Contemporary College Mathematics

BRETT · CONTEY · SENTLOWITZ

Contemporary College Mathematics

WILLIAM F. BRETT
LOUIS C. CONTEY
MICHAEL SENTLOWITZ

Rockland Community College, Suffern, New York

WEST PUBLISHING CO. | St. Paul · New York · Boston
Los Angeles · San Francisco

Library of Congress Cataloging in Publication Data

Brett, William F.

Contemporary college mathematics.

Includes index.

1. Mathematics—1961– I. Contey, Louis C., joint author. II. Sentlowitz, Michael, joint author. III. Title.

QA39.2.B74 510 74–32492

ISBN 0–8299–0038–1

Contemporary College Mathematics

To our families.

Preface

Many mathematicians believe that one should study mathematics from a theoretical point of view. Although this method of study appeals to some people, it has become increasingly apparent that students who are not majoring in mathematics are interested in a more practical and less theoretical approach to the subject. They wish to use mathematics to solve problems that arise in the real world. This book recognizes this need and is designed to meet it. However, this is not a vocationally oriented text. Its scope is broad enough to allow the student to move in many different directions after completion of the material.

The book is divided into four major units of study.

Part I: Selected Topics in Modern Mathematics
Part II: Probability and Statistics
Part III: Algebra
Part IV: Mathematics of Finance and Insurance

Once the chapter on sets is mastered and the student is familiar with the material in Chapter 9 (Review of Algebra), the units of study are independent and may be studied in any order.

The text is directed mainly to students who are interested in humanities, social science, business and accounting, or government services. The specific needs of any of the above mentioned areas may be met by careful selection of topics from the text. For example:

For a liberal arts-oriented course the suggested chapters are 1 through 12.

For a business-oriented or social science-oriented finite mathematics course, the suggested chapters are 1 and 6 through 15.

We wish to acknowledge our appreciation to all those who assisted in the preparation of this textbook. We are particularly indebted to Professor Margaret Trivisone of Rockland Community College for her editorial work. We also wish to thank Professor Robert Stephens of Moorpark College and Professor Calvin Latham of Monroe Community College for their helpful comments.

In addition we would like to thank Dean Gerald Anders and Professor Judi McFatter of Rockland Community College for their encouragement and cooperation and Yolanda Sardone for her patience and support in typing the manuscript.

We are also indebted to George Morris for his artwork and layout and to Merl Miller of West Publishing Company for his over-all editorial direction.

Contents

†

PART I

Selected Topics in Modern Mathematics

1

Sets

1-1. Introduction

The concept of set theory is fundamental to the study of mathematics. Through the language of sets, we can describe and express many mathematical ideas. However, communication through symbols and special terms is not limited to mathematics. In recent years educators and social scientists, among others outside the field of mathematics, have been able to describe problems arising in their disciplines using sets. In this chapter, and in other parts of this book, we will make reference to the importance and applicability of sets in mathematics and in other disciplines.

What is a set? Intuitively we may associate the noun "set" with the more familiar noun "collection." If a set is simply a collection of things, then all of the following must be examples of sets:

1. The collection of vowels in our alphabet
2. The collection of counting numbers
3. The collection of small fractions
4. The collection of important presidents of the United States
5. The collection of valuable mathematics textbooks

While each of the above examples identifies a collection of things, there is a very important difference between the first two and the last three examples. The difference can be illustrated by asking yourself whether each of the following statements is true or false.

1. e is a vowel in our alphabet.
2. 1/4 is a counting number.
3. 3/5 is a small fraction.
4. William McKinley was an important president of the United States.
5. This book is a valuable mathematics textbook.

The correct responses for the first two statements are true and false respectively. However, it is not possible to determine whether the last three statements are true or false. There are no universally accepted lists of small fractions, important presidents of the United States, or valuable mathematics textbooks. When we talk about a set, we are talking about a well-defined collection of elements. To be well defined, we must be able to determine whether an element is or is not in the set. Therefore, the collection of vowels in our alphabet and the collection of counting numbers are sets. The collections of small fractions, important presidents of the United States and valuable mathematics textbooks are not well-defined collections, and thus are not sets.

Sets will be used extensively in the section on probability in this book. In order to discuss probability using sets, each set must be well defined. When introducing probability to their students, teachers often ask the question, "What is the probability of getting a head from one toss of a coin?" Most often the students will answer 1/2, or 1 out of 2, or 50-50. Occasionally a student will respond by asking, "Do we include the possibility of a coin landing on its edge?" The student's question may be answered by clearly defining the set of outcomes to include only heads or tails.

When determining whether or not a collection is well defined, beware of words in definitions that contain value judgements such as wealthy, pretty, tall, small, or cute. For example, the collections of wealthy men and pretty girls are not well-defined sets. Some girls may believe that a man must have over $1,000,000 in assets to be considered wealthy, while other girls may include men of more modest means in the collection. Also, your friend may be dating a "pretty" girl according to his criteria, but you may not consider her to be pretty.

Exercises

Determine whether the following collections of elements are sets. That is, is the collection well defined?

1. The collection of counting numbers less than 10,000
2. The collection of large even numbers
3. The collection of tall students in your school
4. The collection of cities in New York State
5. The collection of positive integers less than zero
6. The collection of wealthy Frenchmen
7. The collection of short stories written by Ernest Hemingway
8. The collection of interesting short stories written by Ernest Hemingway
9. The collection of mathematics teachers in small colleges in the United States
10. The collection of racing drivers who have won the Indianapolis 500
11. The collection of strong mixed drinks
12. The collection of your favorite cheeses
13. The collection of great punters in the National Football League
14. The collection of women who are presently presidents of banks in the state of Illinois
15. The collection of good movies produced in 1974

1-2. Methods of Defining Sets

There are two methods which may be used to define the elements of a set. These are the rule method and the roster method. Some sets may be easily described using either method. However, for certain sets, one method may not be practical, and the other must be used.

Using the rule method, we describe the elements of a set without listing them individually. Using the roster method, the elements of the set are listed one by one. If we were to list each student in your class and call that a set, then we would be using the roster method. If we used the statement "The students in math section WO2," we would be using the rule method to describe the same set. We could list the letters in the alphabet as an example of the roster method, or make the statement "The set containing all the letters of the alphabet" as an example of the rule method. There are occasions when it is impossible to describe the same set using both methods. Try to use the rule method to describe the set whose elements are $\Delta, \alpha, \Sigma, \Psi$. Or try to use the roster method to list the set of fractions between 1 and 2. We shall discuss sets like the latter one further on in this chapter, but now let us formally structure the notation used for the roster and rule methods of describing sets.

It is customary to identify each set with a capital letter, and to enclose the elements or the description of the elements with braces. For example:

A = {1,2,3,4}

or

A = {The set of positive integers less than five}.

The use of the capital letter affords us the opportunity to refer verbally to the set {1,2,3,4} as set A, instead of saying, "The set of elements 1,2,3, and 4," or "The set of positive integers less than 5." Thus, the use of the capital letter results in a conservation of words-an objective of any shorthand system. The choice of braces to enclose elements, while arbitrary, must be adhered to. The notation "{1,2}" refers to the set containing the elements 1 and 2. The notation "(1,2)" identifies a point in the Cartesian plane, not a set.

1. The following sets are defined by using the roster method.

   A = {1,2,3,4,5,6}
   B = {a,e,i,o,u}
   C = {3,4,5}

2. The following three sets are defined by using the rule method.

   A = {The set of positive integers less than 7}
   B = {The set of vowels in the English alphabet}
   C = {The set of positive integers between 3 and 5 inclusive}

3. There is a refinement of the rule method, called <u>set-builder notation</u>, which is especially useful for describing sets whose elements are numbers. The basic set-builder model is:

   M ={x|x has a specific property}

   An example using this model is:

   M = {x|x=1,2,3}.

   When broken up into parts, this is read:

{x	"the set of x's"
|	"such that"
x=1,2,3}	"x is equal to 1,2 or 3."

Sets A and C in example 2 are restated below, using set-builder notation. In both examples the inequality symbol "$\leq$" is used for the phrase "less than or equal to."

$A = \{x|x \text{ is a positive integer and } x \leq 6\}$

$C = \{x|x \text{ is a positive integer and } 3 \leq x \leq 5\}$

The first may be read: "A is equal to the set of x's such that x is a positive integer, and x is less than or equal to 6."

4. The rosters for each of the following sets appear on the right.

Set-builder notation	Roster
$E = \{x\|x + 7 = 12\}$	$E = \{5\}$
$F = \{N\| \ 3N - 1 = 4\}$	$F = \{5/3\}$
$G = \{x\|x \text{ is a positive integer } <4\}$	$G = \{1,2,3\}$

To avoid a verbal statement concerning membership in a set, the following shorthand notations can be used.

$\in$ reads "Is an element of."

$\notin$ reads "Is not an element."

For example, if $M = \{1,2,3,5,7,9\}$, then $5 \in M$ and $4 \notin M$.

Exercises

1. Use the roster method, if possible, to identify the elements in each set.

   a. {The set of even integers between 9 and 19}
   b. $\{X \mid 3X + 1 = 13\}$
   c. $\{X \mid X \text{ is a positive integer and } 2X + 1 \leq 10\}$
   d. $\{N \mid 2N + 1 \text{ is an odd positive integer } < 16\}$
   e. $\{N \mid 2N + 1 \text{ is a positive integer } < 21, \text{ divisible by } 5\}$
   f. $\{W \mid W \text{ is a multiple of 7 that is } \leq 50\}$
   g. {The date of the last day of the 20th century}
   h. $\{X \mid 2X + 7 = 10\}$
   i. $\{X \mid X + 1 = 1\}$

2. Use the rule method, if possible, to identify the elements in each set. When there exists only one element, write an equation whose solution is the element. For example, $\{4\} = \{X|2X = 8\}$.

   a. $\{1,2,3,4,5\}$
   b. $\{7\}$
   c. $\{2,4,6,8,10\}$
   d. $\{1,4,9,16,25\}$

e. {1,4,9,16,25,36,49,64,81}

f. {2,3,5,7,11,13,17,19,23,29,31,37,41,43,47}

g. {3,6,9,12,15,18,21,24,27,30,33,36}

h. {0}

i. {1,8,27,64,125}

j. {3}

3. If E = {X|X is divisible by 3}, answer true or false:

a. $40 \in E$.

b. $21 \in E$.

c. $36 \notin E$.

d. $91 \in E$.

e. $51 \notin E$.

f. E contains more than 20 elements.

g. E contains less than 10 million elements.

h. $3 \notin E$.

i. If F = {X|X is divisible by 5}, then E and F have more than 2 elements in common.

j. E and F have less than 10 million elements in common.

1-3. Finite and Infinite Sets

In some of the important applications of elementary set theory, the number of elements contained in a set is more important than the nature of the elements or their properties. This is especially true in the field of probability, where formulas involving the number of elements in a set are used. When we refer to the number of elements contained in a set, we are referring to its cardinality. With respect to cardinality, sets can be divided into two major categories, finite and infinite.

1. A set is finite if its cardinality is a non-negative integer, that is, 0, 1, 2, 3, etc. The notation "n(A)" is read: "the number of elements in set A." Some examples of finite sets, along with their cardinal numbers, are listed below.

Set	Cardinal Number
A = {3,7,2,6}	n(A) = 4
B = {a,e}	n(B) = 2
C = {1,2,3,4,5,6,7,8}	n(C) = 8
D = {101,103,105,107,109}	n(D) = 5
E = {x| x+5 = 17}	n(E) = 1
F = {x|x^2 = 9}	n(F) = 2