years, the area can only grow linearly ($\propto t$) or in arithmetic progression (AP). The consumption depends on the population which, on the other hand, grows exponentially ($\exp[t]$) or in geometric progression (GP). Hence, with time, or year $1, 2, 3, \ldots$, the agricultural production grows as $1, 2, 3, \ldots$, while the consumption demand or population grows in a series like $2, 4, 8, \ldots$. This means that it does not matter how large the area of cultivable land we start with, the population GP series soon overtakes the food production AP series and the population faces a disaster, resulting in famine, war or revolution.

They are inevitable, as an exponentially growing function will always win over a linearly growing function and such disasters will appear almost periodically in time.

Adam Smith (1723–1790) made the first attempt to formulate economic science. He painstakingly argued that a truly many-body system of selfish agents, each having no idea of benevolence or charity towards its fellow neighbors, or having no foresight (views very local in space and time), can indeed reach an equilibrium where the economy as a whole is most efficient; leading to the best acceptable price for each commodity. This "invisible hand" mechanism of the market to evolve towards the "most efficient" (beneficial to *all* participating agents) predates the demonstration of the "self-organization" mechanism in physics or chemistry of many-body systems, where each constituent cell or automata follows very local (in space and time) dynamical rules and yet the collective system evolves towards a globally "organized" pattern (cf. Ilya Prigogine (1917–), Per Bak (1947–2002) and others). This idea of "self-organizing" or "self-correcting economy" by Smith of course contradicted the prescription of the Mercantilists regarding government intervention in the economic activities of the individuals, and argued tampering by an external agency to be counterproductive.

Soon, the problem of price or value of any commodity in the market became a central issue. Following David Ricardo's (1772–1823) formulation of rent and labor theory of value, where the price depends only on the amount of labor put forth by the farmers or laborers, Karl Marx (1818–1883) formulated and advocated emphatically the surplus labor theory of value or wealth in any economy. However, neither could solve the price paradox: why diamonds are expensive, while coal is cheap. The amount of labor in mining is more or less the same for both diamonds and coal. Yet, the prices differ by an astronomical amount. This clearly demonstrates the failure of the labor theory of value. The alternative put forth was the utility theory of price: the more the utility of a commodity, the higher its price. But then, how does one explain why a bottle of water costs less than a bottle of wine? The argument could be made that water is more important for sustaining life and certainly has more utility! The solution identified was marginal utility. According to marginal utility theory, not the utility but rather its derivative with respect to the quantity determines the price: water is cheaper as its *marginal utility* at the present level of its availability is less than that for wine – this will surely change in a desert. This still does not solve the problem completely. Of course increasing marginal utility creates increasing demand for it, but its price must depend on its supply (and will be determined by equating the demand with the supply). If the offered (hypothetical) price p of a commodity increases, the supply will increase and the demand for that commodity will decrease. The price, for which supply S will be equal to demand D, will be the market price of the commodity: $S(p) = D(p)$ at the market (clearing) price. However, there are problems still. Which demand should be equated to which supply? It is not uncommon to see often that price as well as the demand for rice, for example in India, increases simultaneously. This can occur when the price of the other staple alternative, wheat, increases even more.

The solutions to these problems led ultimately to the formal development of economic science in the early twentieth century by Léon Walras (1834–1910), Al-