GAME THEORY AND ECONOMIC ANALYSIS

A QUIET REVOLUTION IN ECONOMICS

EDITED BY CHRISTIAN SCHMIDT

ROUTLEDGE ADVANCES IN GAME THEORY

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London and New York

First published in French in 1995 as
Théorie des jeux et analyse économique 50 ans après (special issue of *Revue d'Economie Politique*, 1995, no. 4, pp. 529–733)
by Éditions Dalloz (Paris)

This edition published 2002
by Routledge
11 New Fetter Lane, London EC4P 4EE

Simultaneously published in the USA and Canada
by Routledge
29 West 35th Street, New York, NY 10001

Routledge is an imprint of the Taylor & Francis Group

Typeset in Times by RefineCatch Limited, Bungay, Suffolk
Printed and bound in Great Britain by
Antony Rowe Ltd, Chippenham, Wiltshire

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloging in Publication Data
Game theory and economic analysis / [edited by] Christian Schmidt.
p. cm. – (Routledge advances in game theory; 001)
Includes bibliographical references and index.
1. Game theory. 2. Economics. I. Schmidt, Christian. II. Series.

HB144.G3727 2002
330′.01′5193 – dc21 2001056890

ISBN 0–415–25987–8

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Introduction

Christian Schmidt

Game theory has already observed the passage of its fiftieth birthday; that is, if one accepts the conventional chronology which places its birth at the publication of *Theory of Games and Economic Behavior* (*TGEB*) by Von Neumann and Morgenstern (1944). This anniversary evidently did not escape the notice of the Academy of Stockholm, which in 1994 awarded the Nobel Prize in Economic Sciences to three game theorists, Nash, Harsanyi, and Selten. A look back at its brief history brings out several troubling similarities with economic science, in places where one might not expect to find them.

Game theory was invented in order to satisfy a mathematical curiosity. The difficulty at the outset was to find a theoretical solution to the problems posed by uncertainty in games of chance. The example of checkers interested Zermelo (1913), and then the first complete mathematical formulation of strategies for games "in which chance (hasard) and the ability of the players plays a role" was sketched out by Borel (1924), who was himself co-author of a treatise on bridge. Nothing about this singular and rather marginal branch of mathematics would at this time have suggested its later encounter with economics.[1] The analogy between economic activity and what goes on in casinos was only suggested much later, in a far different economic environment than that which these two mathematicians would have been able to observe.

One could say that J. Von Neumann was the person who both conferred a sense of scientific legitimacy upon this mathematical construction, and whose work would lead to the connection with economic analysis.[2] The principal stages were as follows:

- *1928*: Von Neumann demonstrates his *minimax theory*. This demonstration occurs within the framework of a category of two-person zero-sum games in which, to use Borel's terminology, chance (hasard) plays no part, at least no explicit part, and in which the results depend solely upon the reason of the players, not upon their ability. "Strategic games" lend themselves naturally to an economic interpretation (Von Neumann 1928)
- *1937*: Pursuing his topological work on the application of the fixed-point theorem, Von Neumann discovers the existence of a connection between

the *minimax* problem in game theory and the saddle point problem as an equilibrium in economic theory (Von Neumann 1937)

- *1940*: Von Neumann chooses the economist O. Morgenstern to assist him in the composition of what would become the first treatise of game theory. The title of their work is explicit: the theoretical understanding of games is presented as relevant to the analysis of economic behavior.

However seductive it may seem, this saga is nonetheless deceptive. To look a little closer, the bonds that connect Von Neumann's mathematical thought to economic theory are more fragile, and partially contingent. The applicability of strategic games, in the sense of the 1928 article, is obviously not limited to the domain of economics. The connection between the *minimax* theorem and the saddle point is the result of a property of convexity, independent of any economic interpretation of it that might be given. The reasons for Von Neumann's collaboration with Morgenstern go beyond the realm of science. Finally and above all, their work together did not in fact culminate in the announced fusion of game mathematics and the analysis of economic situations. Two-thirds of *Theory of Games and Economic Behavior* are devoted to zero-sum games, and non-zero-sum games are handled with recourse to the device of the "fictitious player." As for Böhm-Bawerk's famous example of the horse market, it represents a particular economic situation that offers only a fragile support for the theoretical result it illustrates. One need only change the numerical givens in the auction market bearing on substitutable but indivisible goods (the horses), and one can demonstrate that the "core" of the allocations is empty (cf. Moulin, this volume: Chapter 4).

Contemporaries were not fooled. As evidenced by the long articles that followed the publication of this massive work, economists did not respond to Von Neumann's and Morgenstern's hopes (cf. Dimand and Dimand, this volume: Chapter 1). Indeed, over the course of twenty years, game theory would remain above all, with only a few exceptions, either an object of study for a small group of mathematicians, or a research tool for military strategists. The first category, working with Kuhn and Tucker at Princeton, would refine, deepen, and generalize the formal properties of the theory left behind by Von Neumann. The second category, which benefited from substantial military funding, worked – particularly in connection with the Rand Corporation – to apply these concepts to new strategic realities by linking them to operational research. A last group of applied mathematicians working around the University of Michigan tried to create a bridge between the statistical approach of decision-making theory and the new theory of games through experimental tests. Among them, emerged the names of Thomson and Raiffa.

But the most suggestive aspect of this history is probably the behavior of Von Neumann himself. Working with the Manhattan project, and having left Princeton, he looked skeptically upon applications of game theory to economics. Shortly before his premature death in 1957, he formulated a critical judgment which went beyond a simple statement of facts. According to him, there were more than just empirical difficulties standing in the way of the development of such applications. The application of game theory to economics posed a more fundamental problem due to the distance separating several major concepts articulated in *Theory of Games and Economic Behavior* (rules of the game, game solution, coalition, etc.) from the categories constructed by economic analysis.[3] Whatever the case, the small group of economists who persisted in working on games found themselves faced with serious difficulties. In particular, they had to free themselves from the hypothesis of the transferability of utilities: they had to introduce a dynamic into what had been an essentially static treatment of the interactions between the players, and they had to abandon the unrealistic framework of complete information.

A third point of view on the relations between game theory and economic theory would modify matters further. The publication of Nash's profoundly innovative articles in the early 1950s quickly refreshed the thinking of those few economists who had been seduced by game theory, and thereafter they directed their energies towards retrospective reconstructions. Shubik rediscovered in Cournot's work the premises of Nash's concept of equilibrium (Shubik 1955). Harsanyi compared Nash's model of negotiation with economic analyses beginning with Zeuthen and continuing with Hicks (Harsanyi 1956). Similarities came to light between the problematic of competition laid out by Edgeworth and the laws of the market (Shubik 1959). The way was now open for further comparisons. The question could be asked, for instance, whether Shapley's solution did not simply develop, in axiomatic form, several of the ideas suggested by Edgeworth in his youthful utilitarian phase.[4] Those works are to be considered as a starting point for a kind of archaeology. In the train of these discoveries, a hypothesis took shape. An economic game theory perhaps preceded the mathematical theory elaborated by Von Neumann (Schmidt 1990). It is surely not by chance that several of the problems posed by the application of game theory to economics were resolved in the 1960s by the very scholars who had been the most active in researching the economic roots of game theory. One thinks particularly of the work of Shubik, Harsanyi, Shapley, and Aumann.

In the light of these new developments, the role of the Hungarian mathematical genius in this affair appears more complex. While he remains the undeniable intermediary between the mathematics of games and economics, it is necessary also to recognize that he has contributed, through the orientation he gave to his theory (zero-sum games with two players, extension to *n* players and, only finally, to non-zero-sum games through several fictions), to eclipsing the old strategic approach to economic problems, a tradition illustrated by often isolated economists going back to the nineteenth century. It is true that the tradition always remained hopelessly foreign to his economist collaborator Morgenstern, who was educated in a quite different economic discipline, namely the Austrian school.

At the end of the 1970s, the connections between game theory and economics entered a new phase. The game theory approach had progressively invaded the majority of sectors of economic analysis. Such was the case first of all with industrial economy, which was renewed by the contribution of games. Insurance economics, then monetary economics and financial economics and a part of international economics, all, one by one, were marked by this development. The economy of well-being and of justice have been affected, and more recently the economics of law. It would be difficult today to imagine a course in micro-economics that did not refer to game theory. And at the same time, proportionally fewer and fewer pure mathematicians have been working on game theory; which obviously does not mean that all the mathematical resources applicable to game theory have already been exploited.[5]

The results of the pioneering work of the few economists invoked above have begun to bear fruit. Other, deeper, factors explain this double metamorphosis, of which only one will be mentioned here. In the course of its development, game theory has revealed characteristics that are opposite to those it was initially considered to possess. Far from representing a strait-jacket whose application to the analysis of real phenomena imposed a recourse to extremely restrictive hypotheses, it has shown itself, quite to the contrary, to be a rigorous but sufficiently supple language, able to adapt itself to the particular constraints of the situations being studied. In exchange for this flexibility, game theory seems to have lost its original unity. The diversity of game solution concepts and the plurality of equilibria-definitions susceptible to being associated to a single category of games provide the most significant illustrations of this, to say nothing of the ever-increasing number of game types that enrich the theory. The question today is whether the name "game theory" should remain in the singular, or become "game theories" in the plural. This tendency towards fragmentation represents a handicap in the eyes of the mathematician. But for the economist it offers an advantage, to the degree that it brings game theory closer to the economist's more familiar environment: for the plurality of situations and the diversity of perspectives are both the daily bread of economists.

This particular evolution of game theory contradicts the prophesy of its principal founder. The relations between game theory and economic science is in the process of reversing itself. Economics is today no longer the domain of application for a mathematical theory. It has become the engine of development for a branch of knowledge. Indeed, a growing amount of cutting-edge research in game theory is the work of economists or of mathematicians who have converted to economics. The result has been to place the discipline of economics in an extremely unfamiliar position, and to give a reorientation to its developments (renaissance of micro-economics, expansion of experimental economics, new insights in evolutionary economics, first steps in cognitive economics). The first three chapters of the history have been laid out, but it is not over, and no doubt still holds surprises in store.

The ambition for this special edition is to present an image of the many facets characterizing the variety of current contributions of game theory to economics. The contents reflect several major evolutions observed in this domain.

In the middle of the 1980s, the majority of contributions would have dealt with non-cooperative games. What was called "Nash's research program" (Binmore and Dasgupta 1986, 1987; Binmore 1996) dominated the field. The pendulum has now swung back in the other direction and there is a growing interest in cooperative games. The abstract distinction between these two game categories is now clarified. This does not prevent it from seeming unsatisfying, both from the point of view of the classification of the realms of study of theory, as well as from that of their appropriateness to the economic phenomena being studied (Schmidt 2001). It has long been recognized that the analysis of negotiation could adopt one or other point of view. Industrial economics, on the other hand, had up to the present privileged non-cooperative games; but now it makes reference to cooperative games in order to provide a theoretical substratum to the study of coalitions. In the opposite sense, public economics took up the question of the allocation of resources in terms of cooperative games; now, it has begun to discover the fecundity of non-cooperative games, when it extends that line of inquiry through the analysis of the mechanisms of incentive that allow it to be put into practice (cf. the "theory of implementation"). The complementary nature of these developments must not make us forget the existence of a no-bridge between these two approaches. The current efforts of several theoreticians consists in attempting to join them, through various rather unorthodox means (Roth's semistable partitions, Greenberg's theory of social situations, etc.: cf. Cochinard, this volume: Chapter 5).

The subjects of game theory are the players, and not a supposedly omniscient modeler. Only recently have all the consequences of this seemingly banal observation come to light. How ought one to treat the information possessed by the players before and during the game, and how ought one to represent the knowledge they use to interpret it? This question leads to an enlargement of the disciplines involved. The initial dialogue between mathematics and economics which accompanied the first formulation of the theory is coupled with a taking into consideration of the cognitive dimension, which necessarily involves theories of information, logic, and a part of psychology. Thus the definition of a player cannot be reduced to the identification of a group of strategies, as once thought, but requires the construction of a system of information which is associated with him. Thus game theory requires a deeper investigation of the field of epistemic logic (Aumann 1999). If this layer of semantics in game theory enlarges its perspectives, it also holds in store various logical surprises about the foundations of the knowledge it transmits.

As for the new openness towards experimental psychology, it enriches its domain while complicating the game theoretician's methodological task. Making judgments turns out to be delicate when the experimental results contradict the logical results of the theory, as is the case, for example, with the centipede game.[6] The heart of the difficulty lies in reconciling two different conceptions of the use of game theory. Either one sees it as a storehouse of models for clarifying the economic phenomena one wishes to explain, or one considers it a support for experimentation on interactive behavior in situations close to those studied by economists (cf. Rullière and Walliser, this volume: Chapter 7).

The origin of this volume was a special issue of the *Revue d'Economie Politique* devoted to game theory and published in 1995. From this basis, several papers have been revised and enlarged, some dropped and others added. The chapters that make up this collection fall into two categories. Some lay out in a non-technical way the panorama of a particular branch of the theory, of the evolution of one of its concepts, or of a problem that runs through its development. Others are original contributions bearing on a domain of specific research that, nonetheless, is significant for the field as a whole. All attempt to show how the present situation derives directly or by default from the work of Von Neumann and Morgenstern. The order of arrangement follows the historical chronology of the problem, and its degree of generality in game theory. The contributions are distributed in three parts respectively devoted to historical insight, theoretical content, and applications.

The chapter by R. W. Dimand and M. A. Dimand traces the prehistory, the history, and what one might call the "posthistory" of *TGEB*. In particular, they draw on Léonard's research in shedding light on the role played by Morgenstern. Their presentation leads one to the conviction that, even if the intellectual quality of *TGEB* was assessed favorably, the majority of economists immediately after the war, even in the USA, remained impervious to its message for economic science.

C. Schmidt raises the question of the continuity of game theory between *TGEB* and Nash's contributions during the 1950s. He first captures the aim of the research program contained in *TGEB* and then tries to reconstruct a complete Nash program from his few available papers. Their confrontation shows that Nash, starting from a generalization of Von Neumann's main theorems (1950), quickly developed a quite different framework for studying non-cooperative games, which culminated in his bargaining approach to cooperation (1953). According to this view, Nash obviously appears as a turning point in the recent history of game theory. However, this investigation also reveals an actual gap between the respective programs of Von Neumann and Morgenstern, on one side, and Nash on the other side. Such a gap opens up a domain that remains hardly explored by game theorists until today.

S. Sorin looks at players' strategic use of information. His first concern is to isolate the historic origins of the question which, via Von Neumann and Morgenstern, may be traced back to Borel and Possel. He shows how mixed strategies were conceived of at this period as a strategic use of chance (hasard). He then studies the incidence of the revelation of the players' strategies (both true and false) regarding the unfolding of the game, starting with the example of poker, which, abundantly treated in *TGEB*, sheds light on the possibilities for manipulating information in a bluff. Finally he extends his field of inquiry to contemporary research on the analysis of signals, of credibility, and of reputation, showing that all these are extensions of the strategic recourse to uncertainty.

H. Moulin offers a state of the question on cooperative games and at the same time develops a personal thesis on its role and its place in the literature of games. Considered as a sort of "second best" by Von Neumann and Morgenstern, cooperative games flourished in the 1960s, with the studies on the heart of an economy, before becoming once again the poor relation of the family. Moulin rejects the interpretation that would see cooperative games as a second-rate domain of research. He maintains, on the contrary, that the models of cooperative games lead back to a different conception of rationality whose origin lies in a grand tradition of liberal political philosophy. After having reviewed the problems posed by the application of the concept of the core to the analysis of economic and social phenomena (economies whose core is empty, economies whose core contains a very high number of optimal allocations), he emphasizes the recent renewal of the normative treatment of cooperative games through the comparison and elaboration of axiomatics that are able to illuminate social choices by integrating, in an analytic manner, equity in the allocation of resources and in the distribution of goods.

In an extension of Moulin's text, S. Cochinard takes on the question of the organization and functioning of coalitions. He especially underlines the fact that coalitions present the theoretician with two distinct but linked questions: how is a coalition formed (external aspect)? and how are its gains shared between the members of the coalition (internal aspect)? The examination of the relation between these two problems orients this chapter. He states first of all that this distinction does not exist in the traditional approach to this question via cooperative games (Von Neumann and Morgenstern's solution, Shapley's solution, Aumann and Maschler's solution, etc.). He reviews the different formulae proposed, and shows that none of them responds to the first problem, which requires an endogenous analysis of the formation of coalitions. Next he explores several approaches to the endogenization of coalitions in a game in following the notion of coalition structure due to Aumann and Drèze (1974). Two conclusions emerge from this study: the very meaning of a coalition varies so widely from one model to the next that there results a great variety of responses to the proposed question; and a convergence is traced out in the results obtained between the approach to the problem via cooperative games and the approach via non-cooperative games. Such an observation suggests another look at the borderline between these two components of game theory.

C. Schmidt considers the connections that persist between the mathematical game theory conceived by Von Neumann and the vast domain assigned to him by researchers today. To illustrate his topic, he analyzes the incidence of the information a player holds regarding the other players in the definition of rational strategy. He shows first how this question led Von Neumann to formulate two hardly compatible propositions. On the one hand, each player chooses his strategy in complete ignorance of the strategies chosen by the other players; on the other hand the strict determination of the values of the game requires that players' expectations of the others are quite perfect (Von Neumann 1928, 1969), thanks to auxiliary construction, Von Neumann and Morgenstern succeed in making them consistent in *TGEB*. Thus he explains how the suggestions formulated by Von Neumann and Morgenstern came to be at the origin of such heterodox projects as Howard's theory of metagames and Schelling's idea of focal points. Finally, he examines the extensions that might be given them. Metagames lead to a more general analysis of each player's subjective representations of the game, and focal points lead to an innovative approach to the coordination of players' expectations.

The chapter by J.-L. Rullière and B. Walliser bears on the apprehension of the problem of the coordination of strategic choices between independent players. The two authors maintain that game theory has evolved on this question. It started from a strictly hypothetical-deductive approach that supposed in each player the faculty to mentally simulate the reactions of others, while today game theory insists on the players' handling of received information in the course of the development of the game, and on the effects of apprenticeship it can engender. This way of proceeding succeeds in integrating temporality into the process, but raises other difficulties. The authors emphasize in conclusion the epistemological consequences of this transformation of game theory, which caused it to lose its status as a speculative theory and to draw closer to the sciences of observation.

With the chapter by C. d'Aspremont and L.-A. Gérard-Varet, one encounters original research on more particular points of game theory. The two authors examine a few possible developments of non-cooperative games leading to an illumination of incentive mechanisms that satisfy a criterion of collective efficiency. They introduce a general incomplete information model characterized by a Bayesian game. This model permits a mediator who knows the players' utility configuration, the structure of their beliefs, and a result function, to identify the balanced transfers that satisfy a paretian criterion of collective efficiency. Next they analyze the problem of each player's revelation of his private information, which permits them to reduce equilibrium constraints to incentive constraints. In comparing the conclusions yielded by their model with the results obtained by other methods, they are able to specify the domains in which their research may be applied (public oversight, relation between producers and consumers of public goods, judgment procedures, and insurance contracts). While they confirm that collectively efficient incentive mechanisms exist when the phenomena of moral hasard and of anti-selection manifest themselves, the meeting of individual incentives and of collective efficiency is far from being always guaranteed, on account of the different nature of the content of their information.

J.-P. Ponssard, S. Steinmetz, and H. Tanguy's contribution is devoted to an analysis of strategic problems raised by coordination inside firms. The question is investigated through pure coordination team games, where the players have exactly the same payoff functions. Such a general framing is successively applied to two different situations. The firm is supposed to be completely integrated in the first case and decentralized in the second case. The main interest of the exercise is to associate the definition of a precise policy profile to each Nash equilibrium identified, which gives rise to relevant interpretations according to the structural hypotheses chosen. This theoretical approach is supplemented by the interpretation of some experimental results. Finally, the chapter shows a direction where game theory can provide fruitful insights on problems as crucial as the dual coordination decentralization for firms' management.

Notes

1 Borel, however, pointed out the economic application of his tentative theory of games from the very beginning (Borel 1921) and even sketched out a model of price adjustment in a later publication (Borel 1938).

2 This interpretation of Von Neumann's role as an interface between mathematical research and economic theory is buttressed and developed in Dore (1989).

3 See in particular J. Von Neumann, "The impact of recent developments in science on the economy and on economics," (1955) (Von Neumann 1963: Vol. 6). This original diagnostic by Von Neumann was interpreted by Mirowski as the culmination of a process of realizing the unsuitability of the minimax theory to the economic preoccupations manifested in *TGEB* (Mirowski 1992). We prefer to think that this position, which Von Neumann took for the most part before the work on *TGEB*, was based on the obstacles encountered in the application of the method adopted in *TGEB* for the analysis of economic interactions.

4 Provided the value of Shapley is interpreted as the result of putting into play normative principles guiding an equitable allocation, and provided one does not limit Edgeworth's utilitarian work to *Mathematical Psychics* (1881) but goes back to his earlier works.

5 The possibilities offered by "calculability" in the form of Turing machines only began to be explored in a systematic manner by extending the suggestions of Binmore (1987). On finite automata equilibria see Piccione (1992) and Piccione and Rubinstein (1993).

6 Here it is a question of non-cooperative two-player games which unfold according to finite sequences known in advance by the players. The players alternate turns. With each sequence, the total payments are augmented by a coefficient k but their sharing-out between the two players is reversed, so that the possible gain for each player is always less than for the turn immediately following his choice. The logical solution suggested by backward induction would have the first player stop at the first move. But experimental results show, on the contrary, that hardly any player stops at the first move and that very few follow the game to its end (MacKelvey and Palfrey 1992). Indeed, Aumann has demonstrated that when rationality is common knowledge among the players and the game of perfect information, players' rationality logically implies backward induction (Aumann 1995). And so what? The lesson to be drawn from these counterfactuals results remains far from clear (Schmidt 2001).

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