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Foreign exchange option pricing

A Practitioner's Guide

IAIN J. CLARK

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Iain J. Clark



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For Isabel

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$$\begin{array}{r} 101\,598\,490 \\ + \underline{21\,858\,299} \end{array}$$

This book is for you and for all students, young and old, of the mathematical arts. I wish you all the very best with your studies and your work.

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