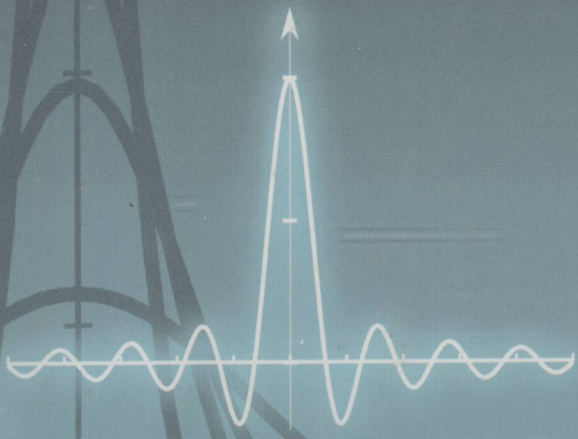
Lokenath Debnath
Dambaru Bhatta

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(\xi)\, e^{-ik\xi} d\xi\right] e^{ikx}\, dk$$

Integral Transforms and Their Applications

Second Edition



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Chapman & Hall/CRC
Taylor & Francis Group
Boca Raton London New York

Chapman & Hall/CRC is an imprint of the
Taylor & Francis Group, an informa business

Chapman & Hall/CRC
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

Printed in the United States of America on acid-free paper
10 9 8 7 6 5 4 3 2 1

International Standard Book Number-10: 1-58488-575-0 (Hardcover)
International Standard Book Number-13: 978-1-58488-575-7 (Hardcover)

Library of Congress Cataloging-in-Publication Data

Debnath, Lokenath.
Integral transforms and their applications. -- 2nd ed. / Lokenath Debnath and Dambaru Bhatta.
p. cm.
Includes bibliographical references and index.
ISBN 1-58488-575-0 (acid-free paper)
1. Integral transforms. I. Bhatta, Dambaru. II. Title.

QA432.D36 2006
515'.723--dc22 2006045638

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Integral Transforms and Their Applications

Second Edition

To my wife **Sadhana** and granddaughter **Princess Maya**

Lokenath Debnath

To my wife **Bisruti** and sons **Rohit** and **Amit**

Dambaru Bhatta

Preface to the Second Edition

> "A teacher can never truly teach unless he is still learning himself. A lamp can never light another lamp unless it continues to burn its own flame. The teacher who has come to the end of his subject, who has no living traffic with his knowledge but merely repeats his lessons to his students, can only load their minds; he cannot quicken them."
>
> Rabindranath Tagore

When the first edition of this book was published in 1995 under the sole authorship of Lokenath Debnath, it was well received, and has been used as a senior undergraduate or graduate level text and research reference in the United States and abroad for the last ten years. We received many comments and suggestions from many students and faculty around the world. These comments and criticisms have been very helpful, beneficial, and encouraging. This second edition is the result of that input.

Another reason for adding this second edition to the literature is the fact that there have been major discoveries of several integral transforms including the Radon transform, the Gabor transform, the inverse scattering transform, and wavelet transforms in the twentieth century. It is becoming even more desirable for mathematicians, scientists and engineers to pursue study and research on these and related topics. So what has changed, and will continue to change, is the nature of the topics that are of interest in mathematics, science and engineering, the evolution of books such as this one is a history of these shifting concerns.

This new and revised edition preserves the basic content and style of the first edition. As with the previous edition, this book has been revised primarily as a comprehensive text for senior undergraduates or beginning graduate students and a research reference for professionals in mathematics, science, and engineering, and other applied sciences. The main goal of this book is on the development of the required analytical skills on the part of the reader, rather than the importance of more abstract formulation with full mathematical rigor. Indeed, our major emphasis is to provide an accessible working knowledge of the analytical methods with proofs required in pure and applied mathematics, physics, and engineering.

We have made many additions and changes in order to modernize the contents and to improve the clarity of the previous edition. We have also taken advantage of this new edition to update the bibliography and correct typographical errors, to include additional topics, examples of applications, exercises, comments, and observations, and in some cases, to entirely rewrite whole section. This edition contains a collection of over 600 challenging worked examples and exercises with answers and hints to selected exercises. There is plenty of material in the book for a year-long course. Some of the material need not be covered in a course work and can be left for the readers to study on their own in order to prepare them for further study and research. Some of the major changes, additions, and highlights in this edition and the most significant difference from the first edition include the following:

1. Chapter 1 on Integral Transforms has been completely revised and some new material on brief historical introduction was added to provide new information about the historical developments of the subject. These changes have been made to provide the reader to see the direction in which the subject has developed and find those contributed to its developments.

2. Chapter 2 on Fourier Transforms has been completely revised and new material added, including new sections on Fourier transforms of generalized functions, the Poisson summation formula, the Gibbs phenomenon, and the Heisenberg uncertainty principle. Many sections have been completely rewritten with new examples of applications.

3. Four entirely new chapters on Radon Transforms, and Wavelets and Wavelet Transforms, Fractional Calculus and its applications to ordinary and partial differential equations have been added to modernize the contents of the book. A new section on the transfer function and the impulse response function with examples of applications was included in Chapters 2 and 4.

4. The book offers a detailed and clear explanation of every concept and method that is introduced, accompanied by carefully selected worked examples, with special emphasis being given to those topics in which students experience difficulty.

5. A wide variety of modern examples of applications has been selected from areas of ordinary and partial differential equations, quantum mechanics, integral equations, fluid mechanics and elasticity, mathematical statistics, fractional ordinary and partial differential equations, and special functions.

6. The book is organized with sufficient flexibility to enable instructors to select chapters appropriate to courses of differing lengths, emphases, and levels of difficulty.

7. A wide spectrum of exercises has been carefully chosen and included at the end of each chapter so the reader may further develop both analytical skills in the theory and applications of transform methods and a deeper insight into the subject.

8. Answers and hints to selected exercises are provided at the end of the book to provide additional help to students. All figures have been redrawn and many new figures have been added for a clear understanding of physical explanations.

9. All appendices, tables of integral transforms, and the bibliography have been completely revised and updated. Many new research papers and standard books have been added to the bibliography to stimulate new interest in future study and research. Index of the book has also been completely revised in order to include a wide variety of topics.

10. The book provides information that puts the reader at the forefront of current research.

With the improvements and many challenging worked problems and exercises, we hope this edition will continue to be a useful textbook for students as well as a research reference for professionals in mathematics, science and engineering.

It is our pleasure to express our grateful thanks to many friends, colleagues, and students around the world who offered their suggestions and help at various stages of the preparation of the book. We express our sincere thanks to Veronica Martinez and Maria Lisa Cisneros for typing the final manuscript with constant changes. In spite of the best efforts of everyone involved, some typographical errors doubtless remain. Finally, we wish to express our special thanks to Bob Stern, Executive Editor, and the staff of CRC/Chapman Hall for their help and cooperation.

Lokenath Debnath
Dambaru Bhatta
The University of Texas-Pan American

Preface to the First Edition

Historically, the concept of an integral transform originated from the celebrated Fourier integral formula. The importance of integral transforms is that they provide powerful operational methods for solving initial value problems and initial-boundary value problems for linear differential and integral equations. In fact, one of the main impulses for the development of the operational calculus of integral transforms was the study of differential and integral equations arising in applied mathematics, mathematical physics, and engineering science; it was in this setting that integral transforms arose and achieved their early successes. With ever greater demand for mathematical methods to provide both theory and applications for science and engineering, the utility and interest of integral transforms seems more clearly established than ever. In spite of the fact that integral transforms have many mathematical and physical applications, their use is still predominant in advanced study and research. Keeping these features in mind, our main goal in this book is to provide a systematic exposition of the basic properties of various integral transforms and their applications to the solution of boundary and initial value problems in applied mathematics, mathematical physics, and engineering. In addition, the operational calculus of integral transforms is applied to integral equations, difference equations, fractional integrals and fractional derivatives, summation of infinite series, evaluation of definite integrals, and problems of probability and statistics.

There appear to be many books available for students studying integral transforms with applications. Some are excellent but too advanced for the beginner. Some are too elementary or have limited scope. Some are out of print. While teaching transform methods, operational mathematics, and/or mathematical physics with applications, the author has had difficulty choosing textbooks to accompany the lectures. This book, which was developed as a result of many years of experience teaching advanced undergraduates and first-year graduate students in mathematics, physics, and engineering, is an attempt to meet that need. It is based essentially on a set of mimeographed lecture notes developed for courses given by the author at the University of Central Florida, East Carolina University, and the University of Calcutta.

This book is designed as an introduction to theory and applications of integral transforms to problems in linear differential equations, and to boundary and initial value problems in partial differential equations. It is appropriate

for a one-semester course. There are two basic prerequisites for the course: a standard calculus sequence and ordinary differential equations. The book assumes only a limited knowledge of complex variables and contour integration, partial differential equations, and continuum mechanics. Many new examples of applications dealing with problems in applied mathematics, physics, chemistry, biology, and engineering are included. It is *not* essential for the reader to know everything about these topics, but limited knowledge of at least some of them would be useful. Besides, the book is intended to serve as a reference work for those seriously interested in advanced study and research in the subject, whether for its own sake or for its applications to other fields of applied mathematics, mathematical physics, and engineering.

The first chapter gives a brief historical introduction and the basic ideas of integral transforms. The second chapter deals with the theory and applications of Fourier transforms, and of Fourier cosine and sine transforms. Important examples of applications of interest in applied mathematics, physics statistics, and engineering are included. The theory and applications of Laplace transforms are discussed in Chapters 3 and 4 in considerable detail. The fifth chapter is concerned with the operational calculus of Hankel transforms with applications. Chapter 6 gives a detailed treatment of Mellin transforms and its various applications. Included are Mellin transforms of the Weyl fractional integral, Weyl fractional derivatives, and generalized Mellin transforms. Hilbert and Stieltjes transforms and their applications are discussed in Chapter 7.

Chapter 8 provides a short introduction to finite Fourier cosine and sine transforms and their basic operational properties. Applications of these transforms are also presented. The finite Laplace transform and its applications to boundary value problems are included in Chapter 9. Chapter 10 deals with a detailed theory and applications of Z transforms.

Chapter 12 is devoted to the operational calculus of Legendre transforms and their applications to boundary value problems in potential theory. Jacobi and Gegenbauer transforms and their applications are included in Chapter 13. Chapter 14 deals with the theory and applications of Laguerre transforms. The final chapter is concerned with the Hermite transform and its basic operational properties including the Convolution Theorem. Most of the material of these chapters has been developed since the early sixties and appears here in book form for the first time.

The book includes two important appendices. The first one deals with several special functions and their basic properties. The second appendix includes *thirteen* short tables of integral transforms. Many standard texts and reference books and a set of selected classic and recent research papers are included in the Bibliography that will be very useful for the reader interested in learning more about the subject.

The book contains 750 worked examples, applications, and exercises which include some that have been chosen from many standard books as well as recent papers. It is hoped that they will serve as helpful self-tests for understanding of the theory and mastery of the transform methods. These exam-

ples of applications and exercises were chosen from the areas of differential and difference equations, electric circuits and networks, vibration and wave propagation, heat conduction in solids, quantum mechanics, fractional calculus and fractional differential equations, dynamical systems, signal processing, integral equations, physical chemistry, mathematical biology, probability and statistics, and solid and fluid mechanics. This varied number of examples and exercises should provide something of interest for everyone. The exercises truly complement the text and range from the elementary to the challenging. Answers and hints to many selected exercises are provided at the end of the book.

This is a *text* and a *reference* book designed for use by the student and the reader of mathematics, science, and engineering. A serious attempt has been made to present almost all the standard material, and some new material as well. Those interested in more advanced rigorous treatment of the topics covered may consult standard books and treatises by Churchill, Doetsch, Sneddon, Titchmarsh, and Widder listed in the Bibliography. Many ideas, results, theorems, methods, problems, and exercises presented in this book are either motivated by or borrowed from the works cited in the Bibliography. The author wishes to acknowledge his gratitude to the authors of these works.

This book is designed as a new source for both classical and modern topics dealing with integral transforms and their applications for the future development of this useful subject. Its main features are:

1. A systematic mathematical treatment of the theory and method of integral transforms that gives the reader a clear understanding of the subject and its varied applications.

2. A detailed and clear explanation of every concept and method that is introduced, accompanied by carefully selected worked examples, with special emphasis being given to those topics in which students experience difficulty.

3. A wide variety of diverse examples of applications carefully selected from areas of applied mathematics, mathematical physics, and engineering science to provide motivation, and to illustrate how operational methods can be applied effectively to solve them.

4. A broad coverage of the essential standard material on integral transforms and their applications together with some new material that is *not* usually covered in familiar texts or reference books.

5. Most of the recent developments in the subject since the early sixties appear here in book form for the first time.

6. A wide spectrum of exercises has been carefully selected and included at the end of each chapter so that the reader may further develop both manipulative skills in the applications of integral transforms and a deeper insight into the subject.

7. Two appendices have been included in order to make the book self-contained.

8. Answers and hints to selected exercises are provided at the end of the book for additional help to students.

9. An updated Bibliography is included to stimulate new interest in future study and research.

In preparing the book, the author has been encouraged by and has benefited from the helpful comments and criticism of a number of graduate students and faculty of several universities in the United States, Canada, and India. The author expresses his grateful thanks to all these individuals for their interest in the book. My special thanks to Jackie Callahan and Ronee Trantham who typed the manuscript and cheerfully put up with constant changes and revisions. In spite of the best efforts of everyone involved, some typographical errors doubtlessly remain. I do hope that these are both few and obvious, and will cause minimal confusion. The author also wishes to thank his friends and colleagues including Drs. Sudipto Roy Choudhury and Carroll A. Webber for their interest and help during the preparation of the book. Finally, the author wishes to express his special thanks to Dr. Wayne Yuhasz, Executive Editor, and the staff of CRC Press for their help and cooperation. I am also deeply indebted to my wife, Sadhana, for all her understanding and tolerance while the book was being written.

Lokenath Debnath
University of Central Florida

About the Authors

Lokenath Debnath is Professor and Chair of the Department of Mathematics at the University of Texas-Pan American, Edinburg, Texas.

Professor Debnath received his M.Sc. and Ph.D. degrees in pure mathematics from the University of Calcutta, and obtained his D.I.C. and Ph.D. degrees in applied mathematics from the Imperial College of Science and Technology, London. He was a Senior Research Fellow at the Department of Applied Mathematics and Theoretical Physics at the University of Cambridge and has had several visiting appointments at the University of Oxford, Florida State University, University of Maryland, and the University of Calcutta. He served the University of Central Florida as Professor and Chair of Mathematics and as Professor of Mechanical and Aerospace Engineering from 1983 to 2001. He was Acting Chair of the Department of Statistics at the University of Central Florida and has served as Professor of Mathematics and Professor of Physics at East Carolina University for a period of fifteen years.

Among many other honors and awards, he has received a Senior Fulbright Fellowship and an NSF Scientist Award to visit India for lectures and research. He was a University Grants Commission Research Professor at the University of Calcutta and was elected President of the Calcutta Mathematical Society for a period of three years. He has served as a Lecturer of the SIAM Visiting Lecturer Program and as a Visiting Speaker of the Mathematical Association of America (MAA) from 1990. He also has served as organizer of several professional meetings and conferences at regional, national, and international levels; and as Director of six NSF-CBMS research conferences at the University of Central Florida, and East Carolina University and the University of Texas-Pan American. He has received many grants from NSF and state agencies of North Carolina, Florida, and Texas. He has also received many university awards for teaching, research, services and leadership.

Dr. Debnath is author or co-author of ten graduate level books and research monographs, including the third edition of *Introduction to Hilbert Spaces with Applications, Nonlinear Water Waves, Continuum Mechanics* published by Academic Press, the fourth edition of *Linear Partial Differential Equations for Scientists and Engineers* published by Birkhauser Verlag, the second edition of *Nonlinear Partial Differential Equations for Scientists and Engineers,* and *Wavelet Transforms and Their Applications* published by Birkhauser Verlag. He has also edited eleven research monographs including *Nonlinear Waves*

published by Cambridge University Press. He is an author or co-author of over 300 research papers in pure and applied mathematics, including applied partial differential equations, integral transforms and special functions, history of mathematics, mathematical inequalities, wavelet transforms, solid and fluid mechanics, linear and nonlinear waves, solitons, mathematical physics, magnetohydrodynamics, unsteady boundary layers, dynamics of oceans, and stability theory.

Professor Debnath is a member of many scientific organizations at both national and international levels. He has been an Associate Editor and a member of the editorial boards of many refereed journals and he currently serves on the Editorial Board of many refereed journals including *Journal of Mathematical Analysis and Applications, Indian Journal of Pure and Applied Mathematics, Fractional Calculus and Applied Analysis, Bulletin of the Calcutta Mathematical Society, Integral Transforms and Special Functions, International Journal of Engineering Science,* and *International Journal of Mathematical Education in Science and Technology.* He is the current and founding Managing Editor of the *International Journal of Mathematics and Mathematical Sciences.*

Dr. Debnath delivered twenty five invited lectures at national and international conferences, presented over 200 research papers at national and international professional meetings, and given over 250 seminar and colloquium lectures at universities and institutes in the United States and abroad.

Dambaru Bhatta is an Assistant Professor of Mathematics at the University of Texas-Pan American, Edinburg, Texas.

Dr. Bhatta received his Ph.D. degree in applied mathematics from Dalhousie University, Halifax, Canada and M.Sc. degree in mathematics from the University of Delhi. He had worked with various companies in Montreal, Ottawa, and Atlanta. His research interests include wave-structure interaction, computational mathematics, finite element method, nonlinear partial differential equations, fractional calculus, and fractional differential equations.

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