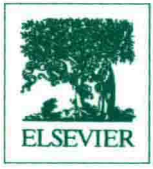

PROGRESS IN HETEROCYCLIC CHEMISTRY

VOLUME 16

EDITORS
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Functional Equations in Applied Sciences

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2005
ELSEVIER
Amsterdam – Boston – Heidelberg – London – New York – Oxford
Paris – San Diego – San Francisco – Singapore – Sydney – Tokyo

ELSEVIER B.V.	ELSEVIER Inc.	ELSEVIER Ltd	ELSEVIER Ltd
Sara Burgerhartstraat 25	525 B Street, Suite 1900	The Boulevard, Langford Lane	84 Theobalds Road
P.O. Box 211, 1000 AE Amsterdam	San Diego, CA 92101-4495	Kidlington, Oxford OX5 1GB	London WC1X 8RR
The Netherlands	USA	UK	UK

First edition 2005

Library of Congress Cataloging in Publication Data
A catalog record is available from the Library of Congress.

British Library Cataloguing in Publication Data
A catalogue record is available from the British Library.

ISBN: 0-444-51788-x
ISSN (Series): 0076-5392

♾ The paper used in this publication meets the requirements of ANSI/NISO Z39.48-1992 (Permanence of Paper).
Transferred to digital printing 2007.

Functional Equations in Applied Sciences

This is volume 199 in
MATHEMATICS IN SCIENCE AND ENGINEERING
Edited by C.K. Chui, *Stanford University*

A list of recent titles in this series appears at the end of this volume.

DEDICATION

To Janos Aczél with admiration.

Note from the Editor

Founded about four decades ago by the visionary distinguished mathematician, the late Richard Bellman, this "red series" Mathematics in Science and Engineering (MISE) has the entrepreneurial tradition of being one of the very first to publish interesting monographs of mathematical topics that may have the potential to make a significant impact to the advancement of sciences and engineering. Since the unfortunate early departure of Professor Bellman, a lot has been happening in mathematics that is beyond the fascination of solutions of "Big Problems" and settlement of well-known conjectures. Various exciting research areas and directions that have more direct applications in the scientific and engineering fields have been introduced. Even the tradition of pursuing individual mathematical research has gradually been adapting to the more common way of carrying out collaborative work in other scientific disciplines. Indeed, it has been a very interesting period of changes, but these changes are both natural and necessary.

The invention of semiconductors, integrated circuits (IC), as well as the exponential rate of technological advancement in IC functionalities and "chip" size, called Moore's law, has been the core and source of this "industrial revolution." Most notably, the unbelievable escalation in computing power, along with the most significant miniaturization of computing devices, not only leads to rapid advancement of all areas in sciences and engineering, but also becomes the source of creating new fields and research directions. At the same time, the tremendous IC capabilities have also significantly advanced such technologies as intelligent sensors, which, in turn, rely on computing. The enormous computing power has also played the key role in advancement and creation of new fields and new directions in other subject areas beyond the traditional science and engineering subjects, including economics, commerce, etc. Hence, the scope of science and engineering applications for MISE, as originally envisioned by Professor Bellman, is to be broadened accordingly as well.

In taking over the editorship of this book series of MISE from Professor William Ames, I am committed to preserving the entrepreneurial spirit of the founder, Professor Bellman, by encouraging publication of mathematics monographs that have the potential to make an impact on the advancement of sciences and engineering technologies, now under a broader scope. It is still the "red series", only with a new artistic design to reflect the closer and more direct relationship between the advancement of mathematics and that of other scientific and engineering fields to be interpreted in broader horizons. We welcome submission of book proposals and manuscripts from all fellow mathematical scientists who share a similar vision of MISE. We need our readers' support to make a lasting impact on the advancement of sciences and engineering technologies.

Charles Chui
Editor-in-Chief
Stanford, California
July, 2004

Note from the Publisher

Having been established in the 1960's by Academic Press, the *Mathematics in Science and Engineering*, "red series", became well-known under the leadership of the founding editor Richard Bellman, many pioneering works being produced.

Almost two hundred volumes were published, up to volume 198 by Igor Podlubny in November 1998, the Editor-in-Chief at this stage being Professor William Ames at Georgia Tech.

After the acquisition of Harcourt General by Elsevier in 2001, which included Academic Press, responsibility for publication of the "red series" passed from the San Diego office of the former AP to Elsevier in Amsterdam, as part of the merging of the two individual publishing programs.

Following the completion of the detailed merger, we are very happy to announce the continuation of the MISE "red series" under the editorship of Prof. Charles Chui, Stanford, USA.

It is our intention to publish around 3 volumes per year, of the highest level of mathematical sciences scholarship, starting with the present vol. 199 by Castillo, Iglesias & Ruíz-Cobo

Keith Jones
Publisher
PMCA-- Physics, Mathematics, Computer Science & Astronomy
Elsevier

Preface

Functional equations is one of the most powerful and beautiful fields of Mathematics we have encountered in our professional life. It was during the summer of 1983, on the occasion of a stay at the ETH (Zürich), when E. Castillo together with A. Fernández-Canteli discovered for the first time the real importance of functional equations. We were trying to model the influence of length and stress range on the fatigue life of longitudinal elements and, when analyzing the inconsistencies of some tentative models, we found a compatibility equation written in terms of a functional equation. Immediately, the 1966 Aczél book on functional equations came to our minds (two or three years earlier, somebody in our library had ordered the book and so it was only by chance that we had the opportunity of taking a look at it without realizing, at first glance, its real importance, yet noting that some powerful methods were behind it). Since then, we have completely changed our minds and incorporated the functional equations' philosophy and techniques to our daily procedures. Even though many years were required to find our first functional equation, many others have appeared since then in our work, and, in fact, today we cannot think of building models or stating problems without using functional equations.

Our experience is that model building in science and engineering is frequently performed based on selecting simple and easily tractable equations that seem to reproduce reality to a given quality level. However, on many occasions these models exhibit technical failures or inconsistencies, such as those we discovered in our fatigue models when we obtained the compatibility equation, and which make them unacceptable. Functional equations is one of the main tools that prevent arbitrariness and allow a rigorous and consistent selection and design of models. In fact, conditions required by many models to be adequate replicas of reality can be written as functional equations.

Functional equations arise in many fields of Applied Science, such as Mechanics, Geometry, Statistics, Hydraulics, Economics, Engineering, etc. However, though the theory of functional equations is very old, not only techni-

cians but many mathematicians are still unaware of the power of this important field of Mathematics. As J. Aczél and J. Dhombres indicate in the preface of their book: *"from their very beginnings, functional equations arose from applications, were developed mostly for the sake of applications and, indeed, were applied quite intensively as soon as they were developed"*. However, most of the recent advances in the theory of functional equations have been published in mathematical journals which are not written in a language that many engineers and scientists can easily understand. This fact, which is common to many other areas of Mathematics, has been the reason why many engineers and applied scientists are still unaware of a long list of these advances and, consequently, they have not incorporated common functional equation techniques into their daily procedures.

Our experience with functional equations was so positive and relevant to applications that we became engrossed in this still relatively unknown field of Mathematics. Impressed by its importance and wishing to share with others this discovery, we decided to write the present book.

One of the aims of this book is to provide engineers and applied scientists with some selected results of functional equations which can be useful in applications. We are aware that this is not an easy task, and that any effort to bring together mathematicians and engineers, as experience shows, has many associated difficulties. We have, intentionally, omitted or simplified many proofs and details of theorems in order to make the text more readable to engineers. However, we wish to go even further, trying to offer the readers a different point of view and offer them a new way of thinking in mathematical modelling. Traditionally, engineers and scientists state practical problems in terms of derivatives or integrals, which lead to differential or integral equations, respectively. With this book we want to offer them the possibility of using functional equations too, as one more alternative, which is at least as powerful as either of the other two.

This book, which is based on lectures delivered by the authors at the University of Cantabria and in the book "Functional Equations in Science and Engineering", published by Marcel Dekker in 1992, focuses primarily upon applications and includes many examples of applications aiming to illustrate how functional equations are the ideal tool to design mathematical models. Thus, special attention is given to the analysis and discussion of the functional equations, in the light of their physical meaning, and to practical examples.

The book is organized in two parts. The first part is devoted to functional equations in general. Chapter 1 is an introduction to functional equations. In it, we use several simple problems to motivate functional equations. The beauty of functional equations becomes apparent when some formulas, such as the area of a rectangle or a trapezoid, or the interest formulas, arise as the only expressions that satisfy some natural conditions. Furthermore, we discover generalized formulas showing that the standard formulas are not sufficient to deal with all practical cases. In Chapter 2 an important effort has been made to identify a list of methods to solve functional equations, and give some illustrative examples to facilitate its understanding. We know of no other book giving this general

methodology to solve functional equations. In Chapters 3 to 6 several functional equations in one or several functions in one or several variables are discussed, and several examples of applications are given. In Chapter 7 we discuss the problem of equivalence of functional, difference, and differential equations and use this equivalence to solve functional equations. The possibility of stating problems as functional equations, as an alternative to the usual statement of problems, based on differential or difference equations, is a new and powerful alternative that deserves special attention. To end this part, Chapter 8 deals with vector and matrix equations.

In the second part we apply functional equations to solve a wide range of practical problems. In Chapter 9 we introduce functional networks, a powerful generalization of neural networks. It is shown how every functional equation or system of functional equations leads to a functional network, and how it can be exploited to solve functional equations numerically. Functional networks have proven to be a powerful technique that allows simple and very efficient networks to be built. In Chapter 10 we deal with some applications to engineering, including the laws of Science, models for fatigue life, and beam equations. In Chapter 11 some applications to Geometry and to computer aided design are presented. Chapter 12 is devoted to applications in the Economic field: taxation functions, price indices, interest formulas, and many other material, including monopoly and duopoly models, are analyzed. Finally, in Chapter 13 some applications to Probability and Statistics are presented. In particular, several families of distributions are characterized.

We would like to thank A. Fernández-Canteli, J. Galambos, Barry C. Arnold, and J.M. Sarabia, with whom we have done joint work related to functional equations, for their invaluable stimulus and encouragement.

We also thank José Antonio Garrido and Iberdrola for partial support of this book.

Special recognition must be given to Janos Aczél. As mentioned before, his 1966 book drew the attention of the authors to the field of functional equations and made possible all their work in this interesting area of Mathematics. Professor Aczél has marked the path to follow for all those who love functional equations. We must also mention the remarkable book of Eichhorn (1978), where extremely interesting applications to Economics were presented.

Finally, we wish to mention the scientific community, mainly those included in the bibliography and those who were, surely but unintentionally, omitted. They, through their life's work, have made this book possible. To all of them, our most sincere thanks.

Enrique Castillo
Andrés Iglesias
Reyes Ruíz–Cobo

Santander, June 10, 2004.

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Part I

Functional Equations