
THIRD EDITION

UNDERSTANDING
SYMBOLIC
LOGIC

VIRGINIA KLENK

Third Edition

***Understanding
Symbolic Logic***

VIRGINIA KLENK

West Virginia University



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Understanding Symbolic Logic

**To my mother,
Helen Crooker Klenk**

Preface

This book is intended as a comprehensive introduction to symbolic logic. It presupposes no prior acquaintance with either logic or mathematics, and it includes all the standard topics through relational predicate logic with identity. The book was written in the conviction that any student can master symbolic logic, and it is designed to give the student as much help as possible in attaining that mastery.

The main part of the book is divided into twenty units, each of which has an introduction and a statement of study objectives so that the student has an overview of what is to come and knows exactly what is required in order to master the unit. The explanatory material for each unit is divided into several subsections, each of which has a specific function and covers one relatively small, clearly defined topic. The clear separation of topics and the division into easily comprehended small “bites” allow the student to master the material step by step without being overwhelmed by an indigestible mass of information.

One-variable predicate logic is developed, in detail, independently of relational predicate logic, and identity is presented in two separate units. The semantics of predicate logic is also developed in a separate unit, as is the semantics for propositional logic. In addition to the basic material,

there are several “extra credit” units, which provide a glimpse into alternative methods of logic and more advanced topics.

I have tried to give as detailed explanations as possible, both for specific techniques, such as drawing up truth tables or constructing proofs, and for the rationale behind these techniques. It seems to me as important for a student to understand *why* things are done in a certain way as to learn the techniques themselves, and in this book I have tried to supply the “why’s” as well as the “how’s.”

The book does, however, supply the “how’s” in abundance. Aside from the detailed explanations, there are numerous examples worked out in the text: various types of truth tables, a great many detailed, step-by-step symbolizations, and over fifty fully worked out proofs. In addition, there are copious exercises, with answers to fully half of these provided at the back of the book. Problems for which answers are given are indicated by stars.

Because of the detailed explanations, the extensive coverage, and the clear division of topics, the book is extremely flexible. It can be used in either freshman courses or upper-division courses and is suitable for quarter, semester, or even two-quarter courses. In one quarter, for instance, one might cover just Units 1 through 14; in a semester course, Units 1 through 15, 17, and 18; and in a two-quarter course one might cover the entire book, including the supplementary units. Because of the step-by-step approach and the numerous examples and exercises, the book can also be used in self-paced classes. Suggestions on how to structure such a course are included in the Instructor’s Manual.

A new edition has given me the opportunity to make numerous changes that should clarify and streamline the presentation. In addition to updating examples and exercises, I have provided new or expanded explanations for dozens of topics that students seemed to find puzzling. I have also cleaned up definitions of several terms that were problematic, have added new definition sections in some units, and have made scores of relatively minor changes that significantly clarify the material. The most substantial changes are in Units 10 through 12, where I have eliminated the discussion of the three senses of “is,” which has allowed for a cleaner presentation of the material. I have also rearranged the material in Unit 11 to provide what seems to me a more natural and direct development of the important concepts. I have added many new exercises, including dozens of new proofs in Units 7, 8, and 9.

It is a great pleasure to acknowledge at this point my considerable debts to the many people who helped make this book what it is. My greatest debt, both in general and in particular, is to Nuel D. Belnap, Jr., from whom I absorbed most of what I know about logic and much of my interest in pedagogy. In addition to these general contributions, the rule system for predicate logic is a slightly modified version of one of his systems.

He also read an earlier version of the entire manuscript and made numerous valuable suggestions, most of which have been incorporated here. Without him the book would not have been written, and without his astute commentary it would not have been as useful as I hope it will be.

I would like to thank Nicholas D. Smith, of the Department of Philosophy of Virginia Polytechnic Institute & State University, for the many excellent comments and suggestions he made in reviewing the manuscript. I would also like to thank numerous colleagues at West Virginia University and students at both WVU and the University of Pittsburgh for many valuable suggestions and for catching many misprints and outright errors. I would especially like to thank Shirley Dowdy, Henry Ruf, and Patricia Long for their time and expertise. Special thanks go to Myrtle Dodge and Kellie Zurzolo for their patient, expert, and good-humored typing of a manuscript that must have been at times enormously frustrating. The Lilly Foundation provided partial summer funding during 1977, and I would like to thank that organization, and Gene D'Amour, for the initial impetus for the book. I would also like to thank the West Virginia University Foundation for making possible in-house publication of an earlier, partial version, which encouraged me to complete the project. Finally, I wish to thank the following for their valuable comments when reviewing the manuscript for this third edition: Richard McCarthy, East Carolina University, and Ronald G. Aichele, University of Southern Colorado.

Finally, this book is dedicated to my mother, Helen Crooker Klenk, who always encouraged my interest in formal studies.

VIRGINIA KLENK

Understanding Symbolic Logic

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PART ONE: SENTENTIAL LOGIC

Unit 1

Introduction to Logic

A. INTRODUCTION

If you have never had a course in logic before, you probably have little idea of what it is all about, and even less idea of what is involved in symbolic logic. You may even wonder what the point is of being “logical”; it sometimes gives the impression of a rather cold, dispassionate approach to life and doesn’t really sound terribly appealing. By the end of the course, however, I hope you will have discovered that the study of logic is not only extremely useful, but can also be fun. I think you will find that many of the procedures you will learn here are intrinsically interesting; you may think of them as puzzles or games, and in the process you will be developing your reasoning ability.

What, then, is reasoning ability, and why should you be concerned with developing it? The ability to reason, or infer, is simply the ability to draw appropriate conclusions from given evidence, and this reasoning ability is extremely important in our daily lives because it is the source of most of our knowledge. Most of our knowledge is inferential; that is, it is gained not through direct observation, but by *inferring* one thing from another. A doctor may *observe*, for instance, that a young patient has a