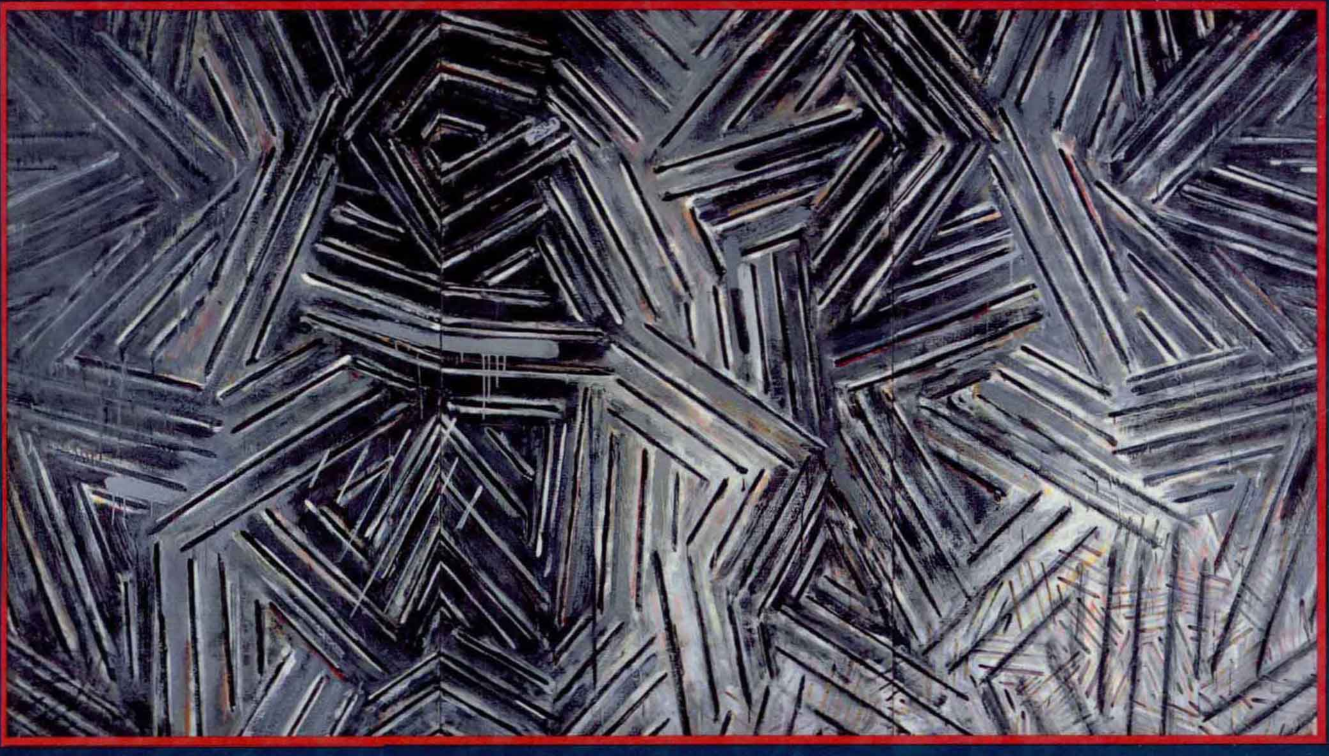
DISCRETE MATHEMATICS AND ITS APPLICATIONS

Third Edition

Kenneth H. Rosen



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PREFACE

In writing this book, I have been guided by my long-standing experience and interest in teaching discrete mathematics. For the student, my purpose was to present material in a precise, readable manner, with the concepts and techniques of discrete mathematics clearly presented and demonstrated. My goal was to show the relevance and practicality of discrete mathematics to students, who are often skeptical. I wanted to give students studying computer science all the mathematical foundations they need for their future studies; I wanted to give mathematics students an understanding of important mathematical concepts together with a sense of why these concepts are important for applications. And I wanted to accomplish these goals without watering down the material.

For the instructor, my purpose was to design a flexible, comprehensive teaching tool using proven pedagogical techniques in mathematics. I wanted to provide instructors with a package of materials that they could use to teach discrete mathematics effectively and efficiently in the most appropriate manner for their particular set of students. I hope that I have achieved these goals.

I have been extremely gratified by the tremendous success of this text. The many improvements in the third edition have been made possible by the feedback and suggestions of a large number of instructors and students at many of the more than 300 schools where this book has been successfully used. There are many enhancements in this edition. The ancillary package has been considerably enriched, making it easier for students and instructors to achieve their goals.

This text is designed for a one- or two-term introductory discrete mathematics course to be taken by students in a wide variety of majors, including mathematics, computer science, and engineering. College algebra is the only prerequisite.

Goals of a Discrete Mathematics Course

A discrete mathematics course has more than one purpose. Students should learn a particular set of mathematical facts and how to apply them; but more importantly, such a course should teach students how to think mathematically. To achieve these goals, this text stresses mathematical reasoning and the different ways problems are solved. Five important themes are interwoven in this text: mathematical reasoning, combinatorial analysis, discrete structures, applications and modeling, and algorithmic thinking. A successful discrete mathematics course should carefully blend and balance all five of these themes.

1. *Mathematical Reasoning:* Students must understand mathematical reasoning in order to read, comprehend, and construct mathematical arguments. This text starts with a discussion of mathematical logic, which serves as the foundation for the subsequent discussions of methods of proof. The technique of mathematical induction is stressed through many different types of examples of such proofs and a careful explanation of why mathematical induction is a valid proof technique.
2. *Combinatorial Analysis:* An important problem-solving skill is the ability to count or enumerate objects. The discussion of enumeration in this book begins with the basic techniques of counting. The stress is on performing combinatorial analysis to solve counting problems, not on applying formulae.
3. *Discrete Structures:* A course in discrete mathematics should teach students how to work with discrete structures, which are the abstract mathematical structures used to represent discrete objects and relationships between these objects. These discrete structures include sets, permutations, relations, graphs, trees, and finite-state machines.
4. *Applications and Modeling:* Discrete mathematics has applications to almost every conceivable area of study. There are many applications to computer science in this text, as well as applications to such diverse areas as chemistry, botany, zoology, linguistics, geography, and business. These applications are natural and important uses of discrete mathematics and are not contrived. Modeling with discrete mathematics is an extremely important problem-solving skill, which students have the opportunity to develop by constructing their own models in some of the exercises in the book.
5. *Algorithmic Thinking:* Certain classes of problems are solved by the specification of an algorithm. After an algorithm has been described, a computer program can be constructed implementing it. The mathematical portions of this activity, which include the specification of the algorithm, the verification that it works properly, and the analysis of the computer memory and time required to perform it, are all covered in this text. Algorithms are described using both English and an easily understood form of pseudocode.

Changes in the Third Edition

The many improvements in this edition are designed to make the book more readable, teachable, flexible, and interesting. However, the basic features of the original edition, as well as the tone and emphases, have been retained. The following are the most noteworthy changes in the third edition.

ENHANCED COVERAGE OF MATHEMATICAL REASONING Many instructors view mathematical reasoning as the central theme of an introductory course in discrete mathematics. In this edition new material on translating sentences into mathematical statements has been added, helping to make the treatment of mathematical reasoning even more comprehensive and understandable.

NEW TOPICS The third edition of the book contains coverage of two topics requested by many instructors, lattices and Turing machines. Material on lattices has been added to Section 6.6 by extending the previous coverage on partial orderings. The application of lattices to modeling information flow helps motivate this topic. Furthermore, a new section on Turing machines has been added to Chapter 10. This makes the coverage of models of computation more comprehensive. This new section explains how Turing machines can be used to recognize languages, how they can be used to compute functions, and how they serve as the most general model for effective computation.

NEW AND IMPROVED EXERCISES More than 200 new exercises have been added. These include routine exercises, many intermediate-level exercises, and selected challenging exercises. Exercises that were unclear or ambiguous have been clarified or deleted. In this edition, exercises of a particular type occur both as odd-numbered and even-numbered exercises. The grading of exercises has been reviewed and revised.

ADDITIONAL BIOGRAPHICAL AND HISTORICAL FOOTNOTES Several new biographical footnotes have been added. Also, new material has been incorporated in some of the biographical footnotes.

IMPROVED DESIGN The design of the book has been enhanced for easier reading. The design now clearly distinguishes between definitions and theorems, with theorems shaded for emphasis.

NEW AND IMPROVED ANCILLARIES A new ancillary, *Exploring Discrete Mathematics and its Applications with MAPLE,* is now available. This ancillary will help students carry out computations and explorations in discrete mathematics using the MAPLE computer algebra system. The *Student Solutions Guide* for the third edition contains suggested references for the writing projects found at the end of each chapter. The *Instructor's Resource Guide* now contains an extensive test bank, which is also available electronically using the McGraw-Hill Testing System. Instructors can use this testing system to create and customize their own tests using a bank of over 1100 questions. A complete description of the supplements package can be found later in the preface.

Features

ACCESSIBILITY There are no mathematical prerequisites beyond college algebra for this text. The few places in the book where calculus is referred to are explicitly noted. Most students should easily understand the pseudocode used in the text to express algorithms, regardless of whether they have formally studied programming languages. There is no formal computer science prerequisite.

Each chapter begins at an easily understood and accessible level. Once basic mathematical concepts have been carefully developed, more difficult material and applications to other areas of study are presented.

FLEXIBILITY This text has been carefully designed for flexible use. The dependence of chapters on previous material has been minimized. Each chapter is divided into sections of approximately the same length, and each section is divided into subsections that form natural blocks of material for teaching. Instructors can easily pace their lectures using these blocks.

WRITING STYLE The writing style in this book is direct and pragmatic. Precise mathematical language is used without excessive formalism and abstraction. Notation is introduced and used when appropriate. Care has been taken to balance the mix of notation and words in mathematical statements.

EXTENSIVE CLASSROOM USE This book has been used at over 300 schools and more than 250 have used it more than once. The feedback from instructors and students at many of the schools has helped make the third edition an even more successful teaching tool than previous editions.

MATHEMATICAL RIGOR AND PRECISION All definitions and theorems in this text are stated extremely carefully so that students will appreciate the precision of language and rigor needed in mathematics. Proofs are motivated and developed slowly; their steps are all carefully justified. Recursive definitions are explained and used extensively.

FIGURES AND TABLES This text contains more than 550 figures. The figures are designed to illustrate key concepts and steps of proofs. Color has been carefully used in figures to illustrate important points. Whenever possible, tables have been used to summarize key points and illuminate quantitative relationships.

WORKED EXAMPLES Over 600 examples are used to illustrate concepts, relate different topics, and introduce applications. In the examples, a question is first posed, then its solution is presented with the appropriate amount of detail.

APPLICATIONS The applications included in this text demonstrate the utility of discrete mathematics in the solution of real-world problems. This text includes applications to a wide variety of areas, including computer science, psychology, chemistry, engineering, linguistics, biology, and business.

ALGORITHMS Results in discrete mathematics are often expressed in terms of algorithms; hence, key algorithms are introduced in each chapter of the book. These algorithms are expressed in words and in an easily understood form of structured pseudocode, which is described and specified in Appendix 2. The computational complexity of the algorithms in the text is also analyzed at an elementary level.

HISTORICAL INFORMATION The background of many topics is succinctly described in the text. Brief biographies of more than 50 mathematicians and computer scientists are included as footnotes. These biographies include information about the lives, careers, and accomplishments of these important contributors to discrete mathematics. In addition, numerous historical footnotes are included that supplement the historical information in the main body of the text.

KEY TERMS AND RESULTS A list of key terms and results follows each chapter. The key terms include only the most important that students should learn, not every term defined in the chapter.

EXERCISES There are over 2600 exercises in the text. There are many different types of questions posed. There is an ample supply of straightforward exercises that develop basic skills, a large number of intermediate exercises, and a good supply of challenging exercises. Exercises are stated clearly and unambiguously, and all are carefully graded for level of difficulty. Exercise sets contain special discussions, with exercises, that develop new concepts not covered in the text, permitting students to discover new ideas through their own work. Exercises that are somewhat more difficult than average are marked with a single star; those that are much more challenging are marked with two stars. Exercises whose solutions require calculus are explicitly noted. Exercises that develop results used in the text are clearly identified with the symbol ☞. Solutions to all odd-numbered exercises are provided at the back of the text. The solutions include proofs in which most of the steps are clearly spelled out.

REVIEW QUESTIONS A set of review questions is provided at the end of each chapter. These questions are designed to help students focus their study on the most important concepts and techniques of that chapter. To answer these questions students need to write long answers, rather than just perform calculations or give short replies.

SUPPLEMENTARY EXERCISE SETS Each chapter is followed by a rich and varied set of supplementary exercises. These exercises are generally more difficult than those in the exercise sets following the sections. The supplementary exercises reinforce the concepts of the chapter and integrate different topics more effectively.

COMPUTER PROJECTS Each chapter is followed by a set of computer projects. The 138 computer projects tie together what students may have learned in computing and in discrete mathematics. Computer projects that are more difficult than average, from both a mathematical and a programming point of view, are marked with a star, and those that are extremely challenging are marked with two stars.

COMPUTATIONS AND EXPLORATIONS A set of computations and explorations is included at the conclusion of each chapter. These exercises (approximately 100 in total) are designed to be completed using existing software tools, such as programs that students or instructors have written or mathematical computation packages such as MAPLE or Mathematica. Many of these exercises give students the opportunity to uncover new facts and ideas through computation.

WRITING PROJECTS Each chapter is followed by a set of writing projects. To do these projects students need to consult the mathematical literature. Some of these projects are historical in nature and may involve looking up original sources. Others are designed to serve as gateways to new topics and ideas. All are designed to expose students to ideas not covered in depth in the text. These projects tie together mathematical concepts and the writing process and help expose students to possible areas for future study. (Suggested references for these projects can be found in the *Student Solutions Guide.*)

APPENDIXES There are three appendixes to the text. The first covers exponential and logarithmic functions, reviewing some basic material used heavily in the course; the second specifies the pseudocode used to describe algorithms in this text; and the third discusses generating functions.

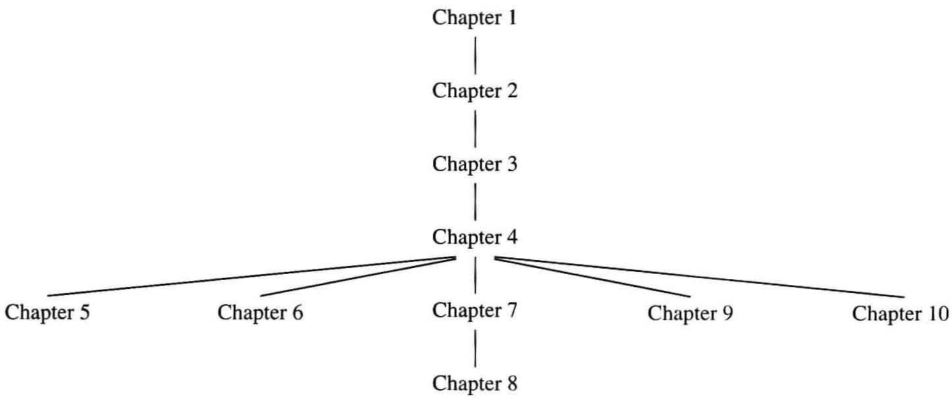
SUGGESTED READING A list of suggested readings for each chapter is provided in a section at the end of the text. These suggested readings include books at or below the level of this text, more difficult books, expository articles, and articles in which discoveries in discrete mathematics were originally published.

How To Use This Book

This text has been carefully written and constructed to support discrete mathematics courses at several levels. The following table identifies the core and optional sections. An introductory one-term course in discrete mathematics at the sophomore level can be based on the core sections of the text, with other sections covered at the discretion of the instructor. A two-term introductory course could include all the optional mathematics sections in addition to the core sections. A course with a strong computer science emphasis can be taught by covering some or all of the optional computer science sections.

Chapter	*Core Sections*	*Optional Computer Science Sections*	*Optional Mathematics Sections*
1	1.1–1.9 (as needed)		
2	2.1–2.3, 2.6 (as needed)	2.4	2.5
3	3.1–3.3	3.4, 3.5	
4	4.1–4.4	4.7	4.5, 4.6
5	5.1, 5.4	5.3	5.2, 5.5
6	6.1, 6.3, 6.5	6.2	6.4, 6.6
7	7.1–7.5		7.6–7.8
8	8.1	8.2–8.4	8.5, 8.6
9		9.1–9.4	
10		10.1–10.5	

Instructors using this book can adjust the level of difficulty of their course by omitting the more challenging examples at the end of sections as well as the more challenging exercises. The dependence of chapters on earlier chapters is shown in the following chart.



Ancillaries

STUDENT SOLUTIONS GUIDE This student manual, available separately, contains *full* solutions to all the odd-numbered problems in the exercise sets. These solutions explain why a particular method is used and why it works. For some exercises, one or two other possible approaches are described to show that a problem can be solved in several different ways. Suggested references for the writing projects found at the end of each chapter are also included in this volume. The guide also includes sample tests and a sample crib sheet for each chapter, both designed to help students prepare for exams. Students find this guide extremely useful.

INSTRUCTOR'S RESOURCE GUIDE This manual contains full solutions to even-numbered exercises in the text. It also provides suggestions on how to teach the material in each chapter of the book, including the points to stress in each section and how to put the material into perspective. Furthermore, the manual contains a test bank of sample examination questions for each chapter, including some sample tests as well as the solutions to the sample questions. Finally, sample syllabi are presented.

APPLICATIONS OF DISCRETE MATHEMATICS This ancillary is a separate text that can be used either in conjunction with the text or independently. It contains more than 20 chapters (each with its own set of exercises) written by instructors who have used the text. Following a common format similar to that of the text, the chapters in this book can be used as a text for a separate course, for a student seminar, or for a student doing independent study. Subsequent editions of this ancillary are planned that will broaden the range of applications covered. Instructors are invited to submit additional applications for possible inclusion in later versions.

TEST BANK New with this edition is an extensive test bank of more than 1100 questions implemented on the McGraw-Hill Testing System, RHTest. This system is available for PCs that run either the DOS or Macintosh operating systems. Instructors can use this software to create their own tests by selecting questions of their choice or by random selection. Instructors can add their own headings and instructions, print scrambled versions of the same test, and edit the existing questions or add their own. A printed version of this test bank, including the questions and their answers, is included in the Instructor's Resource Guide.

EXPLORING DISCRETE MATHEMATICS AND ITS APPLICATIONS WITH MAPLE This ancillary is a separate book designed to help students use the MAPLE computer algebra system to do a wide range of computations in discrete mathematics. For each chapter of this text, this new ancillary includes the following: a description of relevant MAPLE functions and how they are used, MAPLE programs that carry out relevant computations, suggestions and examples showing how MAPLE can be used for the computations and explorations at the end of each chapter, and exercises that can be worked using MAPLE.

COMPUTER PROJECTS SOLUTIONS GUIDE A manual containing solutions to the computer projects is available to instructors who adopt the text. This manual gives the code in Pascal for these projects, including sample input and output. The programs are available on a disc that will run on a DOS PC.

Acknowledgments

I would like to thank the many instructors and students at many different schools who have used this book and provided me with their valuable feedback and helpful suggestions. Their input has made this a much better book than it would have been otherwise. I especially want to thank Jerrold Grossman and John Michaels for their technical reviews of the third edition and their "eagle eyes," which have helped ensure the accuracy of this book.

I thank the many, many reviewers of the first, second, and third editions. These reviewers have provided much helpful criticism and encouragement to me. I hope this edition lives up to their high expectations.

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Kenneth H. Rosen

TO THE STUDENT

What is discrete mathematics? Discrete mathematics is the part of mathematics devoted to the study of discrete objects. (Here *discrete* means consisting of distinct or unconnected elements.) The kind of problems solved using discrete mathematics include: How many ways are there to choose a valid password on a computer system? What is the probability of winning a lottery? Is there a link between two computers in a network? What is the shortest path between two cities using a transportation system? How can a list of integers be sorted so that the integers are in increasing order? How many steps are required to do such a sorting? How can a circuit be designed that adds two integers? You will learn the discrete structures and techniques needed to solve problems such as these.

More generally, discrete mathematics is used whenever objects are counted, when relationships between finite sets are studied, and when processes involving a finite number of steps are analyzed. A key reason for the growth in the importance of discrete mathematics is that information is stored and manipulated by computing machines in a discrete fashion.

There are several important reasons for studying discrete mathematics. First, through this course you can develop your mathematical maturity, that is, your ability to understand and create mathematical arguments. You will not get very far in your studies in the mathematical sciences without these skills.

Second, discrete mathematics is the gateway to more advanced courses in all parts of the mathematical sciences. Math courses based on the material studied in discrete mathematics include logic, set theory, number theory, linear algebra, abstract algebra, combinatorics, graph theory, and probability theory (the discrete part of the subject). Discrete mathematics provides the mathematical foundations for many computer science courses, including data structures, algorithms, database theory, automata theory, formal languages, compiler theory, computer security, and operating systems. Students find these courses much more difficult when they have not had the appropriate mathematical foundations from discrete math. One student has sent me an electronic mail message to tell me that she used the contents of this book in every computer science course she took!

Also, discrete mathematics contains the necessary mathematical background for solving problems in operations research (including many discrete optimization techniques), chemistry, engineering, biology, and so on. In the text, we will study applications to some of these areas.

Finally, I would like to offer some helpful advice to students about how best to learn discrete mathematics. You will learn the most by working exercises. I suggest you do as many as you possibly can, including both the exercises at the end of each section of the text and the supplementary exercises at the end of each chapter. Always attempt exercises yourself before consulting the answers at the end of the book or in the *Student Solutions Guide.* Only after you have put together a solution, or you find yourself at an impasse, should you look up the suggested solution. At that point you will find the discussions in the *Student Solutions Guide* most helpful. When doing exercises, keep in mind that the more difficult exercises are marked as described in the following table.

Key to the Exercises

No marking	A routine exercise
⋆	A difficult exercise
⋆⋆	An extremely challenging exercise
☞	An exercise containing a result used in the text
(requires calculus)	An exercise whose solution requires the use of limits

Kenneth H. Rosen

CONTENTS

5

Advanced Counting Techniques 307

6

Relations 355

7

Graphs 429