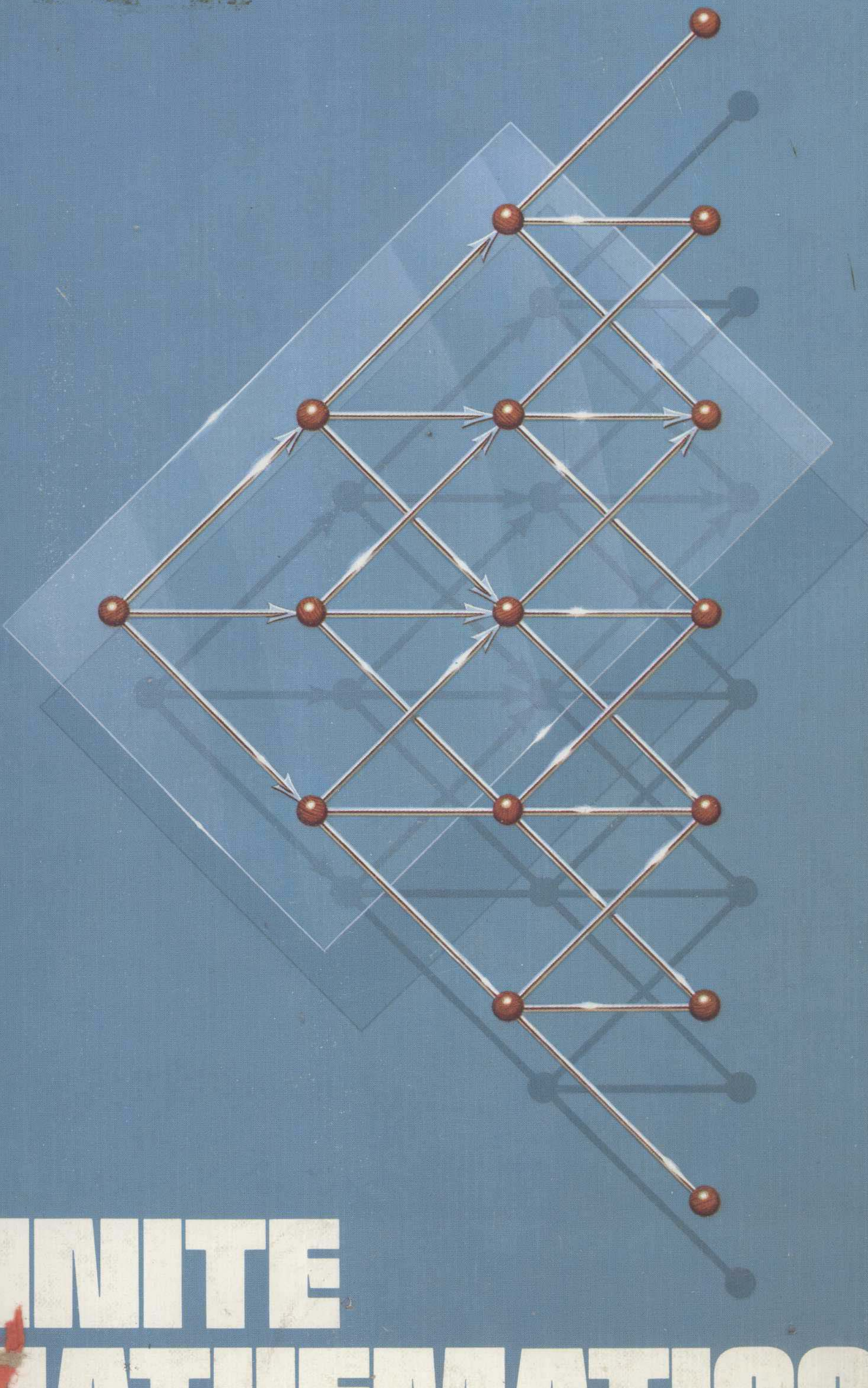
DAVID A. SPRECHER



FINITE MATHEMATICS

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PREFACE

This text offers an intuitive introduction to finite mathematics, suitable for a one-semester or one-quarter course. Finite mathematics courses are often elected by students who seek a basic mathematics course, and my purpose has been to introduce at this level ideas and techniques which, in addition to their intrinsic interest and challenge, are of great importance in many academic and nonacademic areas.

The approach adopted for this text is to teach the subject through active student participation. The conceptual development and the associated problem-solving techniques are, therefore, integrated and treated as inseparable components of the text. The numerous worked-out examples are used to bridge the gap between theory and exercises. They show how theoretical considerations lead to practical tools, and they can be used as models for doing the exercises. The large number of graded exercises should be regarded as an integral part of the text, and a good number of these should be done as a matter of routine. In addition to enhancing the learning process, they will also serve as an excellent indicator of the degree of comprehension of the material.

This text begins where the high school leaves off. For many students, however, it may have been one or more years since their last mathematics course, and like other unused languages and skills, mathematics is easily forgotten. The text therefore begins with a concise review of the needed terminology and facts, together with sets of exercises for regaining facility in computations.

Because of variability in student interests and goals, the chapters have been kept self-contained as much as possible. This gives desired flexibility in omitting material or covering it out of sequence. The extensive cross-referencing will direct the reader in one section to the relevant material elsewhere in the book.

A very basic course can be made up of Chapters 0 (as needed), 1, and 3–6, with the possible omission of Sections 1.4, 3.3, 3.7, 4.4, 6.4, and 6.6. An alternative choice would be to include Chapter 7 but omit Chapter 2 and Sections 3.3, 3.7, and 6.6. Yet another choice for a short course could consist of Chapters 0–6, with the possible omission of Sections 2.5, 2.6, 3.3., and 6.6.

I am indebted to many individuals who, one way or another, extended their help and advice. I wish to express particular appreciation to Jean Mulder and Richard Hull for working the exercises and making useful editorial suggestions; to Sonia Ospina and Devora Sprecher for the expert typing of the manuscript; and to George Telecki, Brenda Goldberg, and the rest of the dedicated staff of Harper & Row for their support and attention during the different stages of the production of this book.

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REVIEW

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The purpose of this material is to provide a quick review of the basic properties of rational numbers, powers, and inequalities. An inadequate understanding of these is a common cause of difficulties in computations, which often makes the difference between success and failure in a mathematics course. Working the exercises below is an effective way of brushing up on the basic manipulative skills required for this text, and many students should benefit from this.

0.1 THE REAL NUMBERS

We begin with an informal discussion of the real numbers for the purpose of recalling basic facts and terminology.

Most familiar to us are the *natural numbers* 1, 2, 3, . . . to which we were introduced early in our education. These numbers, also called *counting numbers*, or *positive integers*, were then augmented to form the *integers* . . ., -3, -2, -1, 0, 1, 2, 3, ... Many problems, such as those involving measures and weights, cannot be solved with integers, and so the *rational numbers* were introduced. The most important discovery at this stage was the fact that numbers can be associated with points on a straight line. Let us briefly describe how this is done.

On a straight line, taken for convenience to be horizontal, we select an arbitrary point as the *origin* and label it 0; a second point is chosen to its right and labeled 1. The line segment $\overline{01}$ is called *unit length* (see Figure 0.1), and with it we mark equally spaced points to the right and to the left of 0. The point lying n units to the right of 0 is labeled n, and the point lying n units to the left of 0 is labeled $-n$.

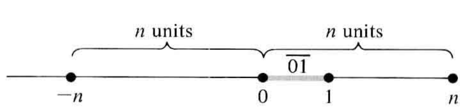

FIGURE 0.1

Dividing each interval so created into two equal parts gives the points associated with $\frac{1}{2}$, $-\frac{1}{2}$, $\frac{3}{2}$, $-\frac{3}{2}$, and so on. In general, for any positive integer n, the points associated with the numbers $1/n$, $-1/n$, $2/n$, $-2/n$, $3/n$, $-3/n$, and so on, are obtained by dividing each of the intervals created by the integers into n equal parts.

Of great significance in mathematics is the fact that infinitely many points on the line have no rational numbers corresponding to them. Numbers which cannot be written as a quotient of two integers are called *irrational*, and examples of such numbers are $\sqrt{2}$, $\sqrt{3}$, and π. Together, the rational and irrational numbers make up the *real numbers*.

Many examples and exercises in subsequent sections involve manipulations with rational numbers. For the purpose of this text, a rational number can be thought of as a fraction a/b, where a and b are integers and $b \neq 0$. We observe that $2/10$, $3/15$, and $6/30$, all represent the same rational number, which in *lowest terms* is $1/5$. Equality of rational numbers is defined as follows:

Equality of rational numbers

$$\frac{a}{b} = \frac{c}{d} \quad \text{if and only if} \quad ad = bc$$

Thus,

$$\frac{2}{10} = \frac{3}{15} \quad \text{since} \quad 2 \times 15 = 10 \times 3$$

and so on.

The arithmetical operations of rational numbers satisfy the following rules.

Rules for operating with rational numbers

(1) $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

(2) $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$

(3) $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

(4) $\frac{a}{b} \Big/ \frac{c}{d} = \frac{a \times d}{b \times c}$

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Example 1

(1) $\frac{2}{3} + \frac{5}{7} = \frac{2 \times 7 + 3 \times 5}{3 \times 7} = \frac{29}{21}$

(2) $\frac{2}{3} - \frac{5}{7} = \frac{2 \times 7 - 3 \times 5}{3 \times 7} = \frac{-1}{21} = -\frac{1}{21}$

(3) $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$

(4) $\frac{2}{3} \Big/ \frac{5}{7} = \frac{2 \times 7}{3 \times 5} = \frac{14}{15}$

The following summation formula is often useful:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \quad \text{for any positive integer } n$$

This formula is derived as follows: Suppose the sum of the first n positive integers equals A. Then we can write down the following scheme:

$$\begin{array}{ccccccccccccc} 1 & + & 2 & + & 3 & + \cdots + & (n-1) & + & n & = A \\ n & + & (n-1) & + & (n-2) & + \cdots + & 2 & + & 1 & = A \\ \hline (n+1) & + & (n+1) & + & (n+1) & + \cdots + & (n+1) & + & (n+1) & = 2A \end{array}$$

$\underbrace{\qquad\qquad\qquad\qquad}_{n \text{ terms}}$

From this we see that

$$2A = n(n+1)$$

and hence

$$A = \frac{n(n+1)}{2}$$

Example 2

$$1 + 2 + 3 + \cdots + 100 = \frac{100 \times 101}{2} = 5050$$

EXERCISES

1. Insert one of the symbols $=$ or $\neq$ into each box to produce a true statement.

(a) $1\Big/\frac{a}{b}\ \square\ \frac{b}{a}$

(b) $\frac{a}{b} - c\ \square\ \frac{bc}{a}$

(c) $\frac{2a + 3b}{4a + 9b}\ \square\ \frac{a + b}{2a + 3b}$

(d) $\frac{a + b - c}{b}\ \square\ 1 + \frac{a - c}{b}$

(e) $\frac{1}{b}\Big/1\ \square\ b$

(f) $\frac{a}{b} \times \frac{c}{d} \times \frac{d}{a}\ \square\ \frac{d}{b}$

(g) $2 + 4 + 6 + 8 + 10 + 12\ \square\ 2 \times \frac{6 \times 7}{2}$

(h) $5 + 10 + 15 + \cdots + 100\ \square\ 5 \times \frac{20 \times 21}{2}$

Simplify the following fractions as much as possible.

2. $\frac{\frac{1}{2}}{2}$

3. $\frac{12}{-\frac{1}{3}}$

4. $\frac{\frac{13}{1}}{\frac{1}{13}}$

5. $\frac{\frac{1}{13}}{\frac{13}{1}}$

6. $\frac{-\frac{5}{2}}{\frac{3}{2}}$

7. $\frac{\frac{7}{2} - 1}{\frac{7}{2} + 1}$

8. $\frac{9 - \frac{1}{3}}{\frac{1}{3}}$

9. $\frac{-\frac{1}{4} + 2}{\frac{1}{4} - 2}$

10. $\frac{\frac{1}{2} - \frac{1}{3}}{\frac{1}{2} + \frac{1}{3}}$

11. $\frac{\frac{2}{3} - \frac{3}{5}}{\frac{3}{5} - \frac{2}{3}}$

12. $\frac{\frac{1}{2}}{\frac{1}{7} - \frac{1}{9}}$

13. $\frac{1}{2} + \frac{1}{3} - \frac{1}{7}$

14. $\frac{\frac{1}{2}}{\frac{1}{3}}$

15. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

16. $\dfrac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{ab}}$

17. $1 - \dfrac{a}{a+b}$

18. $\dfrac{4a+b}{a} - \dfrac{b}{a}$

0.2 INTEGRAL POWERS

Powers (exponents) play a very important role in computations. Here we require only integral powers, but powers a^x can be defined for any real number x.

DEFINITION OF INTEGRAL POWERS

(1) If a is any real number, then

$$a^m = \underbrace{a \times a \times a \times \cdots \times a}_{m \text{ times}} \quad \text{for} \quad m = 1, 2, 3, \ldots$$

(2) $a^0 = 1 \quad \text{for} \quad a \neq 0$

(3) $a^{-m} = \dfrac{1}{a^m} \quad \text{for} \quad a \neq 0$

Example 1

(1) $4^3 = 4 \times 4 \times 4 = 64$

(2) $4^0 = 1$

(3) $4^{-2} = \dfrac{1}{4^2} = \dfrac{1}{16}$

Properties of powers

Let m and n be any integers. Then

(1) $a^m a^n = a^{m+n}$

(2) $\dfrac{a^m}{a^n} = a^{m-n}$

(3) $(a^m)^n = a^{mn}$

(4) $(ab)^m = a^m b^m$

(5) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

Example 2

(1) $4^5\left(\frac{1}{4}\right)^5 = \left(4 \times \frac{1}{4}\right)^5 = 1^5 = 1$

(2) $\left(\frac{2}{3}\right)^{-2}\left(\frac{3}{2}\right)^2 = \frac{1}{(\frac{2}{3})^2}\left(\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2\left(\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$

(3) $\frac{10^{-7} \times 10^{12}}{10^{-1}} = 10^{-7+12+1} = 10^6$

The following summation formula is often useful in computations:

$$1 + a + a^2 + a^3 + \cdots + a^n = \frac{1 - a^{n+1}}{1 - a} \quad \text{whenever} \quad a \neq 1 \quad (1)$$

This formula is verified by multiplying both sides of the equation by $1 - a$, multiplying out $(1 - a)(1 + a + a^2 + a^3 + \cdots + a^n)$ and canceling.

Example 3

(1) $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{64} = \frac{1 - \frac{1}{128}}{1 - \frac{1}{2}} = \frac{1 - \frac{1}{128}}{\frac{1}{2}}$

$$= 2\left(1 - \frac{1}{128}\right) = 2 - \frac{1}{64}$$

(2) $1 + 3 + 9 + 27 + 81 + 243 = \frac{1 - 729}{1 - 3} = \frac{-728}{-2} = 364$

EXERCISES

1. Insert one of the symbols $=$ or $\neq$ into each box to produce a true statement.

   (a) $(-1)^{-1} \square\ 1$

   (b) $\left(\frac{1}{a}\right)^{10} \square\ \frac{1}{a^{10}}$

   (c) $a^5a^{-5} \square\ 0$

   (d) $(-1)^n + (-1)^{n+1} \square\ 0$

   (e) $b^{-4}\left(\frac{a}{b}\right)^4 \square\ a^4b^{-8}$

(f) $a \cdot a^2 \cdot a^3 \square a^6$

(g) $(a^{-2}b)^2 \square a^4b^2$

(h) $[(-2)^{-1}]^{-1} \square 2$

Simplify the numbers on Exercises 2–20 as much as possible, but leave your answers expressed with positive powers.

2. $12\left(\frac{1}{2}\right)^3$

3. $\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)$

4. $\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^4$

5. $\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2$

6. $\left(\frac{5}{4}\right)^2\left(\frac{1}{5}\right)^3$

7. $\left(\frac{8}{9}\right)\left(\frac{1}{9}\right)^3$

8. $\left(\frac{9}{10}\right)^3\left(\frac{10}{12}\right)^{-1}$

9. $10^4 \times 10^{-7}$

10. $\dfrac{10^4 \times 10^{-6}}{10^{-4} \times 10^6}$

11. $(a^{-3})^{-3}$

12. $\left(\frac{a}{b}\Big/\frac{c}{d}\right)^2$

13. $\dfrac{10^{-3}}{10^{-5}}$

14. $\dfrac{8^5}{2^{12}}$

15. $3^3 \times 3^{-4} \times 3^5$

16. $\left(\frac{1}{2}\right)^{-6}$

17. $\left(\frac{1}{5}\right)^4\left(\frac{2}{5}\right)^4$

18. $(2^2 \times 16)^8$

19. $(a^3b)(ab^3)$

20. $a^{2(m-1)}(a^{m-1})^2$

Find the sums in Exercises 21–25.

21. $1 + \frac{1}{2} + \cdots + \frac{1}{2^{10}}$

22. $1 + \frac{1}{5} + \cdots + \frac{1}{5^8}$

23. $1 + \frac{1}{10} + \cdots + \frac{1}{10^6}$

24. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots - \frac{1}{2^9}$

Hint:
Use Formula (1) with $a = -\frac{1}{2}$

25. $1 + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots + \left(\frac{2}{3}\right)^{10}$

0.3 INEQUALITIES

A basic property of real numbers is their order. This is expressed with the familiar symbols $>$ (greater than), $=$ (equal to), and $<$ (less than), and we use the assumption that any two numbers a and b are related by exactly one of the relations

$$a < b \qquad a = b \quad \text{or} \quad a > b$$

The symbols $\geq$ and $\leq$ are used for the following abbreviations:

$a \geq b$ is the same as "$a > b$ or $a = b$"
$a \leq b$ is the same as "$a < b$ or $a = b$"

Example 1

The statement "$4 < 4$ *or* $4 = 4$" is true because $4 = 4$. Hence, we can write $4 \leq 4$.

The statement "$-1 < 0$ *or* $-1 = 0$" is true because $-1 < 0$. Hence, we can write $-1 \leq 0$.

Terminology

$a > b$ is read "a is greater than b."
$a \geq b$ is read "a is greater than or equal to b."
$a < b$ is read "a is less than b."
$a \leq b$ is read "a is less than or equal to b."

It should be noted that $a < b$ and $b > a$ are two different ways of saying the same thing. Similarly, $a \leq b$ and $b \geq a$ convey the same information.

Properties of inequalities

(1) If $a < b$ and $b < c$, then $a < c$.
(2) If $a < b$, then $a + c < b + c$.
(3) If $a < b$ and $c > 0$, then $ac < bc$.
(4) If $a < b$ and $c < 0$, then $ac > bc$.

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