

Fundamentals of
CONDENSED MATTER
AND CRYSTALLINE
PHYSICS



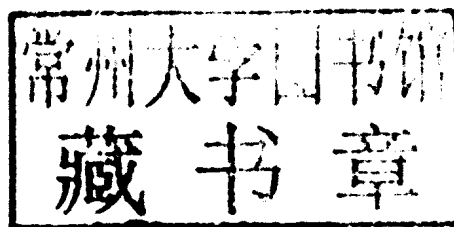
David L. Sidebottom

Fundamentals of Condensed Matter and Crystalline Physics

An Introduction for Students of
Physics and Materials Science

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Fundamentals of Condensed Matter and Crystalline Physics

This undergraduate textbook merges traditional solid state physics with contemporary condensed matter physics, providing an up-to-date introduction to the major concepts that form the foundations of condensed materials. The main foundational principles are emphasized, providing students with the knowledge beginners in the field should understand. The book is structured in four parts, and allows students to appreciate how the concepts in this broad area build upon each other to produce a cohesive whole as they work through the chapters. Illustrations work closely with the text to convey concepts and ideas visually, enhancing student understanding of difficult material, and end-of-chapter exercises, varying in difficulty, allow students to put into practice the theory they have covered in each chapter, and reinforce new concepts. Additional resources including solutions to exercises, lesson plans and pre-lecture reading quiz questions are available online at www.cambridge.org/sidebottom.

David L. Sidebottom is Associate Professor in the Physics Department at Creighton University. He is an experienced teacher and has taught a wide variety of courses at both undergraduate and graduate level in subject areas including introductory physics, thermodynamics, electrodynamics, laser physics and solid state physics. He has taught a course on solid state physics since 2003, adapting and revising its content to reflect the broader themes of condensed matter physics beyond those of the conventional solid state. This textbook stems from that course.

Preface

Purpose and motivation

This textbook was designed to accompany a one-semester, undergraduate course that itself is a hybridization of conventional solid state physics and “softer” condensed matter physics.

Why the hybridization? Conventional (crystalline) solid state physics has been pretty much understood since the 1960s at a time when non-crystalline physics was still a fledgling endeavour. Some 50 years later, many of the foundational themes in condensed matter (scaling, random walks, percolation) have now matured and I believe the time is ripe for both subjects to be taught as one. Moreover, for those of us teaching at smaller liberal arts institutions like my own, the merging of these two subjects into one, better accommodates a tight curriculum that is already heavily laden with required coursework outside the physics discipline.

Why the textbook? For some years now I have taught a one-semester course, originally listed as “solid state physics”, which evolved through each biannual reincarnation into a course that now incorporates many significant condensed matter themes, as well as the conventional solid state content. In past offerings of the course, a conventional solid state textbook was adopted (Kittel’s *Introduction to Solid State Physics*) and students were provided with handouts for the remaining material. This worked poorly. Invariably, the notation and style of the handouts clashed with that of the textbook and the disjointed presentation of the subject matter was not only annoying to students, but a source of unnecessary confusion. Students were left with the impression that solid state and condensed matter were two largely unrelated topics being crammed into a single course. Frustrated, I opted to spend a portion of a recent sabbatical assembling all of the material into a single document that would better convey the continuity of these two fields by threading both together into a seamless narrative.

So if you are looking for a reference-style textbook that provides a comprehensive coverage of the entire field of condensed matter, read no further because this is not it. This textbook was not written for practitioners, but rather for novices. It was designed to help students comprehend, not so much the details, but the major concepts that form the foundations of condensed matter and crystalline physics. At the very least I want students to leave the course able to comprehend the meaning behind terminology used by solid state physicists (e.g., “symmetry

operations”, “Brillouin zones”, “Fermi surfaces”) and condensed matter physicists (e.g., “mean field theory”, “percolation”, “scaling laws”, “structure factors”) so that they might rapidly acclimate to current research in either field.

Layout and use

I confess that my inspiration for the textbook style was Kittel’s *Introduction to Solid State Physics*, which has been a valuable guide for maintaining the development at a level appropriate for an undergraduate audience. Although criticized by some, his text is now in its eighth edition and has remained a popular choice for many undergraduate courses on solid state physics (including my own). Those familiar with Kittel, will find that this hybrid textbook incorporates most of the same subject matter (albeit abbreviated in places and arranged in a different order due to the way it is now interwoven with other non-crystalline topics) as is found in the first twelve chapters of Kittel.

Students will need a limited exposure to both quantum mechanics and statistical mechanics. The level of quantum mechanics that is provided in an introductory sophomore-level course on modern physics (1D wave mechanics, particle in a box, harmonic oscillators) should be sufficient. Beyond that, statistical mechanics and thermodynamics (specifically, Boltzmann statistics and free energies) are introduced periodically throughout the text and this is more likely to be the deficiency for some students. In an effort to help alleviate this and other potential deficiencies, an appendix is included which provides an introduction to such things as statistical mechanics, Fourier transforms and the use of Dirac delta functions.

The text is divided into four major parts: Structure, Scattering, Dynamics, and Transitions. Within each part are anywhere from four to six chapters designed more to delineate topics than to represent equal amounts of material. Although a common rule of thumb would be to allot three, 50-minute lecture periods per chapter, several chapters (e.g., 2, 3, 7, 10, 14, 15) could be adequately discussed in just two periods and Chapter 5 could likely be addressed in a single period. The lesson plan that I have adopted looks something like this:

	Lecture #1 (50 min)	Lecture #2 (50 min)	Lecture #3 (50 min)
Week	Chapter	Chapter	Chapter
1	1	1	1
2	2	2,3	3
3	4	4	5
4	6	6	6,7
5	7	7,8	8
6	8	9	9
7	10	10	11

	Lecture #1 (50 min)	Lecture #2 (50 min)	Lecture #3 (50 min)
Week	Chapter	Chapter	Chapter
8	11	11	12
9	12	12,13	13
10	13	13	13
11	14	14	15
12	15	16	16
13	16,17	17	17
14	18	18	18

Can all these topics be covered in a semester? Maybe. In my experience, I have so far only managed to cover about 85%. Topics to skip are really a matter of preference. I had no reservations about skipping the subject of bonds and cohesion (Chapter 3) and only modest discomfort at skipping the subject of bulk dynamics (Chapter 14). Others that are less interested in amorphous solids could skip glass structure (Chapter 2), but I would advise not to skip the material on scattering from self-similar objects (Chapter 8), as this contributes an important conceptual foundation for much of the materials in the last four chapters (Chapters 15–18) of the text.

Some might be tempted to skip the development of scattering theory presented in Chapter 5, so let me petition against this. In my experience, students struggle with the concept of reciprocal space primarily because of how most conventional solid state textbooks mysteriously introduce it directly after discussing Bragg's law. Students rarely grasp the significance of this abstract space and probably question why it is introduced at all, given how Bragg's law seems sufficient. By first introducing the fundamentals of scattering in Chapter 5, the reciprocal space appears more naturally as the discrete set of scattering wave vectors for which non-zero scattering occurs. Bragg's law is only presented as a consequence.

Anywhere from five to ten exercises can be found at the end of each chapter. These come in a variety of difficulty levels and are designed mostly to help students digest the material and develop skills. Many of the easier problems are derived from the text itself and ask students to complete the missing steps in a derivation. Although some may see this as aimless "busy work", for many undergraduate students (in my experience) these exercises represent a challenging skill yet to be mastered.

For students

Good luck and I hope this textbook helps you. Please let me know what you do and don't like about the textbook so that I can improve it in the future.

Acknowledgements

Let me start by thanking the many students that have taken my course in the past several years, and in particular the 2011 class (David, Jamison, Clifford, Nathan, Stan, Tri and Yuli), who braved an early prototype of the textbook and provided many valuable suggestions for revision and improvement. I thank also several close colleagues, Chris Sorensen, Jeppe Dyre and Per Jacobsson whose positive feedback on an early draft inspired me considerably and eventually prompted me to seriously consider publication. I am especially indebted to Chris who has been a mentor to me throughout the years and who helped immensely by giving an early draft of the textbook a thorough read.

Naturally, the support of Creighton University in the form of employment and a sabbatical leave is gratefully acknowledged. Grateful too am I for support from the faculty and staff in my department who have tolerated my erratic behavior during the past two years. I am especially grateful for advice given to me by Robert Kennedy about the publishing process.

And last, but certainly not least, I want to acknowledge the support and encouragement from my best friend and wife, Lane. Thank you, my love.

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Contents

<i>Preface</i>	<i>page</i> xiii
<i>Permission Disclosures</i>	xvii

Part I: Structure

1 Crystal structure	3
1.1 Crystal lattice	3
1.1.1 Basis set	5
1.1.2 Primitive cells	6
1.2 Symmetry	8
1.2.1 Conventional cells	10
1.3 Bravais lattices	10
1.3.1 Cubic lattices	12
1.3.2 Hexagonal lattices	16
Summary	17
Exercises	18
2 Amorphous structure	20
2.1 A statistical structure	20
2.1.1 Ensemble averaging	21
2.1.2 Symmetry	21
2.1.3 The pair distribution function	22
2.2 Two amorphous structures	26
2.2.1 Random close packed structure	26
2.2.2 Continuous random network	28
Summary	31
Exercises	32
3 Bonds and cohesion	35
3.1 Survey of bond types	35
3.1.1 The van der Waals bond	35
3.1.2 Ionic, covalent and metallic bonds	38
3.1.3 The hydrogen bond	41
3.2 Cohesive energy	41
3.2.1 Crystals	41
3.2.2 Amorphous materials	46

Summary	47
Exercises	47
4 Magnetic structure	50
4.1 The ordering process	50
4.1.1 Correlations and pattern formation	51
4.2 Magnetic materials	53
4.2.1 Magnetic moments	53
4.2.2 Diamagnetism	55
4.2.3 Paramagnetism	57
4.2.4 Ferromagnetism	60
Summary	64
Exercises	65
 Part II: Scattering	
5 Scattering theory	69
5.1 The dipole field	69
5.1.1 The scattering cross section	71
5.2 Interference	72
5.2.1 Scattering from a single atom	73
5.3 Static structure factor	76
5.3.1 A relevant scattering length scale	77
5.3.2 A Fourier relationship: the density–density correlation function	79
Summary	80
Exercises	80
6 Scattering by crystals	82
6.1 Scattering by a lattice	82
6.1.1 A set of allowed scattering wave vectors	84
6.2 Reciprocal lattice	85
6.3 Crystal planes	86
6.3.1 Miller indices	86
6.3.2 Bragg diffraction	88
6.3.3 Missing reflections	89
Summary	92
Exercises	92
7 Scattering by amorphous matter	95
7.1 The amorphous structure factor	95
7.1.1 Equivalence for liquids and glasses	97
7.1.2 Investigating short-range order	97
7.1.3 Rayleigh scattering	100

7.2	Light scattering by density fluctuations	101
7.2.1	The van Hove space correlation function	103
7.2.2	Intermediate-range order: SAXS and SANS	105
	Summary	107
	Exercises	107
8	Self-similar structures and liquid crystals	109
8.1	Polymers	109
8.1.1	The random walk	110
8.1.2	Swollen polymers: self-avoiding walks	114
8.2	Aggregates	115
8.2.1	Fractals	117
8.2.2	Example: soot formation	118
8.3	Liquid crystals	124
8.3.1	Thermotropic liquid crystals	125
8.3.2	Lyotropic liquid crystals: micelles and microemulsions	129
	Summary	132
	Exercises	132
Part III: Dynamics		
9	Liquid dynamics	139
9.1	Dynamic structure factor	139
9.1.1	The van Hove correlation function	141
9.1.2	Brownian motion: the random walk revisited	142
9.1.3	Hydrodynamic modes in liquids	145
9.2	Glass transition	150
9.2.1	Kauzmann paradox	151
9.2.2	Structural relaxation	153
9.3	Polymer liquids	154
9.3.1	Rouse model	155
9.3.2	Reptation	157
	Summary	160
	Exercises	160
10	Crystal vibrations	163
10.1	Monatomic basis	163
10.1.1	Dispersion relation	166
10.1.2	Brillouin zone	167
10.1.3	Boundary conditions and allowed modes	169
10.1.4	Phonons	170

10.2	Diatomic basis	173
10.2.1	Long wavelength limit	174
10.2.2	Waves near the Brillouin zone	176
10.2.3	Acoustical waves, optical waves and energy gaps	176
10.3	Scattering from phonons	177
10.3.1	Elastic (Bragg) scattering: The Debye–Waller factor	178
10.3.2	Inelastic scattering by single phonons	179
	Summary	180
	Exercises	181
11	Thermal properties	182
11.1	Specific heat of solids	182
11.1.1	Einstein model	184
11.1.2	Debye model	186
11.2	Thermal conductivity	189
11.2.1	Phonon collisions	191
11.3	Amorphous materials	193
11.3.1	Two-level systems	194
11.3.2	Phonon localization	197
	Summary	199
	Exercises	200
12	Electrons: the free electron model	201
12.1	Mobile electrons	201
12.1.1	The classical (Drude) model	202
12.2	Free electron model	204
12.2.1	Fermi level	206
12.2.2	Specific heat	207
12.2.3	Emission effects	210
12.2.4	Free electron model in three dimensions	210
12.2.5	Conduction in the free electron model	212
12.2.6	Hall effect	214
	Summary	216
	Exercises	216
13	Electrons: band theory	218
13.1	Nearly free electron model	218
13.1.1	Bloch functions	218
13.1.2	Bragg scattering and energy gaps	220
13.2	Kronig–Penney model	221
13.2.1	Energy bands and gaps	223
13.2.2	Mott transition	226

13.3 Band structure	226
13.4 Conductors, insulators, and semiconductors	230
13.4.1 Holes	232
13.4.2 Intrinsic semiconductors	233
13.4.3 Extrinsic semiconductors	235
13.5 Amorphous metals: the Anderson transition	241
Summary	243
Exercises	244
14 Bulk dynamics and response	246
14.1 Fields and deformations	246
14.1.1 Mechanical deformations	247
14.1.2 Electric and magnetic deformations	249
14.1.3 A generalized response	251
14.2 Time-dependent fields	252
14.2.1 Alternating fields and response functions	253
14.2.2 Energy dissipation	256
14.3 The fluctuation–dissipation theorem	257
Summary	261
Exercises	261

Part IV: Transitions

15 Introduction to phase transitions	267
15.1 Free energy considerations	267
15.2 Phase diagrams for fluids	269
15.2.1 PT diagram	269
15.2.2 PV diagram	270
15.2.3 TV diagram	272
15.2.4 Order parameter	272
15.3 Supercooling/heating and nucleation	274
15.4 Critical phenomena	276
15.4.1 A closer look: density fluctuations	278
15.5 Magnetic phase transitions	282
15.5.1 Exchange interaction	283
15.5.2 Magnetic phase diagrams	284
15.6 Universality: the law of corresponding states	285
Summary	287
Exercises	287

16 Percolation theory	289
16.1 The percolation scenario	289
16.1.1 Percolation threshold: the spanning cluster	292
16.1.2 A closer look: cluster statistics	293
16.2 Scaling relations	297
16.2.1 Finite-sized scaling	298
16.2.2 Renormalization	300
16.2.3 Universality and the mean field limit	303
16.3 Applications of percolation theory	305
16.3.1 Orchard blight and forest fires	305
16.3.2 Gelation	305
16.3.3 Fractal dynamics: anomalous diffusion	307
Summary	310
Exercises	310
17 Mean field theory and renormalization	313
17.1 Mean field theory	313
17.1.1 The mean field approximation	313
17.2 The mean field equation of state	316
17.2.1 Fluids: the van der Waals model	316
17.2.2 Magnets: the Ising model	318
17.3 Law of corresponding states	319
17.4 Critical exponents	321
17.4.1 Compressibility and susceptibility	323
17.4.2 Order parameter	323
17.5 Landau theory	324
17.6 Renormalization theory	326
17.6.1 A matter of perspective	327
17.6.2 Kadanoff spin renormalization	327
17.6.3 Scaling relations	330
Summary	332
Exercises	333
18 Superconductivity	334
18.1 Superconducting phenomena	334
18.1.1 Discovery	334
18.1.2 Meissner effect	335
18.1.3 Critical field	340
18.1.4 Specific heat	340
18.1.5 Energy gap	343
18.1.6 Isotope effect	344
18.2 Cooper pairs and the BCS theory	345
18.2.1 Cooper pairs	346
18.2.2 Flux quantization	348