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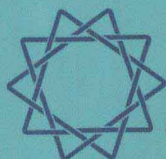
Volume 1

Edited by

Zoé Chatzidakis, Dugald Macpherson, Anand Pillay
and Alex Wilkie

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Model Theory with Applications to Algebra and Analysis

Volume 1

ZOÉ CHATZIDAKIS
CNRS – Université Paris 7

DUGALD MACPHERSON
University of Leeds

ANAND PILLAY
University of Leeds

ALEX WILKIE
University of Manchester



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Managing Editor: Professor N.J. Hitchin, Mathematical Institute,
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Preface

These two volumes contain both expository and research papers in the general area of model theory and its applications to algebra and analysis. The volumes grew out of the semester on “Model Theory and Applications to Algebra and Analysis” which took place at the Isaac Newton Institute (INI), Cambridge, from January to July 2005. We, the editors, were also the organizers of the programme. The contributors have been selected from among the participants and their papers reflect many of the achievements and advances obtained during the programme. Also some of the expository papers are based on tutorials given at the March–April 2005 training workshop. We take this opportunity, both as editors of these volumes and organizers of the MAA programme, to thank the Isaac Newton Institute and its staff for supporting our programme and providing a perfect environment for mathematical research and collaboration.

The INI semester saw activity and progress in essentially all areas on the “applied” side of model theory: o-minimality, motivic integration, groups of finite Morley rank, and connections with number theory and geometry. With the exception of motivic integration and valued fields, these topics are well represented in the two volumes.

The collection of papers is more or less divided into (overlapping) themes, together with a few singularities. Aspects of the interaction between stability theory, differential and difference equations, and number theory, appear in the first six papers of volume I. The first paper, based on Pillay’s workshop tutorial, can also serve as a fast introduction to model theory for the general reader, although it quickly moves to an account of Mordell–Lang for function fields in characteristic 0. The

“arithmetic of differential equations” figures strongly in Pillay’s paper on the Grothendieck-Katz conjecture and its nonlinear generalizations, as well as in Bertrand’s paper which initiates the investigation of versions of Ax-Schanuel for nonisoconstant semiabelian varieties over function fields. The Galois theory of difference equations is rather a hot topic and the Chatzidakis-Hardouin-Singer paper compares definitions and concepts that have arisen in algebra, analysis, and model theory.

Interactions of complex analytic geometry with model theory and logic (in the form of stability, o-minimality, as well as decidability issues) appear in papers 7 to 10 of volume 1. The papers by Peterzil-Starchenko and Moosa-Pillay (on nonstandard complex analysis and compact Kähler manifolds respectively) are comprehensive accounts of important projects, which contain new results and set the stage for future research. In the first, o-minimality is the model-theoretic tool. In the second it is stability. Wilkie’s paper characterizes the holomorphic functions locally definable from a given family of holomorphic functions, and Macintyre’s paper is related to his work on the decidability of Weierstrass functions. They are both set in the o-minimal context.

The o-minimality theme is continued in papers 12 and 13 of volume 1 from a (real) geometric point of view. In particular Rolin’s paper is a comprehensive account of the most modern techniques of finding o-minimal expansions of the real field.

In recent years Zilber has been exploring connections between model theory and noncommutative geometry, and in his paper in volume I he succeeds in interpreting certain “quantum algebras” as Zariski structures. Fesenko’s short note contains a wealth of speculations and questions, including the use of nonstandard methods in noncommutative geometry.

Definable groups of “finite dimension” in various senses (finite Morley rank, finite SU-rank, o-minimal) figure strongly in papers 1 to 5 of volume II. Papers 1 and 2 contain new and striking general results on groups of finite Morley rank, coming out of techniques and results developed in work on the Cherlin-Zilber conjecture. Paper 3 gives an overview of a model-theoretic approach to asymptotics and measure stimulated by the analogous results and concepts for finite and pseudofinite fields. The article by Hrushovski and Wagner, on the size of the intersection of a finite subgroup of an algebraic group with a subvariety, generalizes a theorem of Pink and Larsen. Otero’s paper gives a comprehensive

description of work since the 1980's on groups definable in o-minimal structures. This includes an account of the positive solution to "Pillay's conjecture" on definably compact groups which was proved during the Newton semester.

Hilbert's 10th problem and its generalizations, as well as first order properties of function fields, appear in papers 6 to 8 of volume II. The Pheidas-Zahidi and Eistenträger papers are based on tutorials given at INI, and give a comprehensive account of work on Hilbert's 10th problem for the rational field and for various rings and fields of functions. Paper 8 proves among other things definability of the constant field in function fields whose constant field is "large". The three papers together give a good picture of an exciting and very active subject at the intersection of logic and number theory.

The volumes are rounded off by important papers on Hrushovski constructions, ordered abelian groups, and continuous logic. In particular the paper 10 in volume II (based again on a tutorial) is an elementary and self-contained presentation of "continuous logic" or the "model theory of metric structures" which is fast becoming an autonomous area of model theory with links to both stability and functional analysis.

Zoé Chatzidakis
 Dugald Macpherson
 Anand Pillay
 Alex Wilkie

Contributors

Itai Ben Yaacov
Institut Camille Jordan
Université Claude Bernard (Lyon-1)
43 boulevard du 11 novembre 1918
69622 Villeurbanne cédex
France

Alexander Berenstein
Departamento de Matematicas
Universidad de los Andes
Carrera 1 Nro 18A-10
Bogotá
Colombia

Daniel Bertrand
Institut de Mathématiques de Jussieu
Université Paris 6
Boite 247
4 place Jussieu
75252 Paris cedex 05
France

Alexandre Borovik
School of Mathematics
University of Manchester
Oxford Road
Manchester M13 9PL
UK

Zoé Chatzidakis
UFR de Mathématiques
Université Paris 7 - Case 7012
2 place Jussieu
75251 Paris cedex 05
France.

Gregory Cherlin
Department of Mathematics
Rutgers University
110 Frelinghuysen Rd
Piscataway, NJ 08854
USA

Kirsten Eisenträger
Department of Mathematics
The Pennsylvania State University
109 McAllister Building
University Park, PA 16802
USA

Richard Elwes
School of Mathematics
University of Leeds
Leeds LS2 9JT
UK

Ivan Fesenko
Department of Mathematics
University of Nottingham
Nottingham NG7 2RD
UK

Olivier Frécon
Laboratoire de Mathématiques et Ap-
plications
Université de Poitiers
Téléport 2 - BP 30179
Boulevard Marie et Pierre Curie
86962 Futuroscope Chasseneuil cedex
France

Charlotte Hardouin
IWR
Im Neuenheimer Feld 368
69120 Heidelberg
Germany

C. Ward Henson
Department of Mathematics
University of Illinois
1409 W. Green St.
Urbana, IL 61801
USA

Ehud Hrushovski
Einstein Institute of Mathematics
The Hebrew University of Jerusalem
Jerusalem 91904
Israel

Eric Jaligot
Institut Camille Jordan
Université Claude Bernard (Lyon-1)
43 boulevard du 11 novembre 1918
69622 Villeurbanne cédex
France

Angus Macintyre
School of Mathematics
Queen Mary, University of London
Mile End Road
London E1 4NS
UK

Dugald Macpherson
School of Mathematics
University of Leeds
Leeds LS2 9JT
UK

Rahim Moosa
Department of Pure Mathematics
200 University Avenue West
Waterloo, Ontario N2L 3G1
Canada

Margarita Otero
Departamento de Matemáticas
Universidad Autónoma de Madrid
28049 Madrid
Spain

Ya'acov Peterzil
Department of Mathematics
University of Haifa
Haifa
Israel

Thanases Pheidas
Department of Mathematics
University of Crete
Knossos Avenue
71409 Iraklio, Crete
Greece

Anand Pillay
School of Mathematics
University of Leeds
Leeds LS2 9JT
England

Bjorn Poonen
Department of Mathematics
University of California
Berkeley, CA 94720-3840
USA

Florian Pop
Department of Mathematics
University of Pennsylvania, DRL
209 S 33rd Street
Philadelphia, PA 19104
USA

Jean-Philippe Rolin
Université de Bourgogne
I.M.B.
9. Avenue Alain Savary
BP 47870
21078 Dijon Cedex
France

Damian Rössler
Institut de Mathématiques de Jussieu
Université Paris 7 Denis Diderot
Case Postale 7012
2, place Jussieu
F-75251 Paris Cedex 05
France

Thomas Scanlon
University of California, Berkeley
Department of Mathematics
Evans Hall
Berkeley, CA 94720-3840
USA

Philip Scowcroft
Department of Mathematics
and Computer Science
Wesleyan University
Middletown, CT 06459
USA

Michael F. Singer
North Carolina State University
Department of Mathematics
Box 8205
Raleigh, North Carolina 27695-8205
USA

Sergei Starchenko
Department of Mathematics
University of Notre Dame
Notre Dame, IN 46556
USA

Alexander Usvyatsov
UCLA Mathematics Department
Box 951555
Los Angeles, CA 90095-1555
USA

Frank Wagner
Institut Camille Jordan
Université Claude Bernard (Lyon-1)
43 boulevard du 11 novembre 1918
69622 Villeurbanne cedex
France

A.J. Wilkie
School of Mathematics
Alan Turing Building
The University of Manchester
Manchester M13 9PL
UK

Karim Zahidi
Dept of Mathematics, statistics and
actuarial science
University of Antwerp
Prinsenstraat 13
B-2000 Antwerpen
Belgium

Martin Ziegler
Mathematisches Institut
Albert-Ludwigs-Universität Freiburg
D79104 Freiburg
Germany

B. Zilber
Mathematical Institute
24 - 29 St. Giles
Oxford OX1 3LB
UK

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Model theory and stability theory, with applications in differential algebra and algebraic geometry

Anand Pillay[†]

University of Leeds

This article is based around parts of the tutorial given by E. Bouscaren and A. Pillay at the training workshop at the Isaac Newton Institute, March 29 - April 8, 2005. The material is treated in an informal and free-ranging manner. We begin at an elementary level with an introduction to model theory for the non logician, but the level increases throughout, and towards the end of the article some familiarity with algebraic geometry is assumed. We will give some general references now rather than in the body of the article. For model theory, the beginnings of stability theory, and even material on differential fields, we recommend [5] and [8]. For more advanced stability theory, we recommend [6]. For the elements of algebraic geometry see [10], and for differential algebra see [2] and [9]. The material in section 5 is in the style of [7]. The volume [1] also has a self-contained exhaustive treatment of many of the topics discussed in the present article, such as stability, ω -stable groups, differential fields in all characteristics, algebraic geometry, and abelian varieties.

1 Model theory

From one point of view model theory operates at a somewhat naive level: that of point-sets, namely (definable) subsets X of a fixed universe M and its Cartesian powers $M \times \cdots \times M$. But some subtlety is introduced by the fact that the universe M is “movable”, namely can be replaced by an elementary extension M' , so a definable set should be thought of more as a functor.

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Subtlety or no subtlety, model theory operates at a quite high level of generality.

A (1-*sorted*) structure M is simply a set (also called M) together with a fixed collection of distinguished relations (subsets of $M \times \cdots \times M$) and distinguished functions (from $M \times \cdots \times M$ to M). We always include the diagonal $\{(x, x) : x \in M\} \subset M \times M$ among the distinguished relations. (Example: Any group, ring, lattice, ... is a structure under the natural choices for the distinguished relations/functions.) These distinguished relations/functions are sometimes called the *primitives* of the structure M . From the collection of primitives, one constructs using the operations composition, finite unions and intersections, complementation, Cartesian product, and projection, the class of \emptyset -*definable* sets and functions of the structure M . Let us call this class $Def_0(M)$, which should be seen as a natural “category” associated to the structure M : the objects of $Def_0(M)$ are the \emptyset -definable sets (certain subsets of $M \times \cdots \times M$) and the morphisms are \emptyset -definable functions $f : X \rightarrow Y$ (i.e. $graph(f)$ is \emptyset -definable). The category $Def(M)$ of definable (with parameters) sets in M is obtained from $Def_0(M)$ by allowing also fibres of \emptyset -definable functions as objects: if $f : X \rightarrow Y$ is in $Def_0(M)$ and $b \in Y$ then $f^{-1}(b)$ is a definable set (defined with parameter b). For A a subset of the (underlying set of M) $Def_A(M)$ denotes the category of definable sets in M which are *defined over* A , namely defined with parameter which is a tuple of elements of A . By convention, by a definable set we mean a set definably possibly with parameters. By a uniformly definable family of definable sets we mean the family of fibres of a definable map $f : X \rightarrow Y$.

We give a couple of examples.

The reals.

Consider the structure consisting of \mathbb{R} with primitives $0, 1, +, -, \cdot$. Then the natural total ordering on \mathbb{R} is a 0-definable set, being the projection on the first two coordinates of $\{(x, y, z) \in \mathbb{R}^3 : y - x = z^2 \text{ and } x \neq y\}$. Tarski’s “quantifier elimination” theorem says that the definable sets in $(\mathbb{R}, 0, 1, +, -, \cdot)$ are precisely the *semialgebraic sets*, namely finite unions of subsets of \mathbb{R}^n of the form

$$\{x \in \mathbb{R}^n : f(x) = 0 \text{ and } g_1(x) > 0 \text{ and } \dots g_r(x) = 0\}$$

where f and the g_i are polynomials over \mathbb{R} .