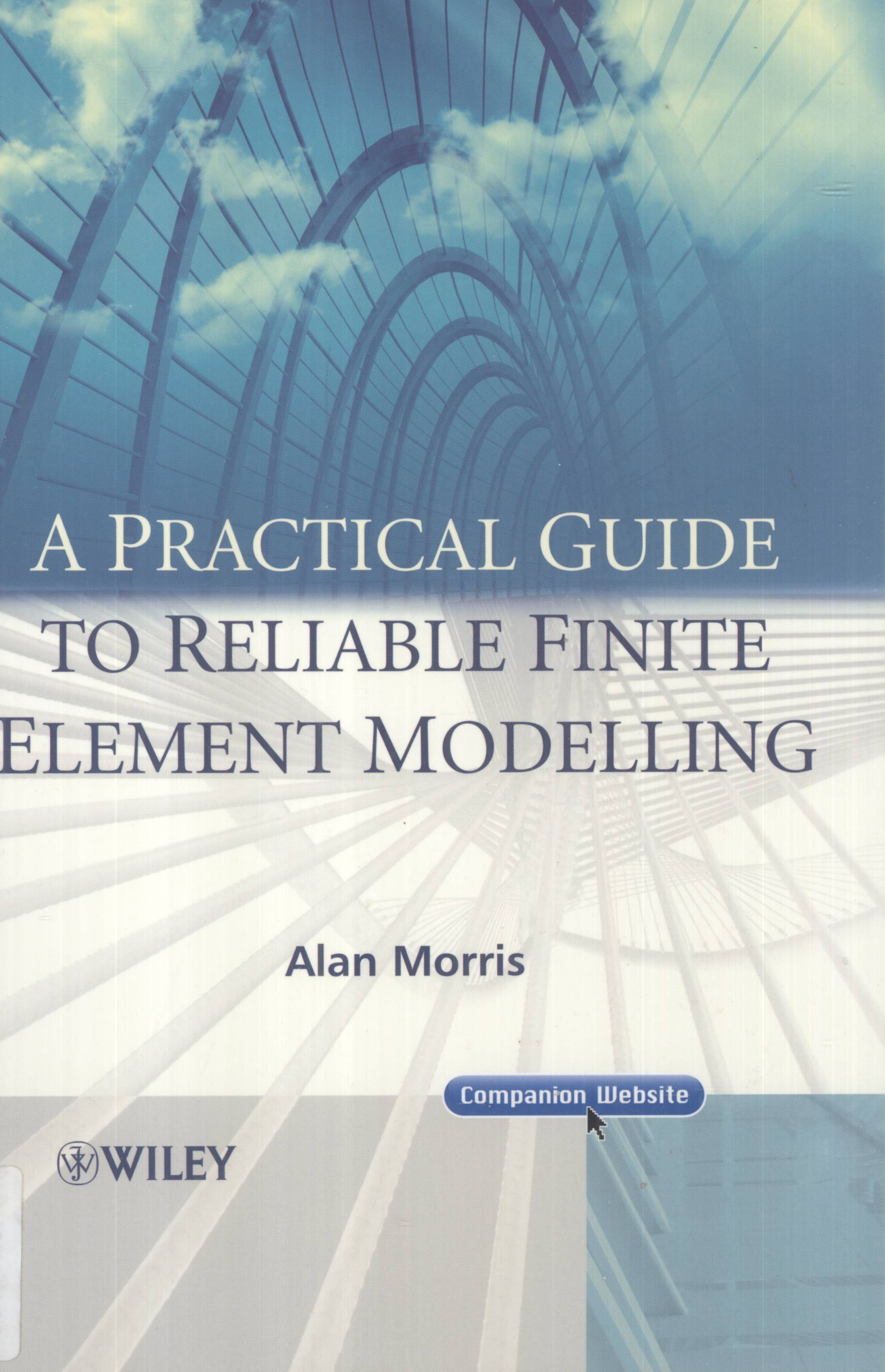

A Practical Guide to Reliable Finite Element Modelling

Alan Morris

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A Practical Guide to Reliable Finite Element Modelling

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To Lilian,
for endless patience.

Preface

This book is not a standard finite element text that can be used to provide all the information required to obtain a well-grounded understanding of all aspects of the Finite Element Method. There are already many excellent books on this topic and a number of these are referenced in Chapter 1, Introduction. The question that the book attempts to answer is 'How can an error-controlled finite element analysis be performed?' It is tempting to think that with the development of comprehensive finite element packages there is no need to worry about errors and uncertainties. Sadly this is not the case.

The Sleipner oilfield within the Norwegian sector of the North Sea is one of the major sources of oil and gas for Europe. The Sleipner A platform is a concrete gravity base structure consisting of 24 cells that rest on the sea bed at a depth of 82 m with a total base area of 16,000 m^2. Four of these cells are elongated so that they reach above the surface of the sea and support a deck that weighs 57,000 tons and drilling equipment weighing 40,000 tons. On 23 August 1991, while being prepared for deck mating through a controlled ballasting operation in the Gandsfforden outside Stravanger, the first Sleipner A platform sprang a leak and sank. The crash caused a seismic event of 3.0 on the Richter scale, left a pile of debris at a depth of 220 m and an economic loss of £700 million. The cause of the crash was traced to an inaccurate finite element analysis that underestimated the shear stress in the cells by 47%, so that certain concrete walls were not thick enough. After the accident, a more careful finite element analysis was performed on the original Sleipner A platform which predicted a structural failure at 62 m matching well with the actual failure depth of 64 m. Had an effective quality system been in place that allowed the analysis team to control the errors and uncertainties in the analysis, this failure could have been avoided. It may be thought that because the event took place some time ago the current situation would be much better, but, to the author's personal knowledge, other serious analysis failures have taken place recently. These have not been publicised as legal action was taken but were resolved at the courtroom door following an agreed compensation package.

In order to avoid such distressing consequences an analyst needs to have both sufficient basic knowledge of the Finite Element Method and a procedure for systematically performing a finite element analysis. This book aims to satisfy both these needs by providing essential background knowledge and information and a sequential application process. The book draws on two sources. One is information from lectures developed at Cranfield University and given to postgraduate aeronautics students and industrial short courses given both at Cranfield and in-house at international aerospace companies in the UK and elsewhere. The second source is the research output from a major UK government-funded project, within the Safety Critical Systems initiative, entitled SAFESA™, under contract DTI/EPSRC project 9034. Five organisations were involved in the project: Cranfield University, Lloyds Register, W.S. Atkins, Nuclear Electric (now British Nuclear Group) and Assessment Services (now Siemens). The author is particularly grateful to a number of colleagues involved in the SAFESA project who worked for these companies: Dr Mike Fox, Dr John Maguire, Dr Nigel Knowles and Professor Rade Vignjvec. Through creative and innovative thinking these engineers came forward with concepts and ideas that have significantly influenced the contents of this book.

Although Chapter 9 draws on the output of the SAFESA project, the method presented therein is distinctive. Nevertheless, the reader may wish to take advantage of the earlier work and this can be done, at one level, by consulting references [1] and [2] which present a synopsis of the main SAFESA results. A fuller description can be found in the *SAFESA Technical Manual* that was issued to the technical community, at the conclusion of the project, by the Minister then in charge of the Department of Industry and Science, the Rt Hon. Michael Heseltine MP (now Lord Heseltine). The SAFESA project team subsequently gave the NAFEMS organisation permission to reprint the *Manual* and copies can be obtained from that organisation through its offices in East Kilbride, Glasgow, UK.

The companion website for the book is http://wiley.com/go/morrisfem

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Contents

1

Introduction

1.1 AIM OF THE BOOK

There are many excellent text books on finite element theory incorporating the development of specific types of finite elements and describing the associated solution processes. This book has a different purpose from these standard texts as it provides a practical guide for the reliable use of the Finite Element Method in supporting the design of complex structures. Within this broad framework it gives an introduction to the Finite Element Method and links it to the problems associated with creating an effective and relatively error-free finite element model for solving a real-world structural design problem. By error is meant the difference between the finite element analysis's predicted behaviour and response of a structure subjected to applied loads and that which occurs when the structure enters service where the in-service loads come into play.

In practical terms the book is intended to assist engineers and companies involved with finite element analysis on a regular basis to operate in a manner that:

1. Reduces the possibility that any type of error is introduced into a finite element analysis.
2. Ensures that analyses undertaken by an individual analyst or analysis team are performed to a consistent and reliable standard.
3. Provides documentary evidence of having adhered to a consistent error control process as a basis for a defence in legal proceedings

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should a structural failure occur after a finite element analysed product has entered service.

Clearly one of the key aspects of the book is the provision of a methodology that allows a finite element analysis of a structure to be undertaken in such a manner that potential differences between the values for specific behaviour parameters obtained from the analysis and the measured values from operational use are identified and controlled. This requires that the analyst is not only able to identify the sources of error that may give rise to such differences, but also able to provide bounds on their maximum likely value. The targeted parameters should be selected by a process that clearly and explicitly defines the qualification criteria that, when satisfied, allow the structure to be constructed and enter service in a manner that renders it fit for purpose. In essence, the process is attempting to generate a procedure that places analysis as the primary route for the qualification of a structure. This creates a new environment in which testing is analysis controlled and is employed to support the analyst, providing information for the bounding or control of potential errors. In this situation, testing is a subservient activity because the analyst defines specific requirements for test data to compensate for identified deficiencies in the finite element analysis. If a test is now used in the proof of a structure, it is there simply to validate the analysis which has become the actual validating machine.

In attempting to satisfy the requirements listed above the book offers a basis for constructing a logical approach to finite element analysis. This is ambitious and it is not claimed that it provides a complete and totally comprehensive method for satisfying this requirement. Rather it provides a door through which the reader is invited to step and after crossing the threshold develop the ideas presented herein into a more comprehensive and authoritative method that is personal to an individual analyst or analysis team. In the case of an inexperienced or new finite element analyst, it provides a starting point. For an experienced analyst or a company that regularly undertakes finite element analyses, it should be taken as an input into what should be a regular review of their finite element qualification process.

In order to keep the length and complexity of the book under control the problem domain is restricted to linear static and linear dynamic structural analyses. Nevertheless, the broad approach adopted in the chapters devoted specifically to error control and treatment has general applicability.

Finally, it is worth noting that this book is not intended as a broad introduction to the use of finite element analysis in engineering design; this is covered by Adams and Askenazi [1]. Nor does it focus on the development of internal error bounds and the use of this type of bounding process in h- and p-type adaptive meshing codes. However, the use of such codes is touched on as they provide one component in a total error and uncertainty control methodology. Details of error estimation techniques based on internal and self-referencing procedures are covered in the excellent book by Szabó and Babuška cited as reference [2] and, in more detail, by reference [3].

1.2 FINITE ELEMENT TYPES – A BRIEF OVERVIEW

The underlying principle of the Finite Element Method is that a physical structure is modelled as an assemblage of individual elements as outlined in Chapters 2 and 3 but more fully in books addressing the mathematical fundamentals such as references [4], [5], [6]. All finite element models employ polynomial approximations to at least one of the main fields employed in describing the physical phenomena that are the focus of the analysis. In this book, attention is restricted to the analysis of loaded structures responding in a manner that can be modelled using elasticity theory. For this class of modelling problems there are three basic element types: displacement elements, equilibrium elements and hybrid elements. All commercially available finite element packages and systems employ displacement finite elements, many employ some hybrid elements and a few have equilibrium elements. Chapters 2 and 3, in outlining some of the fundamentals of the method, use displacement elements. However, most of the arguments advanced in this book apply equally to all three types.

A schematic of a displacement finite element is shown in Figure 1.1. The displacement on the interior of the element is approximated using relatively low-order polynomials. These polynomials must have a form that ensures the displacements at the edge or edge surfaces of the element can link up with adjacent elements in such a way that certain components of the displacement field are continuous across adjacent element interfaces. In the case of plates and shells the polynomials must be able to ensure continuity of the appropriate rotation terms. The

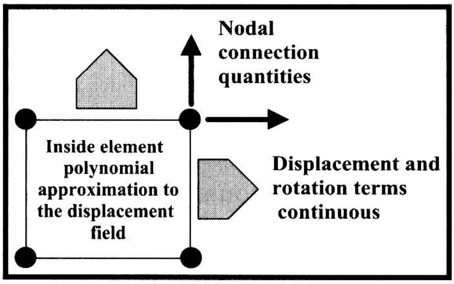

Figure 1.1 Schematic of a displacement element.

polynomials are then defined in terms of nodal values that can be specified at a vertex, as shown in Figure 1.1, or at specific points along element edges or surfaces. Adjacent elements are now connected to each other through these nodes and because of this the nodal displacements or rotations are called connection quantities. Loads are applied to the finite element model through these same nodes. It is worth noting that the displacement finite element formulation degenerates the structure under analysis into a set of points distributed through the space occupied by the structure and there is no longer any explicit representation of the actual structure nor any explicit representation of the physically distributed load system. As shown in Chapters 2 and 3, the resulting nodal model is then solved in terms of the initially unknown nodal connection quantities and terms such as element stresses are derived from this solution.

The formulation for an equilibrium finite element is similar, in principle, to that of the displacement element as shown in Figure 1.2. In the case of an equilibrium element, the interior stress field is approximated by polynomials and the connection from one element to the next is via side or surface forces that are distributed along the element edges and surfaces. As shown in Chapter 4, the displacement

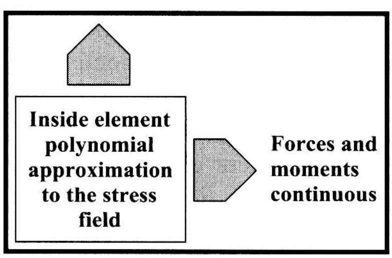

Figure 1.2 Schematic of an equilibrium element.