# Topics in Current Physics

# Real-Space Renormalization

**Editors:**
**T. W. Burkhardt and J. M. J. van Leeuwen**

| | |
|---|---|
| T. W. Burkhardt<br>J. M. J. van Leeuwen | Progress and Problems in Real-Space Renormalization |
| T. W. Burkhardt | Bond-Moving and Variational Methods in Real-Space Renormalization |
| R. H. Swendsen | Monte Carlo Renormalization |
| G. F. Mazenko<br>O. T. Valls | The Real-Space Dynamic Renormalization Group |
| P. Pfeuty R. Jullien<br>K. A. Penson | Renormalization for Quantum Systems |
| M. Schick | Application of the Real-Space Renormalization to Adsorbed Systems |
| H. E. Stanley<br>P. J. Reynolds<br>S. Redner F. Family | Position-Space Renormalization Group for Models of Linear Polymers, Branched Polymers, and Gels |

Springer-Verlag
Berlin Heidelberg New York

# Real-Space Renormalization

Edited by
T. W. Burkhardt and J. M. J. van Leeuwen

With Contributions by
T. W. Burkhardt F. Family R. Jullien
J. M. J. van Leeuwen G. F. Mazenko K. A. Penson
P. Pfeuty S. Redner P. J. Reynolds M. Schick
H. E. Stanley R. H. Swendsen O. T. Valls

With 60 Figures

Springer-Verlag Berlin Heidelberg New York 1982

Dr. Theodore W. Burkhardt

Institut Laue-Langevin, 156 X, F-38042 Grenoble Cédex, France, and
Department of Physics, Temple University, Philadelphia, PA 19122, USA

Professor Dr. J. M. J. van Leeuwen

Technische Hogeschool Delft, Laboratorium voor Technische Natuurkunde, 2600 6A Delft, The Netherlands

ISBN 3-540-11459-9 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-11459-9 Springer-Verlag New York Heidelberg Berlin

Library of Congress Cataloging in Publication Data. Main entry under title: Real-space renormalization. (Topics in current physics ; 30). Includes bibliographies and index. 1. Renormalization (Physics) I. Burkhardt, T. W. (Theodore W.), 1940– . II. Leeuwen, J. M. J. van. III. Series. QC20.7.R43R43 530.4 82-5763 AACR2

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, reuse of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to "Verwertungsgesellschaft Wort", Munich.

© by Springer-Verlag Berlin Heidelberg 1982
Printed in Germany

The use of registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Offset printing and bookbinding: Konrad Triltsch, Graphischer Betrieb, Würzburg.
2153/3130-543210

Topics in
Current Physics

30

# Topics in Current Physics

Founded by Helmut K. V. Lotsch

1 **Beam-Foil Spectroscopy**
Editor: S. Bashkin

2 **Modern Three-Hadron Physics**
Editor: A. W. Thomas

3 **Dynamics of Solids and Liquids by Neutron Scattering**
Editors: S. W. Lovesey and T. Springer

4 **Electron Spectroscopy for Surface Analysis**
Editor: H. Ibach

5 **Structure and Collisions of Ions and Atoms**
Editor: I. A. Sellin

6 **Neutron Diffraction**
Editor: H. Dachs

7 **Monte Carlo Methods** in Statistical Physics
Editor: K. Binder

8 **Ocean Acoustics**
Editor: J. A. DeSanto

9 **Inverse Source Problems** in Optics
Editor: H. P. Baltes

10 **Synchrotron Radiation**
Techniques and Applications
Editor: C. Kunz

11 **Raman Spectroscopy** of Gases and Liquids
Editor: A. Weber

12 **Positrons in Solids**
Editor: P. Hautojärvi

13 **Computer Processing of Electron Microscope Images**
Editor: P. W. Hawkes

14 **Excitons** Editor: K. Cho

15 Physics of **Superionic Conductors**
Editor: M. B. Salamon

16 **Aerosol Microphysics I**
Particle Interactions
Editor: W. H. Marlow

17 **Solitons**
Editors: R. Bullough, P. Caudrey

18 **Magnetic Electron Lenses**
Editor: P. W. Hawkes

19 Theory of **Chemisorption**
Editor: J. R. Smith

20 **Inverse Scattering Problems** in Optics
Editor: H. P. Baltes

21 **Coherent Nonlinear Optics**
Recent Advances
Editors: M. S. Feld and V. S. Letokhov

22 **Electromagnetic Theory of Gratings**
Editor: R. Petit

23 **Structural Phase Transitions I**
Editors: K. A. Müller and H. Thomas

24 **Amorphous Solids**
Low-Temperature Properties
Editor: W. A. Phillips

25 **Mössbauer Spectroscopy II**
The Exotic Side of the Method
Editor: U. Gonser

26 **Crystal Cohesion and Conformational Energies** Editor: R. M. Metzger

27 **Dissipative Systems in Quantum Optics**
Resonance Fluorescence, Optical Bistability, Superfluorescence
Editor: R. Bonifacio

28 **The Stratospheric Aerosol Layer**
Editor: R. C. Whitten

29 **Aerosol Microphysics II**
Chemical Physics of Microparticles
Editor: W. H. Marlow

30 **Real-Space Renormalization**
Editors: T. W. Burkhardt and J. M. J. van Leeuwen

31 **Hyperfine Interactions of Radioactive Nuclei**
Editor: J. Christiansen

32 **Superconductivity in Ternary Compounds I**
Structural, Electronic, and Lattice Properties
Editors: Ø. Fischer and M. B. Maple

33 **Molecular Collision Dynamics**
Editor: J. M. Bowman

34 **Superconductivity in Ternary Compounds II**
Editors: M. B. Maple and Ø. Fischer

此为试读,需要完整PDF请访问: www.ertongbook.com

# Preface

The renormalization-group approach is largely responsible for the considerable success which has been achieved in the last ten years in developing a complete quantitative theory of phase transitions. Before, there was a useful physical picture of phase transitions, but a general method for making accurate quantitative predictions was lacking. Existent theories, such as the mean-field theory of Landau, sometimes reproduce phase diagrams reliably but were known to fail qualitatively near critical points, where the critical behavior is particularly interesting because of its universal character.

In the mid 1960's Widom found that the singularities in thermodynamic quantities were well described by homogeneous functions. Kadanoff extended the homogeneity hypothesis to correlation functions and linked it to the idea of scale invariance. In the early 1970's Wilson showed how Kadanoff's rescaling could be explicitly carried out near the fixed point of a flow in Hamiltonian space. He made the first practical renormalization-group calculation of the flow induced by the elimination of short-wave-length Fourier components of the order-parameter field. The universality of the critical behavior emerges in a natural way in this approach, with a different fixed point for each universality class. The discovery by Wilson and Fisher of a systematic expansion procedure in $\varepsilon$ for a system in $d = 4 - \varepsilon$ dimensions was followed by a cascade of calculations of critical quantities as a function of d and of the order-parameter dimensionality n. Using field-theoretic techniques, Brêzin, Le Guillou, Zinn-Justin, Nickel, and others have turned the $\varepsilon$ expansion into an astonishingly accurate tool for computing the critical behavior of three-dimensional systems.

Another class of renormalization procedures has been developed, in which the renormalization is carried out in real space rather than Fourier space. In the real-space renormalization of a system of spins on a lattice, lengths are rescaled by replacing cells of spins by single spins, following the intuitive scaling picture of Kadanoff rather closely. Unlike the Fourier-space methods, real-space renormalization works directly with a microscopic Hamiltonian and attempts to calculate non-universal information, for example phase diagrams, in addition to universal quantities. It is not a priori clear that renormalization will work on a microscopic level. There is no unique prescription for renormalizing in real space, and a con-

siderable number of approximate renormalization techniques have been developed. There have been applications to a wide variety of different physical systems. The domain of applicability is by no means restricted to critical phenomena. Striking successes have been achieved, but there are also some general problems with the approach. This volume is a review of the present state of the art.

The literature on real-space renormalization is already too large to be reviewed in a single volume. We have included contributions on five or six general areas where, in our opinion, particularly significant developments have taken place. We decided not to repeat at length material already covered in the 1976 review article by Niemeijer and van Leeuwen in Vol.6 of the Domb-Green series *Phase Transitions and Critical Phenomena*.

In the introductory chapter we give an overview of real-space renormalization and discuss some of the fundamental problems. Griffiths-Pearce singularities, exact differential renormalization, and the phenomenological renormalization related to finite-size scaling are also considered. Chapter 2, by Burkhardt, is devoted to bond-moving and variational approximation techniques. In Chapter 3 Swendsen reviews Monte Carlo renormalization methods. Real-space renormalization procedures have been developed for quantum systems and for dynamic critical phenomena. These two topics are discussed by Pfeuty, Jullien, and Penson and by Mazenko and Valls in Chapters 4 and 5, respectively. Chapters 6 and 7 review two specific areas of application where real-space renormalization is making a significant contribution. In Chapter 6, by Schick, two-dimensional adsorbed systems are considered, and in Chapter 7, by Stanley, Reynolds, Redner, and Family, applications to percolation and polymers.

The suggestion for a Topics volume on real-space renormalization originally came from K. Binder, whom we thank for his encouragement. We also thank the contributors and H. Lotsch of Springer-Verlag for their cooperation in preparing a review of this rapidly developing field.

Delft and Philadelphia,
March 1982

*T.W. Burkhardt*
*J.M.J. van Leeuwen*

# List of Contributors


Burkhardt, Theodore W.
Institut Laue-Langevin, 156 X, F-38042 Grenoble Cēdex, France, and
Department of Physics, Temple University, Philadelphia, PA 19122, USA

Family, Fereydoon
Center for Polymer Studies and Department of Physics,
Boston University, Boston, MA 02215, USA, and
Department of Physics, Emory University, Atlanta, GA 30322, USA

Jullien, Rémi
Laboratoire de Physique des Solides, Université de Paris-Sud,
Centre d'Orsay, F-91405 Orsay, France

Leeuwen van, J.M.J.
Technische Hogeschool Delft, Laboratorium voor Technische Natuurkunde,
Lorentzweg 1, 2600 GA Delft, The Netherlands

Mazenko, Gene F.
The James Franck Institute, University of Chicago, Chicago, IL 60637, USA

Penson, Karol A.
Institut für Theoretische Physik, Freie Universität Berlin,
Arnimallee 3, D-1000 Berlin 33, Fed. Rep. of Germany

Pfeuty, Pierre
Laboratoire de Physique des Solides, Université de Paris-Sud,
Centre d'Orsay, F-91405 Orsay, France

Redner, Sidney
Center for Polymer Studies and Department of Physics,
Boston University, Boston, MA 02215, USA

Reynolds, Peter J.
Center for Polymer Studies and Department of Physics,
Boston University, Boston, MA 02215, USA, and
MMRD, Lawrence Berkely Lab., University of California, Berkeley, CA 94720, USA

Schick, Michael
Department of Physics, University of Washington, Seattle, WA 98195, USA

Stanley, H. Eugene
Center for Polymer Studies and Department of Physics,
Boston University, Boston, MA 02215, USA

Swendsen, Robert H.
I.B.M. Research Laboratory, Säumerstraße 4, CH-8803 Rüschlikon, Switzerland

Valls, Oriol T.
School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA

# Contents


1. **Progress and Problems in Real-Space Renormalization**

By T.W. Burkhardt and J.M.J. van Leeuwen (With 5 Figures)

1.1 Introduction ..... 1
1.2 Review of Real-Space Renormalization ..... 3
1.3 New Renormalization Methods ..... 6
1.3.1 Bond-Moving and Variational Methods ..... 6
1.3.2 Monte Carlo Renormalization ..... 7
1.3.3 Exact Differential Transformations ..... 7
1.3.4 Phenomenological Renormalization ..... 7
1.4 New Applications ..... 8
1.4.1 Adsorbed Systems ..... 8
1.4.2 Applications to Quantum Systems ..... 8
1.4.3 Percolation and Polymers ..... 9
1.4.4 Dynamic Real-Space Renormalization ..... 9
1.4.5 The Kosterlitz-Thouless Transition ..... 9
1.4.6 Field-Theoretical Applications ..... 9
1.5 Fundamental Problems ..... 9
1.5.1 Choice of the Weight Function ..... 11
1.5.2 Griffiths-Pearce Peculiarities ..... 13
1.6 Exact Differential Real-Space Renormalization ..... 17
1.6.1 The Two-Dimensional Ising Model ..... 18
1.6.2 Discussion ..... 22
1.7 Phenomenological Renormalization ..... 23
1.7.1 Description of the Method ..... 23
1.7.2 Applications ..... 27
1.8 Concluding Remarks ..... 28
References ..... 29

2. **Bond-Moving and Variational Methods in Real-Space Renormalization**

By T.W. Burkhardt (With 6 Figures)

2.1 Introduction ..... 33
2.2 Variational Principles ..... 35

2.2.1 Lower-Bound Property of Bond-Moving Approximations ..... 35
2.2.2 Upper-Bound Property of the First-Order Cumulant Approximation ..... 37
2.3 The Migdal-Kadanoff Transformation ..... 37
2.3.1 Application to the Ising Model with Nearest-Neighbor Interactions ..... 37
2.3.2 Inclusion of a Magnetic Field ..... 41
2.3.3 The Bond-Moving Prescription of EMERY and SWENDSEN ..... 42
2.3.4 Inconsistent Scaling of the Correlation Function ..... 42
2.3.5 Relation to Exactly Soluble Hierarchical Models ..... 43
2.3.6 Applications ..... 43
2.3.7 Modifications of the Migdal-Kadanoff Procedure ..... 44
2.4 Variational Transformations ..... 44
2.4.1 The Kadanoff Lower-Bound Variational Transformation ..... 44
2.4.2 The Kadanoff Criterion for the Optimal Variational Parameter 47
2.4.3 Problems with the Lower-Bound Variational Transformation ... 49
2.4.4 Determination of an Optimal Sequence of Variational Parameters ..... 50
2.4.5 Applications of the Lower-Bound Variational Transformation 51
2.4.6 Other Variational Methods ..... 52
2.5 Conclusion ..... 53
References ..... 55

3. **Monte Carlo Renormalization**
By R.H. Swendsen (With 4 Figures)
3.1 Introduction ..... 57
3.2 Basic Notation and Renormalization-Group Formalism ..... 58
3.3 Large-Cell Monte Carlo Renormalization Group ..... 63
3.4 MCRG ..... 65
3.4.1 Calculation of Critical Exponents ..... 65
3.4.2 Calculation of Renormalized Coupling Constants ..... 74
3.5 MCRG Calculations for Specific Systems ..... 78
3.6 Other Approaches to the Monte Carlo Renormalization Group ..... 83
3.7 Conclusions ..... 84
References ..... 84

4. **The Real-Space Dynamic Renormalization Group**
By G.F. Mazenko and O.T. Valls (With 12 Figures) ..... 87
4.1 Introduction ..... 87
4.2 Dynamic Problem of Interest ..... 88
4.3 RSDRG - Formal Development ..... 91
4.4 Implementation of the RSDRG Using Perturbation Theory ..... 96

4.4.1 General Development ..... 96
4.4.2 Expansion for H and $D_\sigma$ ..... 98
4.4.3 Solution to the Zeroth-Order Problem ..... 101
4.4.4 Renormalization to First Order ..... 103
4.4.5 Recursion Relations for the Correlation Functions ..... 104
4.5 Determination of Parameters ..... 107
4.5.1 General Comments ..... 107
4.5.2 The Parameters $K_o$ and $K_R^o$ ..... 107
4.5.3 The Dynamic Parameters $\alpha_o$ and $\Delta$ ..... 109
4.6 Results ..... 111
4.7 Discussion ..... 115
References ..... 116

5. **Renormalization for Quantum Systems**
By P. Pfeuty, R. Jullien and K.A. Penson
5.1 Background ..... 119
5.2 Application of the Niemeijer-van Leeuwen Renormalization Group Method to Quantum Lattice Models ..... 121
5.3 The Block Method ..... 125
5.3.1 Principles ..... 125
5.3.2 Applications ..... 126
a) The Ising Model in a Transverse Field in One Dimension .. 126
b) The Free Fermion Model in One Dimension ..... 130
5.3.3 Extensions of the Method ..... 133
a) Extension to Large Blocks ..... 133
b) Extension by Increasing the Number $n_L$ of Levels Retained 133
c) Other Extensions ..... 133
5.4 Applications of the Block Method ..... 134
5.4.1 Spin Systems ..... 134
a) The Spin 1/2 Ising Model in a Transverse Field (ITF) .... 134
b) The XY Heisenberg Spin 1/2 Chain ..... 134
c) The XY Model in a Z Field for d = 2, 3 ..... 135
d) The Spin 1 XY Model with an Anisotropy Field for d = 1 .. 135
5.4.2 Fermion Systems ..... 135
a) The d = 1 Hubbard Model ..... 135
b) Interacting Fermions in d = 1 ..... 136
c) One-Dimensional Model of f and d Electrons with Hybridization V and fd Interaction $U_{fd}$ ..... 137
5.4.3 Spin Fermion Systems: The Kondo Lattice in d = 1 ..... 137
5.4.4 Quantum Versions of Classical Statistical Mechanics in 1 + 1 Dimension ..... 138
a) The O(n) Model ..... 138

此为试读,需要完整PDF请访问: www.ertongbook.com

b) The P(q) Potts Model ..... 138
c) Tricritical Point for Ising Systems in 1 + 1 Dimensions 139
5.4.5 Applications to Field Theory ..... 139
a) The Thirring Model in One-Space and One-Time Dimension .. 139
b) The U(1) Goldstone Model in Two Dimensions ..... 139
c) Lattice Gauge Theories ..... 140
5.5 Discussion ..... 140
5.5.1 When is the BRG More Suitable? ..... 140
5.5.2 How to Control the Method? ..... 140
a) The Division of the Lattice into Blocks ..... 141
b) Which Levels to Retain for the Truncated Basis? ..... 142
5.5.3 What Has Been Done and What Are the Difficulties Encountered? 143
a) Quantum Properties at T = 0 ..... 143
b) Quantum Properties at T ≠ 0 ..... 143
c) Difficulties ..... 143
5.5.4 Comparison Between Different Methods ..... 143
a) The Real-Space RG Methods for Classical Systems ..... 143
b) Finite-Size Scaling Methods ..... 144
5.6 What to Do Next? ..... 144
5.6.1 Improvement of the Method ..... 144
5.6.2 Applications ..... 144
References ..... 145

6. **Application of the Real-Space Renormalization to Adsorbed Systems**
By M. Schick (With 18 Figures)
6.1 Introduction ..... 149
6.2 The Sublattice Method ..... 152
6.3 The Prefacing Method and Introduction of Vacancies ..... 156
6.4 The Potts Model ..... 161
6.5 Further Applications of the Vacancy ..... 165
6.6 Summary ..... 167
References ..... 167

7. **Position-Space Renormalization Group for Models of Linear Polymers, Branched Polymers, and Gels.** By H.E. Stanley, P.J. Reynolds, S. Redner and F. Family (With 15 Figures)
7.1 Three Physical Systems ..... 169
7.1.1 Linear Polymers ..... 169
7.1.2 Branched Polymers ..... 171
7.1.3 Gels ..... 172
7.2 Three Mathematical Models ..... 174
7.2.1 Percolation ..... 174

7.2.2 Self-Avoiding Walks ..... 177
7.2.3 Lattice Animals ..... 178
7.3 Position-Space Renormalization Group Treatment ..... 180
7.3.1 Percolation ..... 180
a) Basic Approach ..... 180
b) Extensions ..... 183
7.3.2 Self-Avoiding Walks ..... 190
a) Basic Approach ..... 190
b) Extensions ..... 193
7.3.3 Lattice Animals ..... 195
a) Basic Approach ..... 195
b) Extensions ..... 196
7.4 Other Approaches ..... 200
7.4.1 Percolation ..... 200
7.4.2 Self-Avoiding Walks ..... 201
7.4.3 Lattice Animals ..... 201
7.5 Concluding Remarks and Outlook ..... 202
References ..... 203

**Subject Index** ..... 207

# 1. Progress and Problems in Real-Space Renormalization

T. W. Burkhardt and J. M. J. van Leeuwen

With 5 Figures

The real-space renormalization approach to statistical mechanics is reviewed. A brief account of progress in the field is given. Some fundamental difficulties with the approach are discussed, in particular the proliferation of interactions, the ambiguity in the choice of the weight function, and the peculiarities in the thermodynamic limit noted by GRIFFITHS and PEARCE. Two methods which avoid the problem of proliferation of interactions are considered in some detail: differential real-space renormalization and the phenomenological renormalization related to finite-size scaling.

## 1.1 Introduction

Following WILSON's first application of renormalization-group ideas to critical phenomena, there has been an enormous activity in the field [1.1-9]. Renormalization techniques in both momentum and position space have been developed. In the momentum-space or field-theoretical approach, critical exponents and other universal quantities have been calculated in expansions in $\varepsilon = d - d^*$, where $d$ is the dimension of interest and $d^*$ is the upper or lower critical dimension. The $\varepsilon$ expansion has lead to extremely precise estimates [1.10] of the critical exponents of several three-dimensional systems. Momentum-space renormalization-group techniques have also been applied to dynamic critical phenomena [1.11]. The starting point in a momentum-space renormalization-group calculation is a coarse-grained Hamiltonian in which details irrelevant to the universal critical behavior have been eliminated. This Hamiltonian is a functional of the order-parameter field. Renormalization results from an elimination of the short-wavelength Fourier components of the field and subsequent rescaling of lengths and of the field.

Position or real-space renormalization deals directly with the microscopic Hamiltonian. Most of the applications have considered spins on a lattice. In the renormalization, new spin variables which replace blocks of the original spins are defined and their effective interactions determined. Except in a few special applications the real-space renormalization procedures devised thus far are not exact but rely on approximations lacking an expansion parameter. So far the approach

has been less successful in predicting the critical exponents of three-dimensional systems than the extrapolation procedures incorporating exact results of the $\varepsilon$ expansion. However, for systems, particularly those with discrete spins, in low dimensions where the critical fluctuations are strong and the $\varepsilon$ expansion not very useful, real-space renormalization is one of the most powerful of the calculational techniques of comparable simplicity currently available. As the real-space renormalization procedures are directly applicable to microscopic models of spins on a lattice with no prior coarse graining, one can calculate nonuniversal as well as universal quantities, for example, the phase diagram as a function of the interaction parameters, or the specific heat as a function of the temperature. Real-space renormalization also provides a natural description of a first-order transition [1.12, 13]. The approach has an intuitive simplicity and can often be conveniently tailored to a particular application.

This book is primarily devoted to developments in real-space renormalization which have taken place since the preparation of the 1976 review article [1.13] in the Domb-Green series. The reader seeking a detailed introduction to the subject is referred to [1.13]. Section 1.2 of this chapter contains a brief review of the real-space renormalization approach. Sections 1.3,4 summarize some of the more significant recent developments in the field not covered in [1.13]. Certain of these developments are treated in detail in other contributions to this volume. In Sect.1.5 some of the fundamental inherent difficulties in real-space renormalization are discussed, in particular the proliferation of interactions which results from the renormalization operation, the problem of choosing a suitable weight function, and the peculiarities in the thermodynamic limit pointed out by GRIFFITHS and PEARCE [1.14-16]. Sections [1.6,7] describe two successful procedures that manage to avoid these problems. In Sect.1.6 exact differential renormalization transformations [1.17-22] are considered. An exact differential transformation has been devised for the two-dimensional Ising model with nearest-neighbor interactions on a triangular lattice [1.17]. No additional couplings are generated by the renormalization, but the couplings become spatially dependent. Section 1.7 contains a discussion of "phenomenological renormalization" [1.23], a type of finite-size scaling analysis that bears some resemblance to real-space renormalization. No proliferation of interactions occurs in phenomenological renormalization either. The approximation can be systematically improved and the convergence of the results observed. Section 1.8, which completes the chapter, contains concluding remarks.

## 1.2 Review of Real-Space Renormalization

Real-space renormalization methods implement an intuitive picture, proposed by KADANOFF [1.24] several years before the renormalization-group breakthrough, to explain certain universal aspects of critical behavior. In this picture, cells of spins in a nearly critical system behave like individual spins in a system somewhat farther from criticality. A possible assignment of spins into cells is shown in Fig.1.1. The dots represent the original or site spins, which we denote by $\{s\}$ and the crosses the cell spins $\{s'\}$. The central thermodynamic quantity of interest is the free energy

$$f = \lim_{N\to\infty} (1/N) \ln Z \quad , \tag{1.1}$$

where N is the number of s spins and the partition function Z is defined by

$$Z = \sum_{s} \exp H(s) \quad . \tag{1.2}$$

H(s) is the dimensionless Hamiltonian (in which a factor $-(k_B T)^{-1}$ has been incorporated) of the system of s spins. The sum in (1.2) is over all configurations of the s spins.

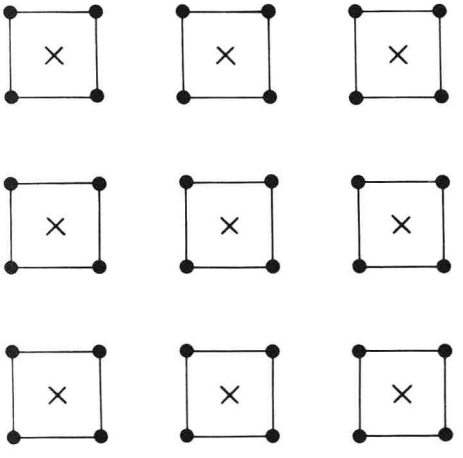

Fig.1.1. Possible assignment of spins to cells. The dots represent site spins and the crosses cell spins. There are four sites per cell

Adding an arbitrary spin-independent constant to H(s) changes the free energy by an additive constant. It is convenient to choose a convention such as $\sum_s H(s) = 0$ or $H = 0$ for a particular spin configuration in order to define the Hamiltonian H(s) uniquely.

In real-space renormalization the interactions between the cell spins are calculated from the interactions between the site spins. A Hamiltonian $G + H'(s')$ for the cell spins (where G is independent of the spin configurations and the zeroes of $H'(s')$ and H(s) are fixed according one of the conventions mentioned above) is

defined by the renormalization transformation

$$\exp[G + H'(s')] = \sum_{s} P(s',s) \exp H(s) \quad . \tag{1.3}$$

The weight function $P(s',s)$, which couples the cell and site spins, satisfies

$$\sum_{s'} P(s',s) = 1 \quad , \tag{1.4}$$

which insures that the partition functions of the cell and site-spin systems are the same. The condition (1.4) is not very restrictive, and a variety of weight functions [1.13] have been tried. Generally (some exceptions are noted below), the transformation is designed so that the s' and s spins are spins of the same type, for example, in applications to the Ising model both the s' and s spins take the values ±1. Evaluating the sum over configurations in (1.3) reduces the number of spins and enlarges the lattice constant. In momentum-space renormalization [1.1-9] degrees of freedom are eliminated and the length scale expanded by integrating the Bolztmann factor exp H of the coarse-grained Hamiltonian over short-wavelength fluctuations of the spin variables.

It is useful to expand $H(s)$ in the form $H(s) = \sum_\alpha K_\alpha \Omega_\alpha(s)$, where the $K_\alpha$ are coupling constants and the $\Omega_\alpha(s)$ a complete set of interactions generated by repeated application of (1.3) from the initial Hamiltonian of interest. Because of the proliferation of interactions under the renormalization operation, the set generally contains an infinite number of elements, but in approximate calculations only a finite subset is taken into account. H' can also be expanded in terms of the $\Omega_\alpha$, with coupling constants $K'_\alpha$. Equation (1. 3) implies a set of equations

$$\underline{K}' = \underline{K}'(\underline{K}) \tag{1.5}$$

relating the coupling constants of the original and renormalized spins. Here $\underline{K}$ and $\underline{K}'$ denote the entire sets of $K_\alpha$ and $K'_\alpha$. The transformation law

$$f(\underline{K}) = g(\underline{K}) + \frac{1}{b^d} f(\underline{K}') \tag{1.6}$$

for the free energy per spin also follows from (1.3,4). In deriving (1.6), one has written $g = G/N$ and $b^d = N/N'$, b being the length rescaling factor, and the thermodynamic limit is implicitly taken to be $N \to \infty$. From (1.6) one sees that $g(\underline{K})$ is the contribution to the free energy from the degrees of freedom eliminated in the renormalization step.

Once the functions $\underline{K}'(\underline{K})$ and $g(\underline{K})$ are known, the free energy may be calculated with the formula

$$f(K) = \sum_{n=0}^{\infty} \frac{1}{b^{nd}} g(\underline{K}^{(n)}) + \lim_{m\to\infty} \frac{1}{b^{md}} f(\underline{K}^{(m)}) \quad , \tag{1.7}$$

obtained by iterating (1.5,6). $\underline{K}^{(n)}$ in (1.7) denotes the set of coupling constants after n iterations. In the renormalization-group approach [1.1-9,13] one can cal-

此为试读,需要完整PDF请访问: www.ertongbook.com