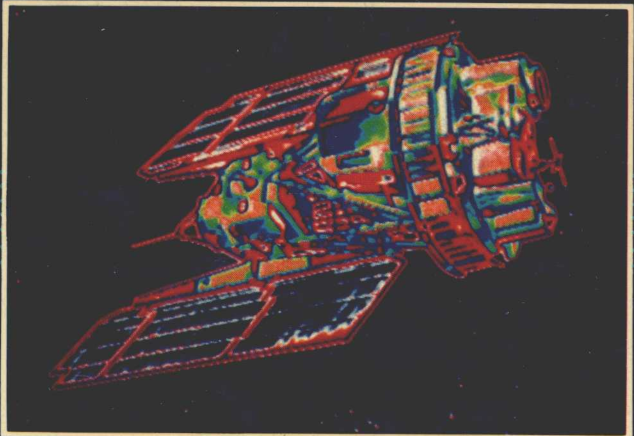
FEEDBACK CONTROL SYSTEMS



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Feedback Control Systems

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To
Bobby, Ellen, Pat, and Doeie (CLP)
My mother, Agnes L. Harbor (RDH)

Preface

This book is intended to be used primarily as a text for junior- and senior-level students in engineering curricula and for self-study by practicing engineers with little or no experience in control systems. For maximum benefits, the reader should have had some experience in linear-system analysis.

This book is based on material taught at Auburn University and at the University of West Florida and in intensive short courses offered in both the United States and in Europe. The practicing engineers who attended these short courses have greatly influenced the material and the descriptions in this book, resulting in more emphasis placed in the practical aspects of analysis and design. In particular, much emphasis is placed on understanding the difference between mathematical models and the physical systems that the models represent. While a true understanding of this difference can be acquired only through experience, the students should understand that a difference does exist.

The material of this book is organized into three principal areas: analog control systems, digital control systems, and nonlinear analog control systems. Chapter 1 in this book presents a brief introduction and an outline of the text. In addition, the Laplace transform is covered to the extent required for the remainder of the book. The mathematical models of some common components that appear in control systems are developed in Chapter 2.

Chapters 3 through 9 cover the analysis and design of linear analog systems; that is, control systems that contain no sampling. Chapter 2 develops the transfer-function model of linear analog systems, and Chapter 3 develops the state-variable model.

Chapter 4 covers typical responses of linear analog systems, including the concept of frequency response. Since many of the characteristics of closed-loop systems cannot be adequately explained without reference to frequency response, this concept is developed early in the book. The authors believe that the frequency-response concept ranks in importance with the time-response concept.

Important control-system characteristics are developed in Chapter 5. Some of the applications of closed-loop systems are evident from these characteristics. The very important concept of system stability is developed in Chapter 6 along with the Routh-Hurwitz stability criterion. Chapter 7 presents analysis and design by root-locus procedures, which are basically time-response procedures. The equally important frequency-response analysis and design procedures are presented in Chapters 8 and 9.

The material of Chapters 3 through 9 applies directly to analog control systems; that of chapters 10 through 12 applies to digital control systems. Essentially all the analog analysis and design techniques of Chapters 3 through 9 are developed again for digital control systems. These topics include typical responses, characteristics, stability, root-locus analysis and design, and frequency-response analysis and design.

Chapter 13 is devoted to modern control-system design. Pole-placement design is developed, and the design of state estimators is introduced.

Nonlinear system analysis is presented Chapter 14. These methods include the describing-function analysis, linearization, and the state-plane analysis.

Three appendices are included. The first appendix reviews matrices, the second appendix develops an analytical design procedure that applies to both root-locus design and to Bode design, and the third appendix is a table of Laplace transforms and z-transforms.

Many examples are given, with an effort made to limit each example to illustrating only one concept. It is realized that in using this approach, many obvious and interesting characteristics of the systems of the examples are not mentioned; however, since this is a book for beginning students in feedback control, making the examples more complex would tend to add confusion.

Usually nonlinear controls are not covered in introductory books in control. However, many of the important characteristics of physical systems cannot be explained on the basis of linear systems. For example, stability as a function of signal amplitude is one of the most common phenomena observed in closed-loop physical systems, and the describing function is included in Chapter 14 to offer an analysis procedure that explains this phenomenon. Lyapunov's first stability theorem is also presented to illustrate some of the pitfalls of linear-system stability analysis.

In general, the material of each chapter is organized such that the more advanced material is placed toward the end of the chapter. This placement is to allow the omission of this material by those instructors who wish to present a less intense course. For example, the material on simulation may be omitted if desired. Also, the PID controllers may be, for the most part omitted, but this is not recommended; most industrial control systems are of this type.

This book may be covered in its entirety as a 3-hour 1-semester course in analog control (Chapters 1 through 9), and a 3-hour 1-semester course in digital control and nonlinear control with an introduction to modern control (Chapters 10 through 14). The material may also be covered in a 2-quarter course sequence, with approximately 5 hours required for each course. With the omission of appropriate material, the remaining material may be covered in courses with fewer credits. If a course in digital control is taught without the coverage of the first nine chapters, some of the material of the first nine chapters must be introduced; Chapters 10 through 12 rely on some on this material. A solutions manual containing the solutions to all problems at the ends of the chapters is available.

We wish to acknowledge the many colleagues, graduate students, and staff members of Auburn University and the University of West Florida who have contributed to the development of this book. We are especially indebted to Professor J. David Irwin, head of the department of Electrical Engineering of Auburn University, and to Professor T. F. Elbert, chairman of the Systems Science Department of the University of West Florida, for their aid and encouragement. Finally, we express our gratitude and love for our families, without whom this undertaking would not have been possible.

PREFACE TO COMPUTER DISKS AND SOFTWARE MANUAL

The availability of small computers, such as the IBM PC®, has expanded the student's educational opportunity. One advantage of the use of these computers is the verification of the examples presented in a textbook and of the student's solutions for problems. The authors of this book have developed a group of programs that allow the verification of almost all of the examples and problem solutions involving analysis and design in this book. The programs were developed with the student in mind, and are menu driven and user friendly. Students are able to use these programs with a minimum of instructions.

The objective of these programs is educational. These programs allow analysis, design, and simulation of both analog and discrete linear systems, by both classical and modern methods. Root-locus and frequency-response procedures are covered for both analog and discrete systems, but modern design methods are limited to analog systems. Certain nonlinearities may be inserted for simulation purposes. All plant models are limited to tenth order, and controllers are limited to second order, except for the modern methods.

The programs will run on the IBM PC and compatible computers, with one floppy disk drive, 256K memory, and a monochrome monitor. A plotting program is included, but requires graphics capability for the computer. In addition, hard copies of the plots may be made on Epson FX® type and compatible dot-matrix printers.

The programs are available from Prentice Hall, on two double-sided double density 5¼″ disks. An order form is included in the rear of this book. A manual describing the software is included with the programs.

Charles L. Phillips, Auburn University

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Chapter 1

Introduction

This book is concerned with the analysis and design of closed-loop control systems. In the *analysis* of closed-loop systems, we are given the system, and we wish to determine its characteristics or behavior. In the *design* of closed-loop systems, we specify the desired system characteristics or behavior, and we must configure or synthesize the closed-loop system so that it exhibits these desired qualities.

We define a closed-loop system as one in which certain of the system forcing signals (we call these *inputs*) are determined, at least in part, by certain of the responses of the system (we call these *outputs*). Hence, the system inputs are a function of the system outputs, and the system outputs are a function of the system inputs. A diagram representing the functional relationships in a closed-loop system is given in Figure 1.1.

An example of a closed-loop system is the temperature-control system in the home. For this system we wish to maintain, automatically, the temperature of the living space in the home at a desired value. To control any physical variable, which we usually call a *signal*, we must know the value of this variable, that is, we must measure this variable. We call the system for the measurement of a variable a *sensor*, as indicated in Figure 1.2. In a home temperature-control system, the sensor is a thermostat, which indicates a low temperature by closing an electrical switch and an acceptable temperature by opening the same switch. We define the *plant* of a control system as that part of the system to be controlled. It is assumed

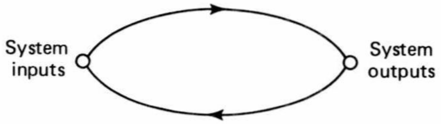

Figure 1.1 A closed-loop system.

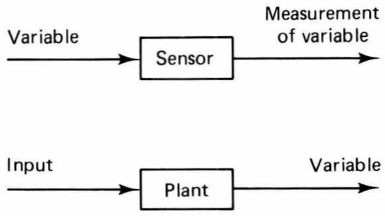

Figure 1.2 Control system components.

in this example that the temperature is increased by activating a gas furnace. Hence the plant input is the electrical signal that activates the furnace, and the plant output signal is the actual temperature of the living area. The plant is represented as shown in Figure 1.2. In the home-heating system, the output of each of the systems is connected to the input of the other, to form the closed loop. However, in most closed-loop control systems, it is necessary to connect a third system into the loop to obtain satisfactory characteristics for the total system. This additional system is called a *compensator*, a *controller*, or simply a *filter*.

The usual form of a single loop closed-loop control system is given in Figure 1.3. The system input is a reference signal; usually we want the system output to be equal to this input. In the home temperature-control system, this input is the setting of the thermostat. If we want to change the temperature, we change the system input. The system output is measured by the sensor, and this measured value is compared with (subtracted from) the input. This difference signal is called the *error signal*, or simply the error. If the output is equal to the input, this difference is zero, and no signal reaches the plant. Hence the plant output remains at its current value. If the error is not zero, in a properly designed system the error signal causes a response in the plant such that the magnitude of the error is reduced. The compensator is a filter for the error signal, since usually satisfactory operation does not occur if the error signal is applied directly to the plant.

Control systems are sometimes divided into two classes. If the object of the control system is to maintain a physical variable at some constant value in the presence of disturbances, we call this system a *regulator*. One example of a regulator control system is the speed-control system on the AC power generators of power utility companies. The purpose of this control system is to maintain the speed of the generators at the constant value that results in the generated voltage having a frequency of 60 Hz in the presence of varying electrical power loads. Another example of a regulator control system is the biological system, which

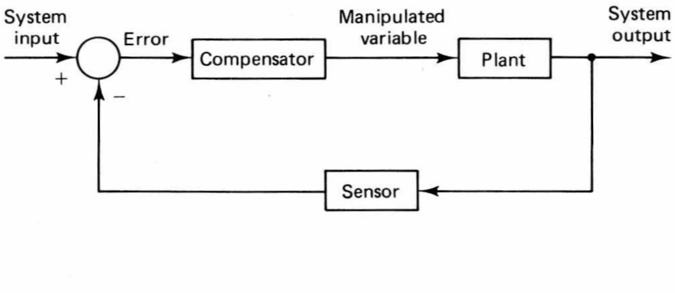

Figure 1.3 Closed-loop control system.

maintains the temperature of the human body at approximately 98.6°F in an environment that usually has a different temperature.

The second class of control systems is the *servomechanism*. Although this term was originally applied to a system that controlled a mechanical position or motion, it is now often used to describe a control system in which a physical variable is required to follow, or track, some desired time function. An example of this type of system is an automatic aircraft landing system, in which the aircraft follows a ramp to the desired touchdown point. A second example is the control systems of a robot, in which the robot hand is made to follow some desired path in space.

The preceding is a very simplified discussion of a closed-loop control system. The remainder of this book improves upon this description. In order to perform either mathematical analysis or design, it is necessary that we have a mathematical relationship between the input and the output for each of the blocks in the control system of Figure 1.3. The purpose of Chapter 2 is to develop these functional relationships for some common physical systems. Chapter 3 presents a different method of expressing these functional relationships.

We examine typical responses that occur in control systems in Chapter 4 and look at control system specifications in Chapter 5. Chapter 6 presents concepts and analysis techniques for system stability. The root locus, one of the principal methods of analysis and design, is developed in Chapter 7. Chapters 8 and 9 present a second principal analysis and design method, the frequency response.

In Chapters 2 through 9 it is assumed that no signals appear in sampled form and in particular that no digital computers are used in the control of the system. The systems considered in these chapters are called *analog* systems, or *continuous-data* systems. Chapters 10 through 12 consider systems in which sampling does occur, and these systems are called *sampled-data* systems. If a digital computer is used in the control of these systems, the systems are then called *digital* control systems. The term *discrete* systems is also used to refer to sampled-data systems and digital control systems. Chapter 13 presents an introduction to a different method of design of analog control systems, which is classified as a modern control procedure.

In the systems of Chapters 2 through 13 all systems are assumed to be linear (linearity is defined in Chapter 2). However, physical systems are not linear, and in general, nonlinear systems are difficult to analyze or design. Throughout this book we discuss the problems of the inaccurate representations in the functional relationships that we use to model physical systems. However, for some systems, nonlinearities must be added to the system model to improve the accuracy of these functional relationships. We consider some common nonlinearities and some analysis methods for nonlinear systems in Chapter 14.

In the analysis of linear systems, we use the Laplace transform for analog systems and the z-transform for discrete systems. The latter part of this chapter presents the concepts and procedures of the Laplace transform, and the z-transform is covered in Chapter 10. However, before the Laplace transform is discussed, the control problem is presented.