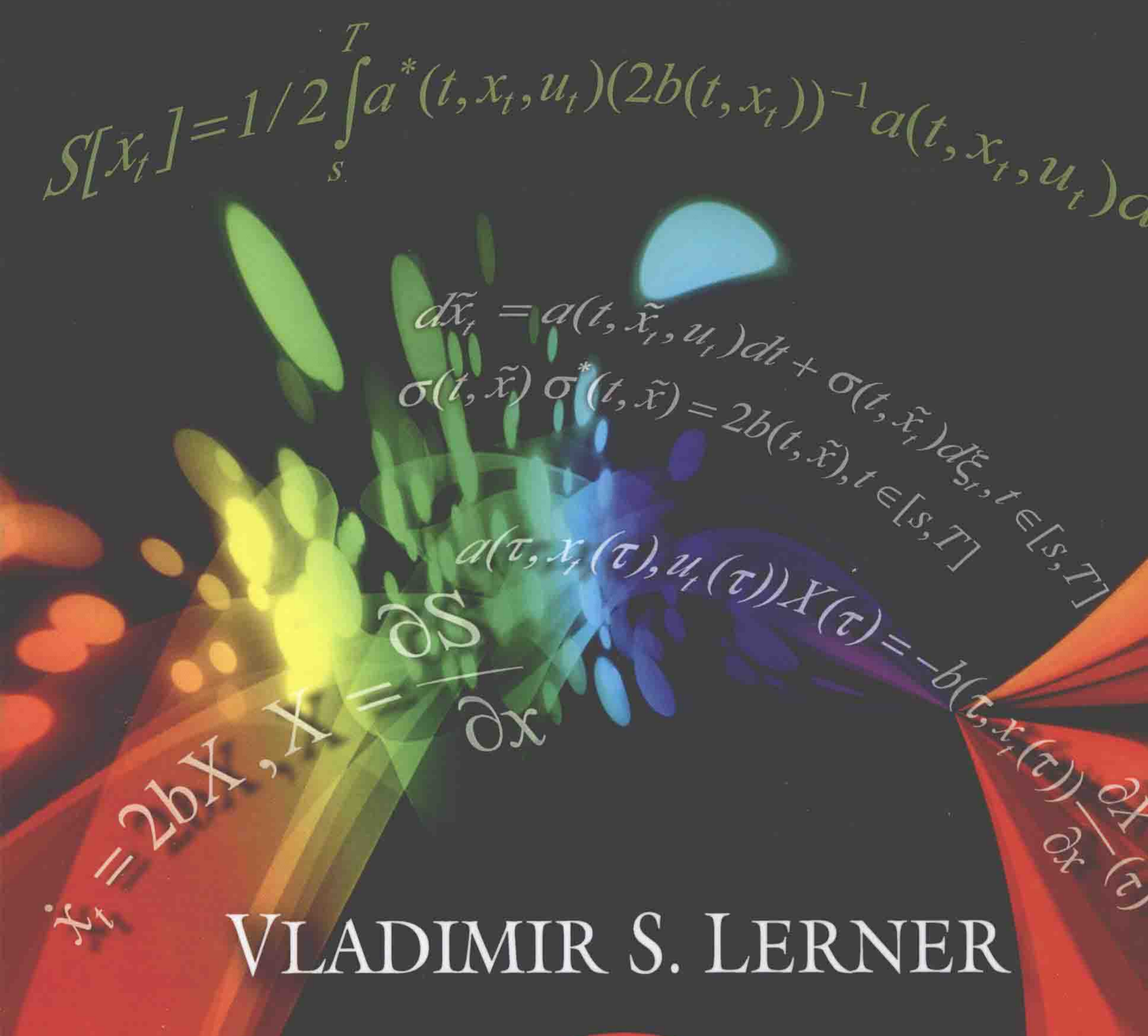
INFORMATION PATH FUNCTIONAL AND INFORMATIONAL MACRODYNAMICS



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ABSTRACT

The book, introduces entropy *functional*, defined on a controllable random process, which serves as the random process' integral information measure, while Shannon's information measure is applicable for the process's selected states.

The random process' *information dynamics* is studied by establishing an *information path functional* (IPF), which, using a variation principle (VP), allows both approximating the entropy functional with a maximal functional probability and finding dynamic trajectories of the random *microprocess* as its *macroprocess*.

The IPF mathematical formalism connects the controlled system's randomness and its dynamic regularities by the informational macrodynamics' (IMD) equations for both concentrated and distributed systems, which describe a *system* of the information dynamic macroprocesses, generated by random observations.

The IMD *provides modeling* of a controlled random *system* by its dynamic macromodel for a disclosure of the system's *information regularities*, expressed by the VP.

The modeling includes the discrete intervals of the process' observation with the identification of the macromodel VP extremal's sequence (segments), and a potential model's renovation (between the VP extremal segments).

The VP information invariants present both dynamic and information measures of each IPF extremal segment.

The invariants allow the process' simple encoding by applying the the IPF or Shannon information measure.

The optimal controls, implementing the VP for the process' integral functional, connect the process exremal segments into a dynamic chain, which is encoded into the process' information network.

Connecting the extremal segment of multi-dimensional process into a *cooperative* chain is accompanied by its states consolidation, which reduces the number of the model independent states.

These allow grouping the cooperative macroparameters and aggregating their equivalent dimensions in an *ordered hierarchical* information network (IN), built on a multidimensional spectrum of the systems operator, which is identified during the VP minimax principle a real-time optimal motion.

The IN synthesis, based on the VP minimax principle, includes the operator eigenvector's ordered arrangement and their aggregation in the elementary cooperative units.

An optimal aggregation, satisfying both a minimal IPF *local* path to each cooperating unit and a stability of the formed cooperative unit, leads to assembling the extremal segments' spectrum sequentially into the IN elementary nodes, composed by a pair of eigenvectors (doublets) and/or by their triple (triplets)-as a maximal *elementary* unit.

The triple cooperation is also able to deliver an elementary quantity of the information contribution, measured by the above information invariants, as the IN elementary triplet's code word.

The IN is formed by ordering and adjoining of the cooperated nodes, which finally move to a single IN ending node, being encoded by the same elementary code word.

This allows both encoding the initial random process in the IN ordered hierarchical macrostructure and compressing the process' encoded information into the IN final node.

The computer procedure, based on the above mathematical formalism, includes the object's identification, combined with optimal control's synthesis, process' consolidation in the cooperative dynamics, and building the IN.

The introduced concept and measure of macrocomplexity (MC) arises in an irreversible macrodynamic cooperative process and determines the process components' ability to assemble into an integrated system.

MC serves as a common indicator of the origin of the *cooperative* complexity, defined by the *invariant information* measure, allowing for both analytical formulation and computer evaluation.

The MC of the IN optimal cooperative triplets' structure is measured by the triplet *code, which* provides the MC *hierarchical* invariant information measure by both its quantity and quality. MC presents a *computable* complexity measure of a *cooperative dynamic irreversible process*, as an opposite to the Kolmogorov complexity's *incomputability*.

The bi-levels renovated macromodel embraces the regularities of the *evolutionary dynamics*, such as creation of an order from stochastics, evolutionary hierarchy, stability, adaptive self-controls and a self-organization with copying information and a genetic code. The equations' regularities follow from the informational path functional's VP as a *mathematical law of evolution*, which is capable of a prognosis of the evolutionary dynamics and its specific components: evolution potentials, diversity, speed, and genetic code.

Book also studies some *physical analogies* related to the information path functional, including its connection to Feynman's path functional, Schrödinger's equation, and Irreversible Thermodynamics.

Process of Cognition, formalized by a minimization of observed process' uncertainty, is described by a piece-wise sequence of the VP dynamic macromodel's externals, identified during observation and built to maximize a reception of information.

The developed computer-based methodology and software were practically used for systems modeling, identification, and control of a diversity of information interactions in some physical (technological) and non-physical (economical, information-biological) objects.

The book on Information Path functional is published for the first time.

PREFACE

"Mathematics Is More Than Just A Language- It Is Language Plus Logic"
R. Feynman

"A new logical basis for information theory as well as probability theory is proposed, based on computing complexity".
A.Kolmogorov (the theory author)

This book introduces an *information path functional* as a basic information measure of a random *process*, whereas Shannon's information measure is applicable to the process' *states*.

The book's mathematical formalism uses a *variation principle* (VP) for the path functional to find the process' *dynamic* trajectories as the VP extremals and to get the process' dynamic equations in the informational macrodynamics (IMD).

The information path functional, defined on a *controlled* random process, extends the obtained dynamic macromodels on a wide class of the *control information systems*. This allows *modeling* (identification) of a controlled random *system* by its dynamic macromodel for a disclosure of the system's *information regularities*, expressed by the VP.

The developed mathematical formalism connects the controlled system's randomnesses and dynamic regularities by the IMD equations, which describe a system of the information dynamic macroprocesses, generated by random observations.

The book's path functional, its VP and the IMD present a *new approach to dynamic information modeling*, applied to a functional on trajectories of a random process and its relation to the random process' entropy functional.

This book focuses on the *mathematical roots* of informational macrodynamics and the recent applications; while broader IMD basics were considered in the author's other books and research papers (please see the References).

The book consists of two parts.

Part1 contains *the foundation of the information path functional and the variation principle with the IMD dynamic model.*

In ch.1.1 we introduce the *initial mathematical models* of the controlled random (microlevel) processes in the forms of a controlled stochastic equation and a random information (entropy) functional, defined on the microlevel process, and present a class of the considered dynamic macroprocesses.

In ch.1.2 we provide the probabilistic evaluation of the micro- and macrolevel processes, using the probabilities of the processes' proximity via the process trajectories' metrical space distances.

Applying these probabilities, we formulate the problem of *dynamic approximation of the random information functional by the information path functional (IPF),* determined through the parameters of the initial stochastic equation. This leads to a variation problem as an extreme of the path functional with a *dynamic constraint*, defined by a maximal closeness of the microprocess' conditional entropy functional to the path functional at the macrotrajectories. The constraint establishes a connection between the micro- and macroprocesses. The functional's structure and the constraint bring the nontraditional solutions of both the extremal's and the control synthesis' problems.

Using both Pontryagin's maximum principle and Lagrange's methods of eliminating constraints, we find in ch.1.3 the *solution to the variation problem* in the form of a dynamic macromodel and the specified equation of constraint, binding the dynamics and stochastics. The solution determines the piece-wise extremal segments, where the macrodynamics act, and the "windows" between the segments, where the microlevel's random information affects the macrolevel dynamics. We use the connection of the micro- and macroprocesses to identify the macromodel operator via the observed random processes, in particular, by measuring and computing the corresponding covariation (correlation) functions.

We also solve the corresponding Bolza problem for *optimal control synthesis*.

We obtain a discrete function for the optimal regular control, applied at each extremal segment, and the optimal "jump" function for the optimal "needle" control, applied between the segments and thereby connecting them.

These controls allow us to build a procedure for optimal control synthesis *combined* with macromodel identification during the optimal control's action along each extremal. Because the above controls also stick the extremals, they sequentially *consolidate* the extremals (of an initially multi-dimensional process) into a process' *cooperative* structure. By this chapter's end we summarize the formal results of chs.1.1-1.3, establishing the IMD math foundation, applied for information modeling of a random *concentrated* system.

Ch. 1.4 introduces the space-time distributed entropy's functional in a random field and the basic path functional's distributed dynamic models (with fixed space coordinates) by the solution of Euler-Ostrogradsky equations for the functional's extremals. These models define the primary macroequations in partial deviations (PDE) of a *distributed* system. Then we consider a family of the space coordinates' *transformations* with the *invariant condition* imposed on the IPF. Searching for the VP's natural limitations, we obtain the IMD extreme model, defined on the admissible space coordinates' variations, which satisfies these transformations. Applying the Noether theorem, we get sufficient conditions for the invariance of the functional at the above transformations and the general forms of the PDE models in a mobile evolving space coordinate system. The invariant conditions bring an *additional* differential constraint to that, imposed by the IPF's VP on the distributed macromodel.

We obtain the IMD controllable distributed macromodel with the optimal controls, operating on space-time discrete intervals, which are found from Erdman-Weierstrass' conditions. We study the IMD macromodel's singular points and the singular trajectories; the

IPF natural variation problem, singular trajectories' control functions, and the field's invariants for the IPF.

In Ch.1.5, analyzing the time-space movement toward the macromodel's cooperation, we use the obtained results for building the *information cooperative network* (IN). The IN nodes integrate the information time-space macromovement along a sequence of extremal segments. The IN dynamic and geometrical structures are studied by applying the VP *information invariants*, following from the solution of the variation problem. The *optimal IN nodes'* cooperation, satisfying to the VP, leads to forming a *set* of triplet's nodes substructures, which are sequentially *enclosed* according to the IN hierarchy.

In ch.1.6 we examine the model *phenomena* and the process' information contributions into the IN triplet's hierarchy, evaluate both the model's reversible and irreversible time courses. We analyze the model information *cellular geometry* and its genetic information *code*, generated by the control processes and the IN triplet's hierarchy. This code, being a specific for each system, is memorized on a *double spiral* cellular time-space trajectory, generated by the IN triplets during the optimal control process.

The optimal *triple digital code* encloses the IN time-space hierarchy of the macromodel's cooperating nodes. The chapter results specify the macromodel's information geometry and its connection to macrodynamics. The path functional's integrated information is revealed through a decoded finite set of the extremal segments, assembled into the IN, which had been encoded by the triplet code.

In ch.1.7 we study the cooperative macrodynamic *complexities* (MC), which determine the process components' (in particular, the segments') ability to assemble into an integrated system in the cooperative informational dynamics.

We introduce the MC notion and the *invariant information measures,* allowing for both the complexities' analytical formulation and computer evaluations. Exploring the formal multi-cooperative mechanism of the IN, we establish the MC *hierarchical* invariant information measure by the *quantity and quality* in the triplet code.

We also consider the MC complexity's connections to Kolmogorov's complexity measures. Applying the *information geometry's space equations*, we determine an *intensity of information attraction* in the cooperative dynamics and its connection to the MC complexity.

Ch.1.8 studies the *regularities of evolutionary dynamics* and the mathematical law of evolution, following from the application of the variation principle for the informational path functional.

The law, applied to Darwinian evolutionary theory, embraces the following bio-regularities' information forms: creation of an order from stochastics via the evolutionary dynamics, described through the gradient of dynamic potential, local evolutionary speeds, and the evolutionary conditions of a fitness; evolutionary hierarchy, stability, potential of evolution; adaptive self-controls and a self-organization with a copying of information and a genetic code. We show that the VP single form of information law, which defines the above regularities, is capable of a prognosis of the evolutionary dynamics and its specific components: potentials, diversity, speed, and genetic code.

In ch.1.9 we study some *physical analogies* related to the information path functional, including its connection to Kolmogorov entropy, Feynman path functional, Schrödinger equation, and Irreversible Thermodynamics.

We also analyze information description for both *superimposition and the controls* by revealing the *information mechanism of the cross phenomena*.

Part 1 is aimed at establishing the IPF *mathematical formalism, describing information regularities, the IMD explicit dynamic features and information mechanisms,* following from the regularities' general VP form.

In part 2, the *formalism is applied* to the information modeling of complex systems with examples, studying the information regularities of *control systems* and the processes' regularities from *biology, technology, and economics.*

The *first goal* of part 2 is to show how these regularities, features, and the information mechanisms work in different areas of applications.

The *second goal* is to show that most of the applications' *specific* regularities (being known and unknown), studied in the particular branches of science, *actually are the concrete realizations* of the *general information regularities* and the IMD mechanisms of their revealing. These results are important for the regularities' understanding and practical use.

Ch.2.1 provides the IMD solution of the *control problems* for a complex system, applied for the system's *joint* identification, optimal control, and consolidation.

The studied in chs. 2.2-2.4 *information complex systems* embrace the case pattern from biology, technology, and economics, which include:

- the information modeling of the encoding-decoding processes, the structure of the information network and its code, applied to *biological and cognitive* systems;
- information modeling and control of some *industrial technology* processes with complex *superimposing phenomena*; and
- the application of the information modeling approach to an elementary *market macroeconomic* system.

All applications focus on disclosing of the information regularities of a studied object using the IMD equations and exploring their specific features for each considered object.

The model's formalism provides a *tool* for developing a *computer-based methodology,* and the *programs* (ch.2.5), which have been applied toward the solutions to the problems of information modeling, identification, optimal control, and consolidation for a wide diversity of complex systems.

The book's *essentials* consist of not only presenting a new theory of the information path functional, but also bringing this math theory up to very practical implementations, allowing the solution of actual problems in biology, technology, economics, and other considered applications.

The book *style* is directed on the initiation and keeping of the audience interest in this new interdisciplinary information science, which enhances both theory and computer applications.

The book starts with basic mathematical statements, followed by their examination and assessments in the comments and in subsequent conceptual reviews.

The book's mathematical results are motivated by means of their potential applications.

The author developed this approach between the years of 1960-1980 using physical models and applying them to technological systems (the References [R], published in Russian, reflect a history of the approach).

The book utilizes the author's scientific and practical experience of more than 40 years. The main research and academic results have been published in more than 250 scientific articles and 6 books, which contain different parts of the developed formalism and its applications.

The author has taught the course “Information Modeling” for graduate students at the University of California at Los Angeles (UCLA) and at West Coast University. Based on the formalism applications, some new advanced courses were given at UCLA and WCU, such as “Information Systems Analysis”, "Advanced Artificial Intellect”, and “Dynamic Cognitive Modeling”.

This book would be interesting for scholars, researchers, and students in applied mathematics, mathematical modeling and control, information and computer sciences, engineering, biology, and economics, as well as other interdisciplinary fields.

Specifically, part 1(chs.1.1-1.2, 1.3-1.4) would be interesting for a reader who wants to understand the basic *mathematical* formalism, which is an essential attribute of dynamic information modeling mechanisms and their correctness.

The reader interested only in an *essence* of the formalism and its conceptual understanding may start reading the book from sec.1.3.5 (by omitting the proofs in chs.1.2-1.3) and then continue with the practical applications in part 2.

The author addresses this book not only to scholars and scientists, but also to curious and searching minds trying to understand a modern World of Information and Uncertainty.

This book provides new ideas for both the theory of the novel path information functional and its applications, which allow the solution of actual practical problems in many areas where other related methods do *not* work.

The results are practically implemented on real objects and demonstrated by the book examples.

Regarding the book's *formulas and references*: Only the book's formulas, cited outside of the part and/or chapter's references, start with part numbers, for example 1.4 for part 1, chapter 4. Within each chapter we use a sequential numbering of formulas starting with the chapter number, and apply them only for the inside references. The same principle we apply also to all figures; the cited references are also related to the corresponding chapters.

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INTRODUCTION

The main subject of this book is mathematical formalism, describing the creation of the dynamic and information regularities from stochastics.

The formalism is based on the introduction of an informational path functional (IPF) via a dynamic approximation of the entropy functional (EF), defined on trajectories of a controlled random process.

Using a *variation principle* with its *dynamic constraint*, connecting both EF and the IPF, we find the IPF extemals, allowing a dynamic approximation of the random *process*.

The solution provides both the *information dynamic model* of a random process and the model of *optimal control* in the forms of differential equations of informational macrodynamics (IMD).

This allows building of a *two-level information model* with a random process at the microlevel and a dynamic process at macrolevel.

Considering a variation principle (VP) as a mathematical form that expresses some regularity, it is assumed that the VP extremals, represented by the solutions of the above dynamic model, describe a movement possessing these regularities.

Such an approach has been used by R. P. Feynman, who introduced the functional on trajectories of an electron's movement and applied the variation principle for this *path functional* to obtain the equations of quantum mechanics.

The same way, we use the IPF and its VP to obtain the IMD equations.

Feynman's path functional is defined on the *dynamic* trajectories and has *not* been applied to *random* trajectories of a *controlled* process.

For an observed multi-dimensional *random controllable process,* affected stochastic perturbations, the mathematical results related to the information path functional (IPF), as well as the variation principle and the following dynamic model have been *unknown*.

For the IPF we use *Shannon's* definition of quantity of information applied to the *functional* probability on the process' *trajectories*.

The path functional's connection to information theory allows bringing a *common information language* for modeling the micro-macro level's processes and their regularities in diverse *interdisciplinary* systems.

For a wide class of random systems, modeled by the Markov diffusion process, and a common structure of the process's information path functional, this approach leads to a *broad-spectrum information* structure of the dynamic macromodel, which we *specify and identify* on a particular system with the applied optimal control functions.

Unlike the existing literature on mathematical modeling, this book introduces a *unified* mathematical-information formalism for *information modeling*, which includes a common computer-based modeling *methodology*, *algorithms, information code*, *and computer software.*

According to the model dynamic regularities, the code arises from stochastics in a form of optimal double and/or triple digits, which compose an information network, formed by a hierarchy of the macromodel cooperating nodes.

The formalism has been *applied* to the information modeling of complex systems, and allows revealing their information regularities, particularly, demonstrated in the book examples from *biology, technology, and economics.*

The book specifics include the following key topics:

The information path functional is found as a *dynamic approximation* of a *random information functional*, defined on the process trajectories, using the probabilistic evaluation of the Markov diffusion process by a functional of action, whose Lagrangian is determined by the parameters of a controlled stochastic equation.

The optimal approximation of the system's random processes by the model's dynamic process leads to the *minimization* of their information difference, evaluated by the information path functional's measure of uncertainty. This problem automatically includes the solution of the *variation* problem (VP) for the system's mathematical model, the *synthesis* of the optimal control, and the solution of an *identification* problem.

The VP solution selects the functional's *piece-wise extremal segments*, where the macrodynamics act, and the *windows* between the segments, where the microlevel random information affects the macrolevel dynamics, and the stochastic-dynamic connection takes place. The VP solution also automatically introduces the model *piece-wise controls*, which operate by both joining the extremals' segments and acting along them.

The model's piece-wise *dependency* upon the observed information allow *identification* of both the controllable stochastic's and dynamic model's operators in *real time under* the optimal control action.

These controls, applied to the random process and its dynamic model, stick the extremals' segments (of an initially multi-dimensional process), sequentially *consolidating* these segments into a *cooperative* process.

The model segments' consolidation leads toward the model's *compressed* representation by the segments' *cooperative macrodynamics with the collective macrostates.* This creates a sequence of the *aggregated macrodynamic processes and the ordered* macrostates, *consolidated* in an *informational network* (IN) of hierarchical macrostructures, which organizes the system's mathematical model. These macrostates, memorized by the controls, create the model's *genetic information code*, which is able to encode the whole macromodel and its IN, allowing the eventual restoration of the system's model.

The microlevel's ability to discretely affect the macrodynamics at the segment's windows brings the model's piece-wise dependency on the randomness, being a source of the model's *renovation and evolution.* The consolidating extremals create the *cooperative complexity of the evolutionary macrodynamics*, initiated by the VP.

The book contains the following key features:

Revealing the dynamic regularities of a random process by applying VP to the process' information functional (as a universal attribute for any natural process) *automatically brings the dynamic constraint*, imposed *discretely* on the random process, which allows selecting the process *quantum states that represent both the process' dynamics and randomness.*

The informational macrodynamics (IMD), resulting from the VP solution for the information path functional, create the Lagrange-Hamiltonian *macromechanics of uncertainty,* which represent an information analog of physical *irreversible* thermodynamics.

The macrolevel's *function of action* portraits a dynamic eqiuvalent of the microlevel's *entropy functional*, which, being sufficient in the theory of dynamic systems, communication theory, and computer science, has not been used before.

A "deterministic impact" of microlevel stochastics on the Hamiltonian mechanics changes the structure and value of the dynamic macromodel operator, carrying its *dynamic piece-wise dependency* upon observed data. The macrodynamic process is characterized by the discrete of time-space intervals (defined by the extremals' segments), which are selected from the Hamilton's solutions and determined by the VP *invariants*. The discrete renovated operator and macrostates (at the window's discrete points (DP)) are sources of new information, new properties, and the macrostructures created by the *state consolidation*.

The Hamilton equations determine the *reversible* dynamic solution within the extremals' time intervals and the *irreversible* solutions that emerge out of these intervals.

The macromodel interacts with the environment at the segments' widows when the model is *open* for external influence. The open system does *not* satisfy the preservation *laws* at the moments of interaction when the external environment is able to change the model structure.

Optimal control functions are an intrinsic part of the model as an inner model's feedback mechanism, which is synthesized by the *duplication* of macrostates at the DPs during the optimal model's movement. These discrete controls are memorized and stored along with the corresponding ordered macrostates.

The *specifics* of the path functional's *controls*, applied at the beginning of the extremal segment, allows the proceeding of the segment's operator identification *under* the optimal control *action*, and, as a result, leads to a joint solution of the object's optimal identification and consolidation problems.

The IPF optimum predicts each extremal's segments movement not only in terms of a *total functional path goal,* but also by setting at each following segment the renovated values of this functional, identified *during the optimal movement*, which currently correct this goal. The *concurrently synthesized optima*l control's actions provide a maximal Markov's probability and optimal filtering of the random process. This optimal dual strategy at each current movement cannot be improved (even theoretically) because it defined by an extremal of a *total path* to a terminal state, which is updated at each optimal control's action.

The IMD system model contains the following main layers: *microlevel stochastics, macrolevel dynamics, and a hierarchical dynamic network of information structures with an optimal code language for the communication and model restoration.*

The system's complex dynamics initiate unique *information geometry and evolution* of the model equations, using the *functional's information mechanisms of ordering, cooperation, mutation, stability, adaptation, and the genetic code.*

In the formalism's applications, the information modeling's *unified language* and systemic categories, such as information entropy, the VP information Hamiltonian and invariants, quantity and quality of information, information flows, forces, and information complexity, *are translated* into the corresponding categories of energy, entropy, temperature, and other physical and/or computer analogies of specific systems.

A computer implementation works directly with the real system's information model that produces its information network and a code. This builds a *bridge* connecting mathematical modeling formalism to the world of information, intelligence, and information technologies, allowing the revelation of the common systemic information regularities across a variety of modeling objects, including a specific information code and complexity for each system, and applying the universal computer-based methodologies, algorithms and programs.

Finally, applying the information path functional allows us to evaluate the *information content* of a random process, build its dynamic macromodel, the corresponding information network, and disclose the process's information code and macrodynamic complexity.

CONTENTS