

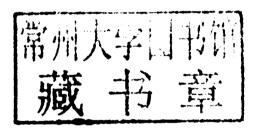


IAIN J. CLARK

# Foreign Exchange Option Pricing

### A Practitioner's Guide

### Iain J. Clark





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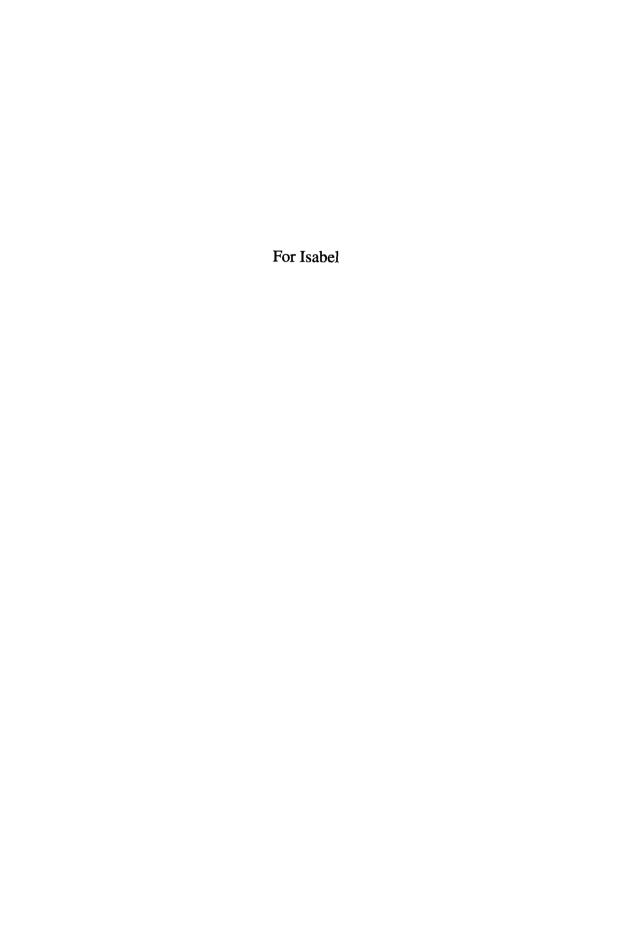
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This book is for you and for all students, young and old, of the mathematical arts. I wish you all the very best with your studies and your work.

#### Web page for this book

www.fxoptionpricing.com

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