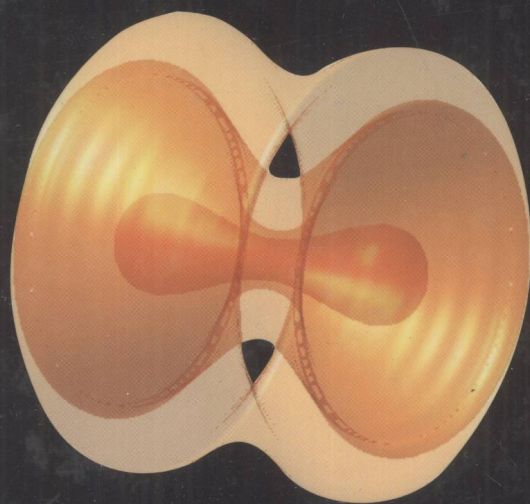


# Localized Waves



Edited by  
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# Preface

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Diffraction and dispersion effects have been well known for centuries and are recognized to be limiting factors in many industrial and technology applications based, for example, on electromagnetic beams and pulses. Diffraction is an always-present phenomenon, affecting two- or three-dimensional waves traveling in nonguiding media. Arbitrary pulses and beams contain plane-wave components that propagate in different directions, causing a progressive increase in their spatial width along propagation. Dispersion is due to the dependence of the material media (refractive index) with frequency: therefore, each pulse's spectral component propagates with a different phase velocity, so that an electromagnetic pulse will suffer a progressive increase in its temporal width along propagation. It is clear that these two effects may be a serious restriction for applications where it is highly desirable that the beam keeps its transverse localization or the pulse keeps its transverse localization and/or temporal width along propagation, which might be desirable, for example, in free-space microwave, millimetric wave, terahertz and optical communications, microwave and optical images, optical lithography, and optical tweezers. As a consequence, the development of techniques capable of alleviating signal degradation effects caused by these two effects is of crucial importance.

*Localized waves*, also known as nondiffractive waves, arose initially as an attempt to obtain beams and pulses capable of resisting diffraction in free space for long distances. Such waves were obtained initially theoretically as solutions to the wave equation in the early 1940s (J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill, New York, 1941), and were demonstrated experimentally in 1987 (J. Durnin, J. J. Miceli, and J. H. Eberly, Diffraction-free beams, *Phys. Rev. Lett.*, vol. 58, pp. 1499–1501, 1987). Nowadays localized waves constitute a growing and dynamic research topic, not only in relation to nondispersive free space (or vacuum), but also for dispersive, nonlinear, and lossy nonguiding media. Taking into account the significant amount of exciting and impressive results published especially in the last five years or so, we decided to edit a book on this topic, the first of its kind in the literature. The book is composed of 13 chapters authored by the most productive researchers in the field, with a well-balanced presentation of theory and experiment.



In Chapter 1, Recami et al. present a thorough review of localized waves, emphasizing the theoretical foundations along with historical aspects and the interconnections of this subject with other technology and scientific areas.

In Chapter 2, Zamboni-Rached et al. discuss in detail the theoretical structure of localized waves, and some applications are presented, among which frozen waves are of particular interest.

In Chapter 3, Besieris and Shaarawi present a hybrid spectral representation method which permits a smooth transition between two seemingly disparate classes of finite-energy spatiotemporally localized wave solutions to the three-dimensional scalar wave equation in free space: superluminal (X-shaped) and luminal (FWM-type) pulsed waves. An additional advantage of the hybrid form is that it obviates the presence of backward wave components, propagating at the luminal speed  $c$ , that have to be minimized in practical applications. A modified hybrid spectral representation method has also been presented which permits a seamless transition from superluminal localized waves to exact luminal splash modes. Within the framework of a certain parametrization, the latter are rendered indistinguishable from the paraxial luminal finite-energy-focused pulsed beam solutions.

In Chapter 4, Jian-yu Lu describes X-waves in depth, providing generalized methods for obtaining such waves through proper transformations, related primarily to the Lorentz transformation. X-wave solutions to Schrödinger and Klein–Gordon equations are also provided. In addition, the potential application of X-waves in medical ultrasound imaging is demonstrated experimentally.

In Chapter 5, Salo and Friberg show theoretically that diffraction-free wave propagation can also be achieved in anisotropic crystalline media. They explicitly analyze the effect of arbitrary anisotropies on both continuous-wave and pulsed nondiffracting fields. Due to beam steering and other effects, generation of nondiffracting waves in anisotropic media poses new challenges, and the authors propose an efficient scheme for the generation and detection of a continuous-wave beam in a crystal wafer.

In Chapter 6, Mugnai and Mochi explore Bessel X-waves' ability to provide localized energy and to exhibit superluminal propagation in both phase and group velocities (as verified experimentally). The authors also describe the ability of such waves to travel through a classically forbidden region (tunneling region) with no shift in the direction of propagation, which makes them different and unique with respect to ordinary waves.

In Chapter 7, Reivelt and Saari focus on the physical nature and experimental implementation or generation of localized waves. The authors demonstrate that the angular spectrum representation and the tilted pulse representation of localized waves are suitable tools for achieving these purposes. They explain the concepts and results of their experiments, where the realizability of Bessel X-waves and focus wave modes was verified for the first time.

In Chapter 8, Porras et al. present an interesting discussion of linear bullets, three-dimensional localized waves or particlelike waves propagating across a host medium, defeating diffraction spreading and dispersion broadening. Special attention is given to the generation of these bullets in practical settings by optical devices or by nonlinear means, showing the intimate relation between linear and nonlinear approaches to wave

bullets, as in light filaments. The advantage of linear bullets with respect to standard wave packets (Gaussian-like) is also demonstrated for a variety of applications, such as laser writing in thick media, ultraprecise microhole drilling for photonic-crystal fabrication, and laser micromachining.

In Chapter 9, the theory of X-waves in nonlinear materials is discussed thoroughly by Conti and Trillo. Potential applications in light-matter interactions at high laser intensities in quantum optics and on the theoretical prediction of X-waves in Bose–Einstein condensates are pointed out.

In Chapter 10, by Kukhlevsky, the problem of spatial localization of light in free space on a nanometer scale is presented in detail. The author shows that a sub-wavelength nanometer-sized beam propagating without diffractive broadening can be produced by the interference of multiple beams of a Fresnel light source of the respective material waveguide. The results demonstrate theoretically the feasibility of diffraction-free subwavelength-beam optics on a nanometer scale for both continuous waves and ultrashort (near-single-cycle) pulses. The approach extends the operational principle of near-field subwavelength-beam optics, such as near-field scanning optical microscopy, to the “not-too-distant” field regime (up to about 0.5 wavelength). The chapter includes theoretical illustrations to facilitate an understanding of the natural spatiotemporal broadening of light waves and the physical mechanisms that contribute to the diffraction-free propagation of subwavelength beams in free space.

In Chapter 11, Grunwald et al. show experimentally that ultraflat thin-film axicons enable the real physical approximation of nondiffracting beams and X-pulses of extremely narrow angular spectra. By self-apodized truncation of Bessel–Gauss pulses (coincidence of first field minimum with the rim of an aperture), needle-shaped propagation zones of large axial extension can be obtained without additional diffraction effects. The signature of undistorted progressive waves was indicated for such needle beams by the fringe-free propagation characteristics and ultrabroadband spatio-spectral transfer functions.

In Chapter 12, Longhi and Janner provide a general overview of wave localization (in a weak sense) for an important and novel class of inhomogeneous periodic dielectric media (i.e., in photonic crystals), which have received increasing attention in recent years. Compared to wave localization in homogeneous media, such as in a vacuum, the presence of a periodic dielectric permittivity strongly modifies the space–time dispersion surfaces and hence the types of localized waves that may be observed in photonic crystals.

In Chapter 13, Bouchal et al. focus on theoretical and experimental problems of nondiffracting and singular optics. Particular attention is devoted to physical properties, methods of experimental realization, and potential applications of single and composed vortex fields carried by a pseudo-nondiffracting background beam. The unique propagation effects of vortex fields are pointed out, and consequences of their spiral phase singularities manifested by a transfer of the orbital angular momentum are also discussed. The complex vortex structures whose parameters and properties are controlled dynamically by a spatial light modulation provide advanced methods of encoding and recording of information and can be utilized effectively in optical manipulations. Spatially localized vortex structures can be extended into

nonstationary optical fields where novel spatiotemporal effects and applications can be expected.

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The editors are grateful to all the contributors to this volume for their efforts in producing stimulating high-quality chapters in an area that is not yet well known outside the community of experts, always with the aim of making the area more easily accessible to interested physicists and engineers. For useful discussions they are grateful to, among others, R. Bonifacio, M. Brambilla, R. Chiao, C. Cocca, C. Conti, A. Friberg, G. Degli Antoni, F. Fontana, G. Kurizki, M. Mattiuzzi, P. Milonni, P. Saari, A. Shaarawi, R. Ziolkowski, M. Tygel, and L. Ambrosio.

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The Editors

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# Contents

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<b>CONTRIBUTORS</b>	<b>xiii</b>
<b>PREFACE</b>	<b>xv</b>
<b>1 Localized Waves: A Historical and Scientific Introduction</b>	<b>1</b>
<i>Erasmo Recami, Michel Zamboni-Rached, and Hugo E. Hernández-Figueroa</i>	
1.1 General Introduction	2
1.2 More Detailed Information	6
1.2.1 Localized Solutions	9
Appendix: Theoretical and Experimental History	17
Historical Recollections: Theory	17
X-Shaped Field Associated with a Superluminal Charge	20
A Glance at the Experimental State of the Art	23
References	34
<b>2 Structure of Nondiffracting Waves and Some Interesting Applications</b>	<b>43</b>
<i>Michel Zamboni-Rached, Erasmo Recami, and Hugo E. Hernández-Figueroa</i>	
2.1 Introduction	43
2.2 Spectral Structure of Localized Waves	44
2.2.1 Generalized Bidirectional Decomposition	46
2.3 Space-Time Focusing of X-Shaped Pulses	54
2.3.1 Focusing Effects Using Ordinary X-Waves	55
2.4 Chirped Optical X-Type Pulses in Material Media	57
2.4.1 Example: Chirped Optical X-Type Pulse in Bulk Fused Silica	62

2.5	Modeling the Shape of Stationary Wave Fields: Frozen Waves	63
2.5.1	Stationary Wave Fields with Arbitrary Longitudinal Shape in Lossless Media Obtained by Superposing Equal-Frequency Bessel Beams	63
2.5.2	Stationary Wave Fields with Arbitrary Longitudinal Shape in Absorbing Media: Extending the Method	70
	References	76
<b>3</b>	<b>Two Hybrid Spectral Representations and Their Applications to the Derivations of Finite-Energy Localized Waves and Pulsed Beams</b>	<b>79</b>
	<i>Ioannis M. Besieris and Amr M. Shaarawi</i>	
3.1	Introduction	79
3.2	Overview of Bidirectional and Superluminal Spectral Representations	80
3.2.1	Bidirectional Spectral Representation	81
3.2.2	Superluminal Spectral Representation	83
3.3	Hybrid Spectral Representation and Its Application to the Derivation of Finite-Energy X-Shaped Localized Waves	84
3.3.1	Hybrid Spectral Representation	84
3.3.2	$(3 + 1)$ -Dimensional Focus X-Wave	85
3.3.3	$(3 + 1)$ -Dimensional Finite-Energy X-Shaped Localized Waves	86
3.4	Modified Hybrid Spectral Representation and Its Application to the Derivation of Finite-Energy Pulsed Beams	89
3.4.1	Modified Hybrid Spectral Representation	89
3.4.2	$(3 + 1)$ -Dimensional Splash Modes and Focused Pulsed Beams	89
3.5	Conclusions	93
	References	93
<b>4</b>	<b>Ultrasonic Imaging with Limited-Diffraction Beams</b>	<b>97</b>
	<i>Jian-yu Lu</i>	
4.1	Introduction	97
4.2	Fundamentals of Limited-Diffraction Beams	99
4.2.1	Bessel Beams	99
4.2.2	Nonlinear Bessel Beams	101
4.2.3	Frozen Waves	101
4.2.4	X-Waves	101
4.2.5	Obtaining Limited-Diffraction Beams with Variable Transformation	102

4.2.6	Limited-Diffraction Solutions to the Klein–Gordon Equation	103
4.2.7	Limited-Diffraction Solutions to the Schrödinger Equation	106
4.2.8	Electromagnetic X-Waves	108
4.2.9	Limited-Diffraction Beams in Confined Spaces	109
4.2.10	X-Wave Transformation	114
4.2.11	Bowtie Limited-Diffraction Beams	115
4.2.12	Limited-Diffraction Array Beams	115
4.2.13	Computation with Limited-Diffraction Beams	115
4.3	Applications of Limited-Diffraction Beams	116
4.3.1	Medical Ultrasound Imaging	116
4.3.2	Tissue Characterization (Identification)	116
4.3.3	High-Frame-Rate Imaging	116
4.3.4	Two-Way Dynamic Focusing	116
4.3.5	Medical Blood-Flow Measurements	117
4.3.6	Nondestructive Evaluation of Materials	117
4.3.7	Optical Coherent Tomography	117
4.3.8	Optical Communications	117
4.3.9	Reduction of Sidelobes in Medical Imaging	117
4.4	Conclusions	117
	References	118
<b>5</b>	<b>Propagation-Invariant Fields: Rotationally Periodic and Anisotropic Nondiffracting Waves</b>	<b>129</b>
	<i>Janne Salo and Ari T. Friberg</i>	
5.1	Introduction	129
5.1.1	Brief Overview of Propagation-Invariant Fields	130
5.1.2	Scope of This Chapter	133
5.2	Rotationally Periodic Waves	134
5.2.1	Fourier Representation of General RPWs	135
5.2.2	Special Propagation Symmetries	135
5.2.3	Monochromatic Waves	136
5.2.4	Pulsed Single-Mode Waves	138
5.2.5	Discussion	142
5.3	Nondiffracting Waves in Anisotropic Crystals	142
5.3.1	Representation of Anisotropic Nondiffracting Waves	143
5.3.2	Effects Due to Anisotropy	146
5.3.3	Acoustic Generation of NDWs	148
5.3.4	Discussion	149
5.4	Conclusions	150
	References	151

<b>6</b>	<b>Bessel X-Wave Propagation</b>	<b>159</b>
	<i>Daniela Mugnai and Iacopo Mochi</i>	
6.1	Introduction	159
6.2	Optical Tunneling: Frustrated Total Reflection	160
6.2.1	Bessel Beam Propagation into a Layer: Normal Incidence	160
6.2.2	Oblique Incidence	164
6.3	Free Propagation	169
6.3.1	Phase, Group, and Signal Velocity: Scalar Approximation	169
6.3.2	Energy Localization and Energy Velocity: A Vectorial Treatment	172
6.4	Space–Time and Superluminal Propagation	180
	References	181
<b>7</b>	<b>Linear-Optical Generation of Localized Waves</b>	<b>185</b>
	<i>Kaido Reivelt and Peeter Saari</i>	
7.1	Introduction	185
7.2	Definition of Localized Waves	186
7.3	The Principle of Optical Generation of LWs	191
7.4	Finite-Energy Approximations of LWs	193
7.5	Physical Nature of Propagation Invariance of Pulsed Wave Fields	195
7.6	Experiments	198
7.6.1	LWs in Interferometric Experiments	198
7.6.2	Experiment on Optical Bessel X-Pulses	200
7.6.3	Experiment on Optical LWs	203
7.7	Conclusions	211
	References	213
<b>8</b>	<b>Optical Wave Modes: Localized and Propagation-Invariant Wave Packets in Optically Transparent Dispersive Media</b>	<b>217</b>
	<i>Miguel A. Porras, Paolo Di Trapani, and Wei Hu</i>	
8.1	Introduction	217
8.2	Localized and Stationarity Wave Modes Within the SVEA	219
8.2.1	Dispersion Curves Within the SVEA	221
8.2.2	Impulse-Response Wave Modes	222
8.3	Classification of Wave Modes of Finite Bandwidth	224
8.3.1	Phase-Mismatch-Dominated Case: Pulsed Bessel Beam Modes	226
8.3.2	Group-Velocity-Mismatch-Dominated Case: Envelope Focus Wave Modes	227

8.3.3	Group-Velocity-Dispersion-Dominated Case: Envelope X- and Envelope O-Modes	229
8.4	Wave Modes with Ultrabroad Bandwidth	231
8.4.1	Classification of SEWA Dispersion Curves	233
8.5	About the Effective Frequency, Wave Number, and Phase Velocity of Wave Modes	236
8.6	Comparison Between Exact, SEWA, and SVEA Wave Modes	238
8.7	Conclusions	240
	References	240
<b>9</b>	<b>Nonlinear X-Waves</b>	<b>243</b>
	<i>Claudio Conti and Stefano Trillo</i>	
9.1	Introduction	243
9.2	NLX Model	245
9.3	Envelope Linear X-Waves	247
9.3.1	X-Wave Expansion and Finite-Energy Solutions	250
9.4	Conical Emission and X-Wave Instability	252
9.5	Nonlinear X-Wave Expansion	255
9.5.1	Some Examples	255
9.5.2	Proof	256
9.5.3	Evidence	257
9.6	Numerical Solutions for Nonlinear X-Waves	257
9.6.1	Bestiary of Solutions	259
9.7	Coupled X-Wave Theory	262
9.7.1	Fundamental X-Wave and Fundamental Soliton	264
9.7.2	Splitting and Replenishment in Kerr Media as a Higher-Order Soliton	264
9.8	Brief Review of Experiments	265
9.8.1	Angular Dispersion	265
9.8.2	Nonlinear X-Waves in Quadratic Media	265
9.8.3	X-Waves in Self-Focusing of Ultrashort Pulses in Kerr Media	266
9.9	Conclusions	266
	References	267
<b>10</b>	<b>Diffraction-Free Subwavelength-Beam Optics on a Nanometer Scale</b>	<b>273</b>
	<i>Sergei V. Kikhlevsky</i>	
10.1	Introduction	273
10.2	Natural Spatial and Temporal Broadening of Light Waves	275
10.3	Diffraction-Free Optics in the Overwavelength Domain	281



10.4	Diffraction-Free Subwavelength-Beam Optics on a Nanometer Scale	286
10.5	Conclusions	292
	Appendix	292
	References	293
<b>11</b>	<b>Self-Reconstruction of Pulsed Optical X-Waves</b>	<b>299</b>
	<i>Ruediger Grunwald, Uwe Neumann, Uwe Griebner, Günter Steinmeyer, Gero Stibenz, Martin Bock, and Volker Kebbel</i>	
11.1	Introduction	299
11.2	Small-Angle Bessel-Like Waves and X-Pulses	300
11.3	Self-Reconstruction of Pulsed Bessel-Like X-Waves	303
11.4	Nondiffracting Images	306
11.5	Self-Reconstruction of Truncated Ultrabroadband Bessel–Gauss Beams	307
11.6	Conclusions	310
	References	311
<b>12</b>	<b>Localization and Wannier Wave Packets in Photonic Crystals Without Defects</b>	<b>315</b>
	<i>Stefano Longhi and Davide Janner</i>	
12.1	Introduction	315
12.2	Diffraction and Localization of Monochromatic Waves in Photonic Crystals	317
12.2.1	Basic Equations	317
12.2.2	Localized Waves	319
12.3	Spatiotemporal Wave Localization in Photonic Crystals	324
12.3.1	Wannier Function Technique	325
12.3.2	Undistorted Propagating Waves in Two- and Three-Dimensional Photonic Crystals	329
12.4	Conclusions	334
	References	335
<b>13</b>	<b>Spatially Localized Vortex Structures</b>	<b>339</b>
	<i>Zdeněk Bouchal, Radek Čelechovský, and Grover A. Swartzlander, Jr.</i>	
13.1	Introduction	339
13.2	Single and Composite Optical Vortices	342
13.3	Basic Concept of Nondiffracting Beams	346
13.4	Energetics of Nondiffracting Vortex Beams	350
13.5	Vortex Arrays and Mixed Vortex Fields	352