

FRACTURE MECHANICS OF ELECTROMAGNETIC MATERIALS

Nonlinear Field Theory and Applications

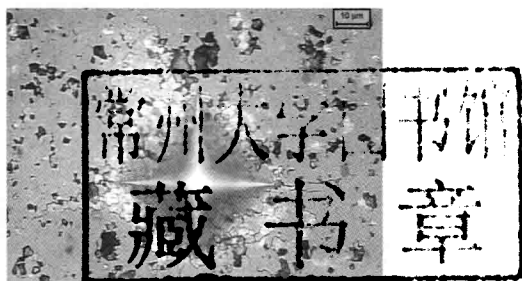


XIAOHONG CHEN ■ YIU-WING MAI

Imperial College Press

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Nonlinear Field Theory
and Applications



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To my parents for their love, understanding, and support.

Xiaohong Chen

To my mother, Yuet-Yau Tsui, and my wife, Louisa Kit-Ling Mai, whose love and unfailing support have enabled me to pursue my academic dreams for many years!

Yiu-Wing Mai

Foreword

I take pleasure in writing this foreword for the excellent new book entitled *Fracture Mechanics of Electromagnetic Materials: Nonlinear Field Theory and Applications*, written by my former colleagues, Drs. Xiaohong Chen and Yiu-Wing Mai, at the Hong Kong University of Science and Technology.

Novel multifunctional materials have tremendous potential for high-performance structural and functional applications in aeronautical, mechanical, and civil engineering, as well as in microelectronic and biomedical devices, due to their versatile actuating, sensing, healing, and other functional properties. The susceptibility of such advanced materials to cracking in service is of fundamental concern and has become a very popular area of research. Attempts to describe the failure behavior of these advanced materials and structures have clearly shown that linear piezoelectric/piezomagnetic fracture mechanics does not adequately explain the crack growth behavior under combined magnetic, electric, thermal, and mechanical loadings. It appears that coupling and dissipative effects play an important role in the growth and propagation of cracks.

Significant discrepancies still exist between theoretical predictions and experimental observations. Both new and modified theories have been proposed to overcome the discrepancies, with only limited success. These failures may be because there is no comprehensive guide to the theoretical basis and application of nonlinear dynamic fracture mechanics, especially in cases involving multiple coupled fields with dissipation effects.

This book is the first monograph on the subject of nonlinear dynamic field theory for piezoelectric/piezomagnetic materials. It provides an overview of the current state of the art of fracture mechanics and some of the authors' recent research outcomes. In developing the theoretical models for application to fracture characterization of materials and structures in the presence of magneto-electro-thermo-mechanical coupling and dissipative effects, the authors emphasize the physical interpretation of the fundamental concepts of fracture mechanics. One of the book's unique contributions is the development of a nonlinear fracture mechanics theory which rigorously treats the dynamic crack problems involving coupled magnetic, electric, thermal, and mechanical fields. By unifying the coupling of these fields, this book fills a gap in the literature of fracture mechanics involving multifield interactions. This book is a valuable resource which sheds light on the still-developing multidisciplinary subject of multifield fracture mechanics.

The book has an extensive list of references reflecting the most recent developments. It can be used as a textbook for graduate students as well as a reference for researchers and engineers studying and/or applying the concepts of advanced fracture mechanics to design and practical applications in the presence of multifield coupling and dissipative effects.

Pin Tong
San Diego
April, 2012

Preface

This book covers the current status of conventional fracture mechanics methodologies and presents a new formulation of a nonlinear field theory of fracture mechanics for electromagnetic materials. The study of classic fracture mechanics problems is concerned with the mechanical state of a deformable body containing a crack or cracks. Nevertheless, recent advances in multifunctional smart materials have created new research frontiers due to the occurrence of magneto-electro-thermo-mechanical coupling and dissipative effects accompanying crack propagation.

Electromagnetic materials have broad civilian and defense applications such as infrastructure health monitoring, microelectronic packaging, novel antenna designs, and biomedical devices owing to their remarkable multifunctional properties. Fracture of these smart material systems has become the subject of active research because of their susceptibility to cracking in service. A major challenge is how to resolve the fundamental discrepancy between theoretical predictions and experimental observations on the fracture behaviors of piezoelectric and piezomagnetic materials.

A highly important question in the development of a fracture mechanics theory for electromagnetic materials is whether there is any particular thermodynamic quantity of a cracked body that can be interpreted as the “driving force” for crack propagation under combined magneto-electro-thermo-mechanical loadings. The answer to this question has been pursued for decades, but no satisfactory agreement has yet been reached. Thus, the establishment of a physically sound fracture criterion becomes the hallmark of an advanced fracture mechanics treatment for electromagnetic materials.

As the first monograph on the subject of nonlinear field theory of fracture mechanics for deformable electromagnetic materials, this book offers postgraduate students, academic researchers, and engineering specialists who are active in this challenging multidisciplinary area a sketch of the history, an overview of the current status, and a description of some most recent research outcomes based on our own work. It gives first priority to physical interpretation of fundamental concepts, development of theoretical models, and exploration of their applications to fracture characterization in the presence of magneto-electro-thermo-mechanical coupling and dissipative effects. A general formulation of a nonlinear field theory of fracture mechanics and a rigorous treatment of dynamic crack problems involving coupled magnetic, electric, thermal, and mechanical fields fill a gap in the literature.

We would like to express our sincere appreciation and gratitude to those who have provided helpful discussions and support to this book project, especially Professors Pin Tong and Tongyi Zhang (Hong Kong University of Science and Technology), Cun-Fa Gao (Nanjing University of Aeronautics & Astronautics), Baolin Wang (Harbin Institute of Technology), Qinghua Qin (Australian National University), and Meng Lu (CSIRO). XHC is also heartily indebted to the late Professor Ren Wang for his guidance and inspiration during her graduate studies in solid mechanics at Peking University and for his care and encouragement throughout all these years. The Centre for Advanced Materials Technology at the University of Sydney, where XHC previously worked and YWM was Founding Director, has provided an intellectually stimulating environment for advanced fracture mechanics research. Special thanks are due to Lance Sucharov, Tasha D'Cruz, Rajesh Babu, Lindsay Robert Wilson, Gregory Lee, and Romén Reyes-Peschl from Imperial College Press and World Scientific Publishing for their commitments to excellence in publishing this book from proposal review to proofreading. Permissions from professional societies and publishers to use cited materials in the book are also gratefully acknowledged.

Xiaohong Chen & Yiu-Wing Mai
San Diego & Sydney
August, 2012

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