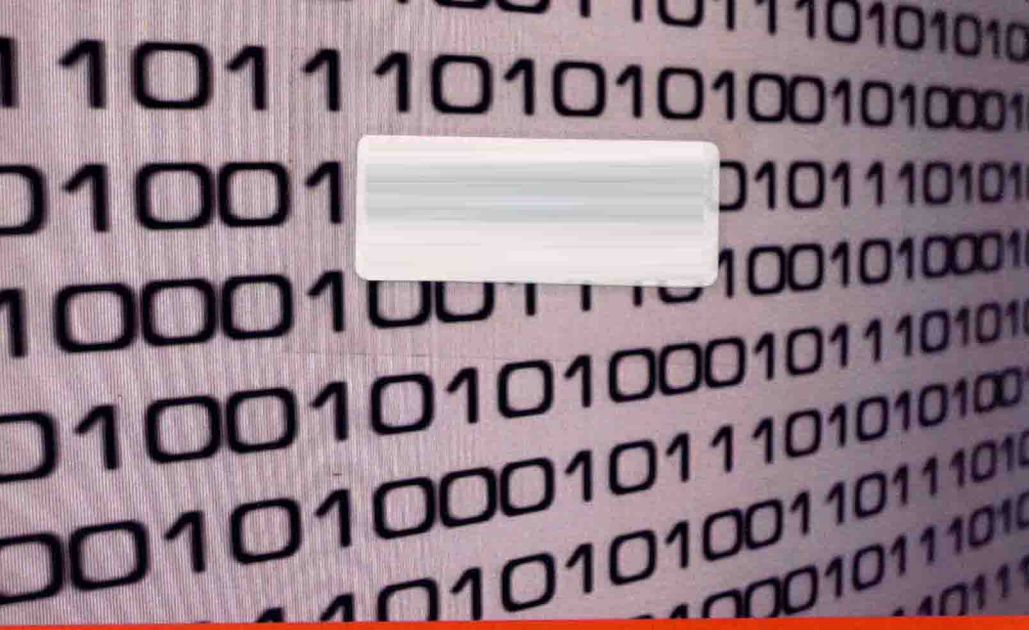

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PRIME FUZZY SUBMODULES (MONOGRAPH)

Prime Submodule, Fuzzy Submodule, Fuzzy Coset of a Fuzzy Submodule

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(Monograph)

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PRIME FUZZY SUBMODULES
(Monograph)

PREFACE

This monograph entitled "Prime Fuzzy Submodules", is divided in to four chapters.

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Satyanarayana Bhavanari,
and
Rama Prasad J.L.

INTRODUCTION

In recent decades interest has arisen in algebraic systems with binary operations addition and multiplication. 'Ring' is one of such systems. A ring (or associative ring) is an algebraic system (R, +, .) satisfying the conditions:

(i) (R, +) is an Abelian group;

(ii) (R, .) is a semigroup; and

(iii) $a(b + c) = ab + ac$ and

$(a + b)c = ac + bc$ for all $a, b, c \in R$.

Moreover, if there exists an element $1 \in R$ such that $1a = a = a1$ for all $a \in R$, then we say that R is a ring with identity. Ring theory occupied an important place in Algebra.

Modern Algebra presently, the basis for developing several new areas mentioned below. The past 40 years have seen an enormous expansion in several new areas of technology. These new areas include Digital Computing, Data Communication, Sequential Machines, Computer Systems and Radar Solar Systems. Work in each of these areas relies heavily on Modern Algebra. This fact has made the study of *Modern Algebra* important to Applied Mathematicians, Engineers and Scientists who use Digital Computers or who work in the other areas of Technology mentioned above.

Let R be an associative ring. An Abelian group (M, +) is said to be a ***module*** over R if there exists a mapping (called *scalar multiplication*) f: $R \times M \to M$ (the image of (r, m) is denoted by rm) satisfying the following three conditions:

(i) $r(a + b) = ra + rb$;

(ii) $(r + s)a = ra + sa$; and

(iii) $r(sa) = (rs)a$ for all $a, b \in M$ and $r, s \in R$.

Moreover, if R is ring with identity 1 and $1m = m$ for all $m \in M$, then M is called a ***unitary R–module***.

Every vector space is a module. Every Abelian group is a module over the ring of integers $\mathbb{Z}$. Every ring R is a module over itself. So the study of module theory include the study of Vector Space Theory, Group Theory and Ring Theory. Thus the Module Theory became an important part of Algebra.

Let R be a fixed (not necessarily commutative) ring with identity. Throughout this monograph, we are concerned with unitary left R-modules M.

It is well known that the concept 'prime ideal' plays an important role in the Theory of Rings. Since every ideal of a ring R is a submodule of the R-module R, the study of submodules of a module is a generalization of the study of ideals in ring theory.

This monograph mainly deals with the concept 'Prime Fuzzy Submodules'.

Success of fuzzy logic in a wide range application inspired much interest in fuzzy logic among Mathematicians. Lotfi. A. Zadeh (a professor in Electrical Engineering and Computer Science at University of California, Berkeley) (July 1964) introduced a theory whose objects called 'fuzzy sets' (are sets with boundaries that are not precise). In a narrow sense fuzzy logic refers to a logical system that generalizes classical two-valued logic for reasoning under uncertainty. Prof. Zadeh believed that all real world problems could be solved with more efficient and analytic methods by using the concept fuzzy sets. The fuzzy boom (1987 to present) in Japan was a result of the close collaboration and technology transfer between Universities and Industries. In 1988 the Japanese Government launched a careful feasibility study about establishing national research projects on fuzzy logic involving both Universities and Industries. As a result Japan is able to manufacture fuzzy vacuum cleaners, fuzzy rice cookers, fuzzy refrigerators, fuzzy washing machines, and others.

After the introduction of Fuzzy set by Zadeh [1], the researchers in mathematics were trying to introduce and study this concept of fuzzyness in different mathematical systems. Fu-Zheng

Pan [1, 2]; Golan [1]; Rajesh Kumar, Bhambri and Prathibha Kumar [1]; Satyanarayana, Godloza and Mohiddin Shaw [1] studied the concept 'fuzzy submodule'. In this monograph, we introduced and studied the concept 'prime fuzzy submodule'. The monograph is divided into four chapters.

Chapter-1 entitled "Fundamentals of Fuzzy Subsets contains a collection of preliminary definitions and results related to the concepts 'fuzzy set' which will be useful in the sequel. In Chapter-2, we provide some fundamental definitions on modules and module homomorphisms.

Chapter-3 entitled "Fuzzyness in Modules and Fuzzy cosets" contains some definitions, examples and results related to the concept "fuzzy submodule". We divide this chapter into four sections. In Section-1, we collect the existing three definitions for 'fuzzy submodule' and we verify that when R is a ring with identity, the concept 'fuzzy submodule' in the sense of Negoita and Ralescue [1] and the concept 'fuzzy submodule' in the sense of Golan [1] are equivalent. In this monograph we study the results related to the concept "fuzzy submodule" in the sense of Rajesh Kumar, Bhambri and Pratibha Kumar [1]. This definition also used in Satyanarayana, Godloza and Mohiddin Shaw [1]. Section-2, is a collection of some fundamental results related to the

concepts "fuzzy submodule" and "level submodule". The characteristic function χ_N of a subset N of M is a fuzzy submodule of M $\Leftrightarrow$ N is a submodule of M. A fuzzy subset μ of M is a fuzzy submodule $\Leftrightarrow$ μ_t is a submodule of M for all $t \in [0, \mu(0)]$. In Section-3, we present the concept "fuzzy coset of a fuzzy submodule" and few related results. If μ is a fuzzy submodule of M, then M/μ is an R-module. In Section-4, we discuss some results related to R-module homomorphisms and fuzzy submodules and we include few examples. If $f: M \rightarrow M^1$ is a module epimorphism, then there is an order preserving bijection between the fuzzy submodules of M^1 and the fuzzy submodules of M that are constant on ker f.

Chapter-4 entitled "Prime fuzzy submodules" contains some recent research work done on "prime submodules" and "prime fuzzy submodules" of a module M over an associative ring R with identity. In this chapter, we present the concept "prime fuzzy submodule" and proofs of some important Theorems related to this concept. Also we include some examples.