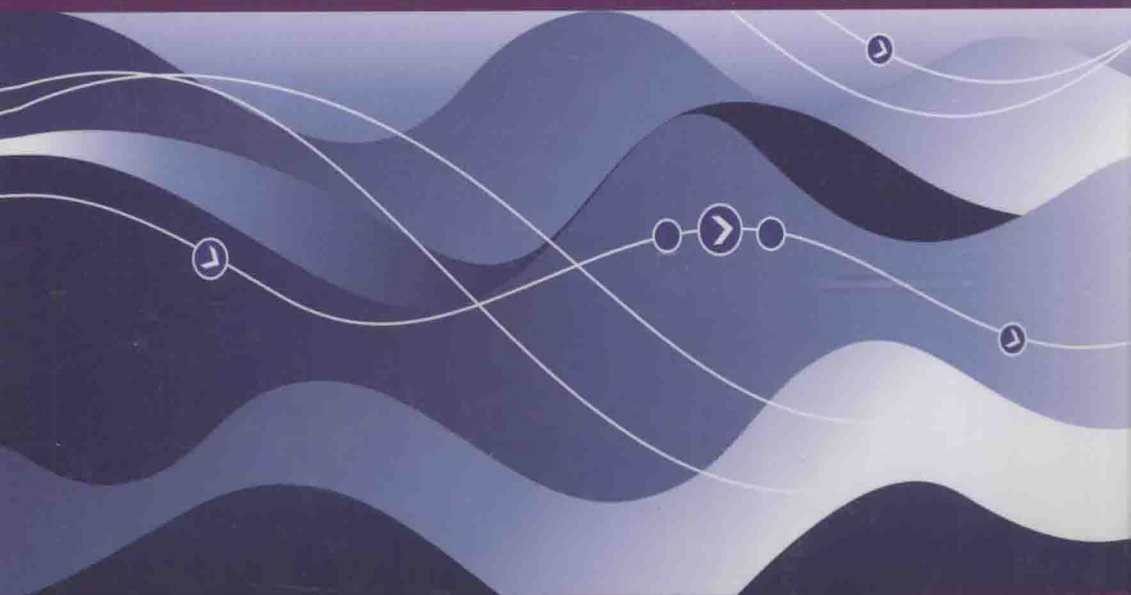


ENVIRONMENTAL HYDRAULICS SERIES

Mathematical Models

Edited by Jean-Michel Tanguy

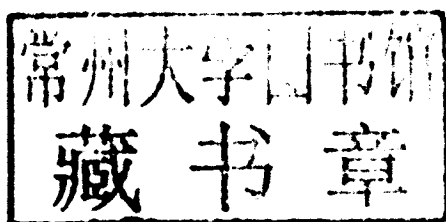


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Mathematical Models



Edited by
Jean-Michel Tanguy

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Mathematical Models

Introduction

This study of environmental hydraulics is made up of five volumes. The first two volumes are concerned with describing the principle processes in physical domains which can be observed and measured.

The first volume is dedicated to mathematical modeling for hydrology and fluvial hydraulics.

It is completed by a volume dedicated to mathematical modelization for oceanic hydraulics for models of transport and conceptual models.

This is followed by two volumes dedicated to numerical modelization. One is on the presentation of operational software and the two volumes are on the various applications of software in case studies.

This present volume draws mathematical theories which enable us to present these processes and simulate them. There is no one unified theory which describes with a unique system the many physical processes occurring within the field and this book.

Most scientific disciplines began with a period of experimentation (be it in hydraulics at the open surface or hydrogeology) in order to understand and research of laws of behavior and draw from these observable correlations.

In contrast there were a number of important theories on the mechanics of fluids which blossomed in the 18th century, notably Euler and his studies. Empirical and theoretical approaches did not go hand in hand until the end of the 19th century.

Today the majority of scientific disciplines we will look at in this book abandoned pure empirical methods and replaced them with theoretical ones. These deal with many different things, some made up from complex mathematics while others retain their roles as very simple models. What they have in common is that they all deal with natural environments adhering to the laws of conservation: mass, quantity of movement and energy.

Our objective in this book is to give the reader a vision which incorporates the many disciplines and laws on water.

The main thread of this book charts the development of numerical models in each scientific discipline. In effect there has been a pendulum movement between complexity and simplicity since the 1960s. With the coming of computer technology, the first objective was to simplify the complex 3D mathematical models as much as possible whilst making sure they could still be used in engineering. As time went on and methods progressed, there was a reverse movement away from the former simplified models and towards more sophisticated ones which now form the basis of the latest models. These days, models are continually being made ever more sophisticated. This iterative step is common to all scientific disciplines, and along the way, a multitude of more or less simplified mathematical models were created. We shall present these later on.

This book is made up of sixteen chapters:

Chapter 1: Reminders for the mechanical properties of fluids

Chapter 2: Navier-Stokes' Equations

Chapter 3: Atmospheric models

Chapter 4: Hydrologic Models

Chapter 5: Fluvial currentology and oceanic models

Chapter 6: Urban hydrologic models

Chapter 7: Tidal model and tide streams

Chapter 8: Swell generation and coastal current models

Chapter 9: Solid transport models and evolution of the seabed

Chapter 10: Oil spill models

Chapter 11: Conceptual, empirical and other models

Chapter 12: Reservoir models in hydrology

Chapter 13: Reservoir models in hydro-geology

Chapter 14: Formal neural network models

Chapter 15: Model coupling

Chapter 16: Different categories of hydrology models

Using a mathematical format, Chapter 1 presents the principles behind mechanical properties of fluids. It introduces the main rules of this discipline in a brief and theoretical manner which can be studied quickly if desired. It recalls the theorem for quantity of movement, the fundamental notations of kinetic energy, enthalpy and the first principles of thermodynamics – Newton's Law concerning the forces in a moving fluid. The theoretical corpus we incorporate establishes the following presentations by discipline.

Chapter 2 deals with Navier-Stokes equations, allowing us to exactify the system of equations which form the basis of great developments in the mechanics of fluids. These are particularly used in their tridimensional form in meteorology as well as for underground flows and swells. It is likely in the next few years that this trend will move progressively towards fluvial hydraulics, water quality and sedimentology.

Naturally this chapter is followed with a presentation of atmospheric models (Chapter 3) which are used by national meteorological services. In order to establish the first models and resolve them by computer, a number of simplifications were necessary to create the barotrope¹ model from 1950 onwards. These consisted of one layer and zero divergence. The coming of calculators enabled more complicated models further removed from the previous simplified hypotheses. Models filtered with several layers of altitude (known as filtered barocline models baroclines²) came into being. It was not until recently that non-hydrostatic 3D models were created, allowing studies of planes with scales of several kilometers. As illustrated by Figure i.1, these models enabled weather forecasting and by consequence provided additional parameters for other models: rain and temperature for hydrologic models and wind and pressure to complete oceanic models looking at the state of the sea.

For hydrologists the task is not so simple. Apart from rain they also need to represent what is happening in the soils under the surface. These two types of flows come together. As we will later see, hydrological models are of a conceptual nature. On the other hand, flows in the soil are well understood from hydrogeological models (Chapter 4). These provide hydrologists and hydraulicists with flows and layers. Hydrogeologists use Navier-Stokes' equations but also draw on Darcy's law of macroscopic behavior. Even in this discipline developments were made

¹ Of a state or a forecasting model in which constant surface pressures (isobares) are parallel to these of constant density. The corresponding models are bidimensional.

progressively, first dealing with saturated then non-saturated flows. The latest are the most sophisticated, but still draw on the empirical laws in representing water retention and permeability.

After presenting hydrogeologic models, we will move on to models for fluvial and oceanic currentology (Chapter 5). In the same way as in meteorology, models began as simple as possible and gradually became more complex. 1D models drawn up by the research offices in the 1960s were replaced, bit by bit with 2D models. These models extended their domains towards the river's upper reaches, as in the case of hydrology, but also towards the lower reaches and ocean where we meet our oceanographer colleagues who use similar, if 3D models. These models draw on empirical relations essentially to represent roughness. The closing models that were used to represent turbulence were again less than satisfactory.

We must then make a detour towards urban hydrogeology (Chapter 6) where we use the same models but in two different manners. Firstly to simulate flow in networks of drainage pipes, and secondly above ground on the roads and sidewalks where excess water flows as a result of rain and overflowing drains. This discipline converges with as hydrogeology as it is concerned with small pools which are strongly impermeable to complex fluid behavior and classic hydraulics. It therefore uses the two types of tools.

By reaching the estuaries, the waters of the rivers get in contact with the *tide* (Chapter 7) and some tidal currents. Oceans and seas are subjected to the movement of the Earth and to the influence of celestial bodies. The tide is considered as the consequence of a succession of actions which get linearly superposed: a hypothesis which remains valid in high waters, but which becomes quite wrong close to the shore. The various models used for the representation of this process are the models in shallow waters belonging to the Saint-Venant type. However, some recent and highly accurate measurements led to the detection of the irregularities and of the anomalies in the propagation of this phenomenon and especially in the vertical profiles of the rates. This suggests then that some 3D models will be developed in the near future.

To complete the consideration of the phenomena which occurs next to the shoreline, the *swell* phenomenon (Chapter 8) will be studied. This phenomenon is quite complex and hard to model. Made of several waves, the swell is quite sensitive to the bathymetry of the beds and to geometry of the coasts generated in high waters by the wind. It develops on large areas and is always changing due to diffraction and reflection phenomena. The first models limited to the refraction of the beds were designed in the 1960s in the research and development offices, but it was only in the 1980s that new and more complex models could be proposed, considering the diffraction processes of the structures and the reflection of the shoreline. Regarding

the generation and the propagation of the swells, the models were still until recently quite empirical but thanks to the international WAM (*W*ave *M*odel) group, some new second and third generation models appeared, so the empirical equations are used less often. Going back to the use of the complex theoretical models has occurred for the last few years in order to finely reproduce the transformation conditions of the swells close to the shoreline. However, some empirical equations are still used, like in the case of the modeling of breaking of waves.

All these fields dealt with the flowing process in the waters of the considered domains as being fixed in time and space. However, it is well known that this is not the truth: everything depends on how things are observed, both in time and space; waters loaded of sediments will become concentrated in the water currents, spreading to the rivers and are then thrown into the sea by the rivers. With a time scale of a few days, the violence of the flowing processes is quite striking during a flood, due to their consequences, such as the breaking of meanders or also by the movement, or even the destruction of some structures in the rivers. In some cases, beaches can get entirely swept away by coastal currents. A presentation of the *fluvial and maritime sedimentology* (Chapter 9) is then necessary. The corresponding models are based on the main conservation principles, but depend more on empirical laws: which is due to the evolution of the quality of the sediments during their transport, to the complex interactions with the bed, the banks and the coasts. After some quite unsatisfactory tests based on a 1D modeling process, 2D models did not lead to much improvement: a new generation of 3D hydrodynamic and sedimentological models is being developed and will be able to better understand the evolutions of the rivers and coastal areas in the future.

To complete this part, the transport models of some materials and especially of *oil spills* (Chapter 10), usually due to some accidental pollutions, will be presented in real time.

A presentation, limited to these physical processes, shows that most of them are quite similar to each other and that the theories used still remain in the frame of the main theories of fluid mechanics. Indeed, this is not really true because some physical mediums are so complex that these theoretical models could not be applied everywhere yet.

For instance, the *evapotranspiration* process (Chapter 11) is a fundamental process which regulates the interactions between the ground and the atmosphere. There actually are some very localized processes, which are necessary for any modeling, but which are analyzed locally by some empirical equations dealing with the exchange process between different mediums.

Another large category of models which ensure the conservation of water within the catchment areas is made of the *conceptual reservoir models in hydrology* (Chapter 12) and *in hydro-geology* (Chapter 13). These models do not deal with the infiltration or superficial flowing models very precisely, but they represent them on a very large scale. Without any knowledge regarding the real path of the water within the catchment areas, and without any adapted and global measuring systems, these models still have to be used. However, some recent developments have enabled us to show that some kinematical wave model from some hydraulic and hydro-geological models are beginning to give good results...

Finally, when the behavior of a physical model is too complex for the description of its own components (these empirical models cannot really represent the dynamics of the basin), the *neural network* model (Chapter 14) can be a really useful tool and can always be improved in new situations.

The last chapters deal with the *coupling process of these models* (Chapter 15) and with the different *categories of codes* (Chapter 16). When a river, an estuary, or a coastal area is studied, water is only one of the components which influence the environment: some behaviors integrate a lot, like pollution, morpho-dynamics, the impact of storms, tsunamis or low waters. Their interactions then have to be modeled. The chapter dealing with this topic specifies the principles and the main issues related to this topic.

Figure i.1 illustrates the various equations and interactions between the models. The upstream meteorological models, simulate the atmosphere and provide the time parameters to the hydrologic, hydraulic and hydro-geological models which deal with the flowing water. In addition, they interact with themselves in most of the natural situations. The third layer model leads to the representation of the flowing effects on the fluvial or maritime medium as well as of the issues of the pollution of water.

We encourage the reader to cover all of these developments and take note of the coherence that exists between the many disciplines. Many draw from the same theories, whilst specializing in specific areas of study. Whether it pools with complex soils, rivers with their changing beds and water levels, urban spaces with heterogeneous occupation of soils, coastal interference from swelling and sediment moving with the current – all these environments are yet to reveal their secrets.

Jean-Michel TANGUY

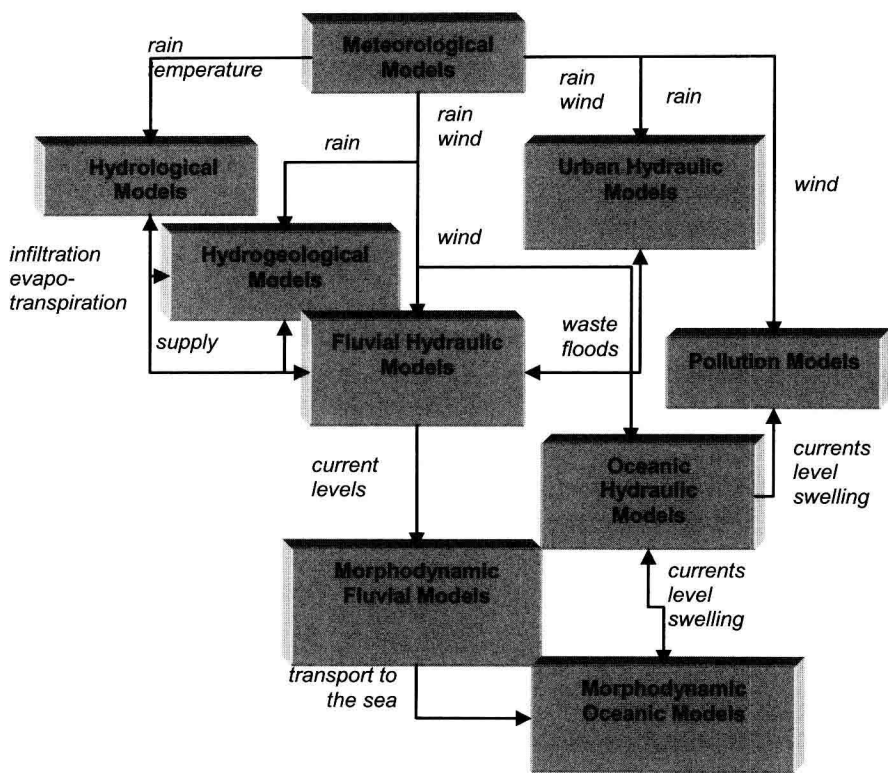


Figure i.1. Diagram of links between models

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