

# **THEORETICAL ACOUSTICS**

**PHILIP M. MORSE**

**K. UNO INGARD**

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## PREFACE

In the thirty years since the textbook "Vibration and Sound" first appeared, the science of acoustics has expanded in many directions. Products of the jet age have added economic incentive to the solution of problems related to the generation and transmission of noise; the behavior of underwater sound now is of commercial as well as of scientific and military importance; new mathematical techniques for the solution of problems involving coupled acoustic systems have been developed and the correspondingly great improvement in acoustical measurement techniques has made the theoretical solutions of more than academic interest. Acoustical measurements are being increasingly used in exploring the properties of matter; the interaction between sound fields and electromagnetic waves is an important part of plasma physics; and magneto-hydrodynamic wave motion is a phenomenon of growing importance in the sciences of meteorology and of astrophysics. The phenomena of acoustics have taken on new importance and significance, both scientifically and technologically.

With these developments in mind, the decisions involved in a revision of "Vibration and Sound" were difficult indeed. The second edition of "Vibration and Sound" is still useful as a fairly elementary text; simple addition of much new material, necessarily more advanced, would destroy its compactness and probably would reduce its utility to beginning students. What has been decided is to write a new book, with a new title. Some of the material in the earlier book has been retained, as introduction to the more advanced portions and to keep the book as self-contained as possible. The result is a graduate, rather than an undergraduate text, and one which we hope will be useful to physical scientists and engineers, who wish to learn about new developments in acoustical theory. As with "Vibration and Sound," each chapter begins with a discussion of the basic physical concepts and the more elementary theory; the later sections can be returned to later for detailed study or for reference.

The choice of new material has had to be limited, of course, by the constraints of size and unity. Experimental equipment and techniques are not discussed; other books cover this field. Specialized aspects of ultrasonics, of underwater sound, and of wave motion in elastic solids are not treated. Coverage is restricted primarily to the subjects of the generation, propagation, absorption, reflection, and scattering of compressional waves in fluid media, in the distortion of such waves by viscous and thermal effects



as well as by solid boundaries, and in their coupling through the induced vibration of walls and transmission panels. New material has been included on the acoustics of moving media, on plasma acoustics, on nonlinear effects, and on the interaction between light and sound. Much of this material has not previously been presented in unified form.

Theoretical techniques, developed for quantum mechanics, field theory, and communications theory, to mention a few, have been adapted to solve these acoustical problems. Autocorrelation functions are introduced to deal with the radiation from randomly vibrating surfaces and with the scattering of sound from turbulence, for example. The Green's function, which enables differential equation and boundary conditions to be combined into an integral equation for the wave motion, is introduced early and is used extensively throughout the rest of the book. Variational techniques are discussed and are used to obtain improved approximations in many complex problems. In all of these subjects we have tried to explain and motivate the theoretical methodology, as well as to use it to elucidate the physics. Problems, illustrating other applications, are given at the end of each chapter. While questions of rigor could not be deeply explored, it is hoped the book will be its own introduction to the more advanced mathematical techniques which have been used. Anyone familiar with calculus and vector analysis should be able to understand the mathematical techniques used here; if he is willing also to spend some time and mental effort, he should be able to master them.

The harmonic oscillator and coupled oscillators are reviewed primarily as a simple means of introducing the concepts of resonance, impedance, transient and steady-state motion, and the techniques of the Fourier transform. Transverse waves on a string are examined in some detail, again as a means of introducing the concepts of wave motion, of the flow of energy and momentum, and as a simple model with which to demonstrate the use of Green's functions and variational methods. There is a short chapter on the vibratory properties of those solid bodies, bars, membranes, and thin plates, which are of importance in later discussions.

The rest of the book is concerned with acoustic wave motion in fluid media. The equations of mass, momentum, and energy flow are discussed in some detail and the interrelations between pressure, density, velocity, temperature, and entropy are examined. The range of validity of the linear wave equation, the origin and consequences of the additional terms present in the Stokes-Navier equation, and the significance of the elements of the stress-energy tensor are developed in a unified manner. Based on these equations, the chapters on radiation, scattering, transmission, and acoustical coupling follow. The final chapters are concerned with more recent ramifications of the theory and with an introductory discussion of nonlinear oscillations and the nonlinear behavior of high intensity sound.

The authors are indebted to Professors Herman Feshbach and Wallace Dean for many suggestions regarding the organization and contents of the textual material. The authors must bear the responsibility for any lapses in clarity and the errors that may remain. We would welcome notice from our readers of such errors they may find, so later printings can be corrected.

*Philip M. Morse*  
*K. Uno Ingard*

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