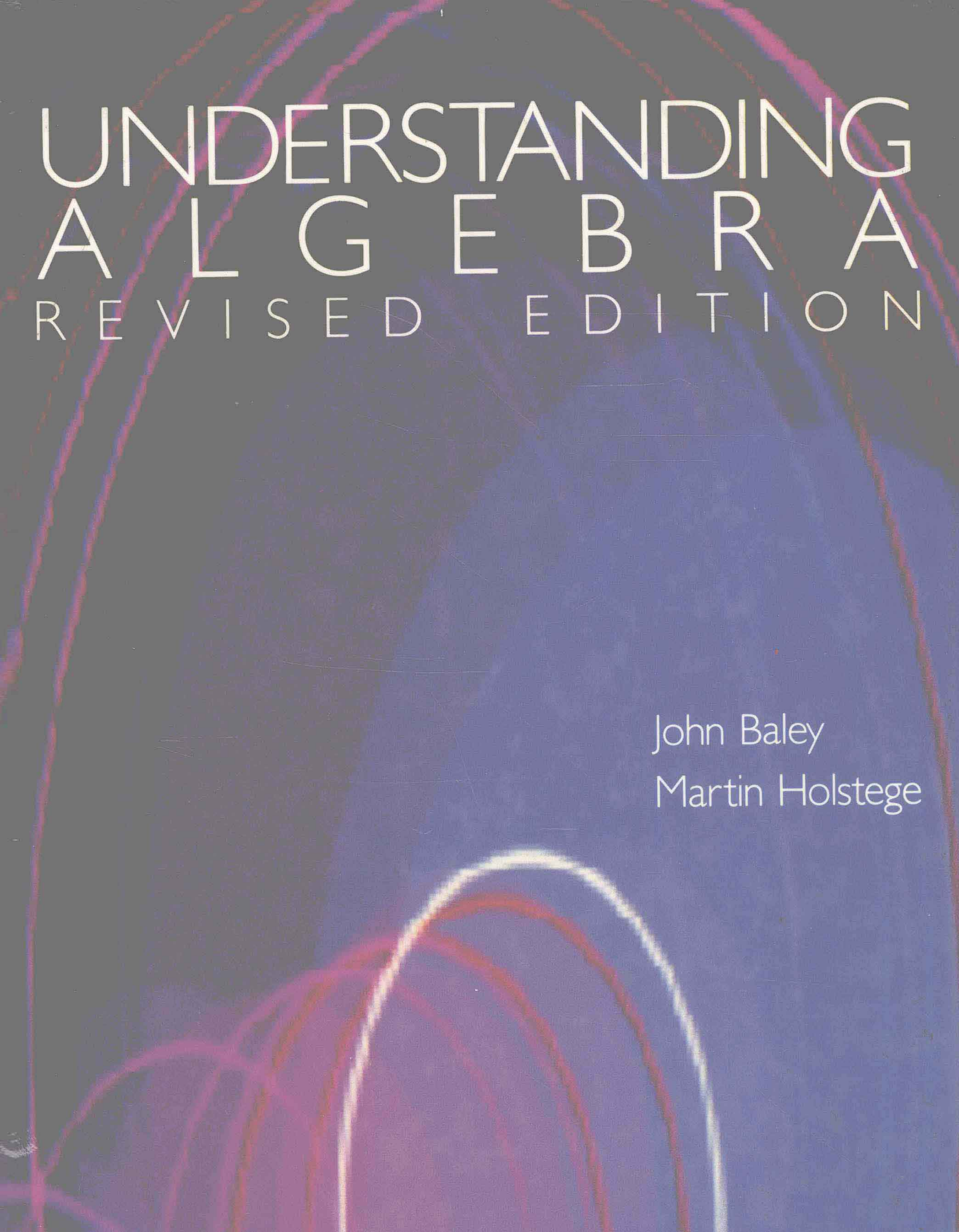

UNDERSTANDING ALGEBRA

REVISED EDITION

John Baley
Martin Holstege

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Cerritos College

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UNDERSTANDING ALGEBRA

Preface

This text is an in-depth second course in algebra. The book was designed to be understood and used by students. Although the development of algebra starts on page one, we, the authors, realize that many students may have completed the prerequisite course in elementary algebra a few years ago. Therefore, a review of elementary algebra has been blended into the early chapters of the text. This book is designed to develop students' skills with algebra so that they will be thoroughly prepared to continue their study of mathematics and science.

Features: Organization and Pedagogy

This book is divided into sixteen chapters to allow instructors greater flexibility in arranging course outlines to conform to academic calendars and student needs.

Each chapter includes:

Preview: This gives the student an overview of how the chapter relates to previous chapters and how the coming chapter will be developed.

Sections: Each chapter is divided into sections of material that approximate one hour of classroom lecture time.

Numerous Examples: Over 400 examples illustrated with graphics demonstrate the concepts presented in the text. Students are given one or more examples of each task they are expected to perform.

Ample Exercises: Over 4,800 problems, usually in matched odd-even sets, give the student the opportunity to apply the concepts and practice the skills taught in the text. Problem sets are a critical part of the learning process because they not only give the student needed practice, but they also show the student the results of subtle variations. By doing problems a student gets an opportunity to internalize mathematics and see the effects of changes in a parameter.

Applications: Whenever possible, this text uses applications in examples and problems to show students that algebra is a powerful branch of mathematics that is used in engineering, electronics, geology, optics, aviation, surveying, construction, forestry, navigation, and physics.

Key Ideas: Each chapter concludes with a summary of key ideas to aid students in organizing their knowledge and help them prepare for exams.

Review Tests: To further insure students that they have mastered the concepts of each chapter, every chapter has a review test that closely approximates the questions that are likely to be asked on an exam.

Special features of this text

Cumulative Reviews: There are cumulative reviews after Chapter 8 and at the end of the book. These reviews are an excellent opportunity for students to get an overview of the course as they study for midterm and final exams.

Conversational Bubbles: These bubbles are spread throughout the book but generally appear in the context of a worked example. Teachers will quickly recognize that these bubbles anticipate and verbalize the questions that students are likely to ask as they are learning the material. Answering bubbles provide the answers that experienced teachers are likely to give.

Highlighted Definitions, Properties, Theorems, and Rules: These features, which are essential to remember and understand, are highlighted in boxes throughout the text, both to draw the students' attention to important concepts and to make it easy for students to reference key ideas.

Pointers for Better Understanding: Fifteen special boxes are spread throughout the text to clarify ideas or give students helpful hints about how to write or visualize certain algebra problems. These pointers deal with many ideas that are needed to work algebra but are frequently never explicitly taught to students. How to write equations, how to deal with denominate numbers, and why you can't divide by zero are examples of this kind of pointer.

Help with Problem Solving: This book makes a special effort to help students develop the skills and understanding needed to apply algebra to practical situations. Throughout the text, it helps the students to:

a. develop skills to express in precise mathematical terms ideas that are imprecisely expressed in English;
b. critically analyze a problem and identify both relevant and irrelevant pieces of information in the problem;
c. build a knowledge-base of formulas that can be applied in this book and throughout life;
d. visualize or draw mental pictures based on information given in a problem.

Sections 1.5, 2.5, 4.2, 7.5, 10.4, and 14.4 are six complete sections devoted to building problem-solving skills. There are also six Pointers for Problem Solving throughout the book that help students build skills of problem organization (p. 172), chosing variables (p. 158), defining exactly what a variable represents (p. 163), model making (p. 166), using variables (p. 406), and visualization (p. 407).

Using Your Calculator: This is a series of special features that show students how to use a scientific calculator efficiently to solve problems. Each calculator feature tells students what they need to know, when they need to know it.

Logarithms: Logarithms are approached with a recognition that scientific calculators exist and are readily available. This approach both reduces the arithmetic involved and allows the student to concentrate on logarithms as functions. This leaves time to introduce many interesting applications of logarithms.

Class Testing

This book has been class tested through five revisions by over 1,500 students in both lecture and semi-independent mathematics classes. The authors are grateful to Professor Ray Battee and Sister Pat Peach, who helped with the class testing of the book. They are also thankful to the many students and instructional aides who provided useful feedback, which helped to improve this text.

Ancillaries

Student Solution Manual

A student solution manual with all the even-numbered problems worked step by step is available. This manual also provides additional hints and explanations about how the problems were solved.

Instructor's Materials

An instructor's printed test manual provides four forms of each chapter test, a midterm exam, a final exam, and suggested tests for Chapters 1–3, 4–6, 7–9, 10–12, and 13–15 to assist teachers who wish to cover multiple chapters on each test. In addition, a separate manual is available that contains answers to all the exercises.

Acknowledgments

The authors wish to thank Debra Pentecost for the level of care and attention she gave to preparing the preliminary editions for class testing. Her efforts have made the development of this book a much more pleasant task for all concerned.

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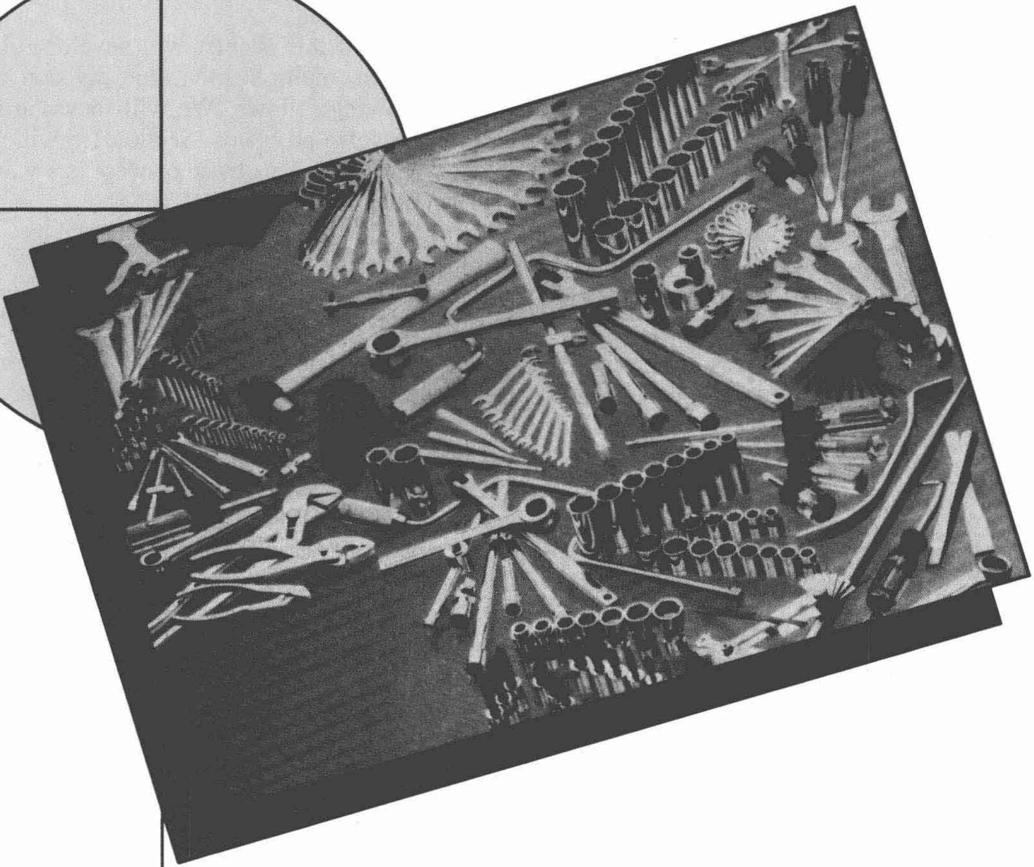
1 Real Numbers and Their Properties



Contents

Preview

As you might expect, algebra is a series of ideas or concepts built into a system of knowledge. Like any system of knowledge it has a language to convey the ideas. It also has its own special vocabulary.

This chapter will start by building your algebra vocabulary with a series of definitions. Then it will list the properties of a mathematical system. All of algebra and most of mathematics are based in these definitions and properties.

Naturally the authors want you to treat them as important. However, it is our experience that students who have difficulty with this unit are trying to read too much into it. Try to accept the first two sections of this chapter just as formal statements of common sense.

The third and fourth sections of this chapter will develop rules for the operations of addition, subtraction, multiplication, and division as used in algebra. A study of problem solving follows. We will focus on *reasoning* rather than on *procedures* as we solve problems. Section 1.5 will show how to find the amount per unit of measure. This basic concept is a valuable tool in solving a wide range of problems.

■ 1.1 Introduction

Any system of knowledge is based upon certain undefined terms. These undefined terms are used to define other ideas in the system. A good system of knowledge has as few undefined terms as possible. One way to study algebra starts with the idea of a set. A **set** is simply a collection of objects. The objects in a set are called **elements.**

There must be some way to tell if a given element is part of the set. One way to determine if an element is in a set is to list all the members of the set. In mathematics, we frequently refer to sets of numbers. Sometimes it is convenient to list the members of the set. In such a listing the elements of the set are usually enclosed in braces. An example is the set containing the letters A, B, and C.

$$\{A, B, C\}$$

1.1A Set Membership

The symbol $\in$ means "is an element of" and $\notin$ means "is not an element of."

1.1B Equal Sets

Set A is **equal to** B if both sets contain the same elements.

If $A = \{1, 2, 3\}$ and $B = \{3, 2, 1\}$ then $A = B$ since $\{1, 2, 3\} = \{3, 2, 1\}$. The order of the elements in a set is not important.

Example 1 ☐ If $A = \{5, 10, 15, 20\}$, use $\in$ or $\notin$ to indicate membership in set A.

$\notin$ $\quad 10 \in A \qquad 3$ ______ A

$\notin, \in$ $\quad 7$ ______ $A \qquad 15$ ______ A ☐

What am I supposed to do with the blanks?

Fill them in. Then check the left-hand column to be sure your answer was correct.

Why not just copy the answers, or better yet have the authors fill in the blanks?

Because learning works best if you're involved. That's why you should think of an answer before you look to the left to see if we agree with you. It doesn't work as well if you merely look in the left column and agree with us.

1.1C Definition: Variable

A variable is a symbol that can represent any element of a set of numbers.

The basic set of numbers we deal with is the set of natural numbers. These are the numbers that are used as a child learns to count.

1.1D Definition: Natural Numbers

The set of natural numbers is the set of counting numbers. They are

$$N = \{1, 2, 3, 4, 5, \ldots\}$$

(. . . means *continue with this pattern.*)

This set of numbers is also called the **counting numbers.**

1.1E Definition: Closure

A set is said to be closed under an operation if performing the operation with any two members of the set always yields a member of the set.

The set of counting numbers is closed under addition because the sum of any two counting numbers is another counting number.

Example 2 ☐ $3 + 5 = 8$ ☐

Both are counting numbers.

The result is a counting number.

Am I missing something? I don't see any big deal.

Like many mathematical ideas, closure is really a very simple concept. Don't try to make it complicated. Do notice that a set can be closed for one operation and not closed for another. Read on.

To show that the set of natural numbers is not closed for subtraction we need only give one counterexample. The answer to $3 - 5$ is not contained in the set of natural numbers. Therefore, the natural numbers are not closed for subtraction.

That means it is not always possible to do subtraction using only natural numbers.

Subtraction is always possible if you subtract the smaller number from the larger.

True, but you're imposing an extra condition. For a set to be self-contained or closed under subtraction you must be able to subtract any number from any other number in the set and get an answer that is in the set.

To define a set of numbers that is closed for subtraction, it is necessary to make a few more definitions.

1.1F Definition: Identity Element for Addition

Zero is the identity element for addition. That is, if you add zero to any number, the result is identical to the original number.

$$4 + 0 = 4 \qquad 0 + 9 = 9$$

Zero isn't a natural number.

True. That's why we have to define whole numbers.

1.1G Definition: Whole Numbers

The set of whole numbers is the set of natural numbers combined with the number zero. They are

$$W = \{0, 1, 2, 3, 4, \ldots\}$$

1.1H Definition: Negative Number

Every natural number n has a negative, $-n$, such that

$$n + (-n) = 0$$

For example,

$$3 + (-3) = 0 \qquad -2 + 2 = 0$$

The negative of a number can be viewed as the opposite of the number. Taking two steps forward can be represented as 2. Taking two steps backward can be represented as -2.

Example 3 □ Give the number you would use to represent each of the following.

−3 Going down three floors in an elevator _______

+15 A gain of 15 yards in football _______

−20 A debt of twenty dollars _______

0 The distance covered by a car that doesn't move _______

−6 The position of a car that backed up 6 feet _______

0 The location of a car that backed up 50 feet then drove forward 50 feet _______ □

The opposite of any number is called its **additive inverse.**

1.1I Definition: Additive Inverse

If two numbers have a sum of zero, they are called **additive inverses of each other.**

$$\underbrace{3 + (-3)}_{\text{Additive inverses}} = 0 \qquad \underbrace{-3 + 3}_{\text{Additive inverses}} = 0$$

Example 4 □ Write the additive inverse of each number below.

−8 The additive inverse of 8 is _______

8 The additive inverse of -8 is _______

−5 The additive inverse of 5 is _______

$-a$ The additive inverse of a is _______

y The additive inverse of $-y$ is _______

0 The additive inverse of 0 is _______ □